FALSE
true

Ch 7: *Non-classical Propositional Logics*

1. System *RP*
   Exercise 7.1
2. Evaluating System *RP*
   Exercise 7.2
3. Modal Propositional Logics
   Exercise 7.3
4. Epistemic and Deontic Propositional Logics
   Exercise 7.4
5. Multi-valued Propositional Logics
   Exercise 7.5

A Worry about Relevance

>

Should we try to distinguish between the following types of argument?

- Bill is 21 years old

  Therefore, Bill is at least 16 years old

A Worry about Relevance

>

Should we try to distinguish between the following types of argument?

- Bill is 21 years old

  ———————————————

  Therefore, Bill is at least 16 years old

- Bill is 21 years old

  ———————————————

  Therefore, Sue is in Montreal

A Worry about Relevance

Should we try to distinguish between the following types of argument?

- Bill is 21 years old

  Therefore, Bill is at least 16 years old

- Bill is 21 years old

  Therefore, Sue is in Montreal

- Bill is 21 years old

  Therefore, Sue is in Montreal or it's not the case that is in Montreal

A Worry about Validity

Recall that in System *P*,

- even conditionals with irrelevant antecedents might be true

A Worry about Validity

Recall that in System *P*,

- even conditionals with irrelevant antecedents might be true
- even arguments with irrelevant premisses might be valid

A Worry about Validity

Recall that in System *P*,

- even conditionals with irrelevant antecedents might be true
- even arguments with irrelevant premisses might be valid

Material Conditional

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

A Worry about Validity

Recall that in System *P*,

- even conditionals with irrelevant antecedents might be true
- even arguments with irrelevant premisses might be valid

Material Conditional

p	*q*	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

Question:

Does "if *p* then *q*" mean "either we're not in a situation where *p* is true, or we are in a situation where *p* is true, but so is *q*"?

A Worry about Paradoxes

Recall the Paradox of the Liar:

Is the following proposition true or false?

This proposition is false

A Worry about Paradoxes

Recall the Paradox of the Liar:

Is the following proposition true or false?

This proposition is false

- If every proposition is either true or false then this proposition will be either true or false

A Worry about Paradoxes

Recall the Paradox of the Liar:

> Is the following proposition true or false?
>
> > This proposition is false

- If every proposition is either true or false then this proposition will be either true or false
- If it is true, then it is true that it is false; so it must be both true and false

A Worry about Paradoxes

Recall the Paradox of the Liar:

> Is the following proposition true or false?
>
> > This proposition is false

- If every proposition is either true or false then this proposition will be either true or false
- If it is true, then it is true that it is false; so it must be both true and false
- If it is false, then it is false that it is false; so it must be true; so it must be both true and false

A Worry about Paradoxes (117)

Recall the Paradox of the Liar:

> Is the following proposition true or false?
>
> > This proposition is false

- If every proposition is either true or false then this proposition will be either true or false
- If it is true, then it is true that it is false; so it must be both true and false
- If it is false, then it is false that it is false; so it must be true; so it must be both true and false
- So in both cases it is both true and false, which is impossible

Reflexivity, Symmetry & Transitivity

A relation that holds between two (possibly distinct) objects is

Reflexivity, Symmetry & Transitivity

A relation that holds between two (possibly distinct) objects is

- *reflexive* if and only if, for all objects *x*, *x* is related to itself; otherwise it is *nonreflexive*

Reflexivity, Symmetry & Transitivity

A relation that holds between two (possibly distinct) objects is

- *reflexive* if and only if, for all objects *x*, *x* is related to itself; otherwise it is *nonreflexive*

- *symmetric* if and only if, for all objects *x* and *y*, if *x* is related to *y*, then *y* is also related to *x*; otherwise it is *nonsymmetric*

Reflexivity, Symmetry & Transitivity

A relation that holds between two (possibly distinct) objects is

- *reflexive* if and only if, for all objects *x*, *x* is related to itself; otherwise it is *nonreflexive*
- *symmetric* if and only if, for all objects *x* and *y*, if *x* is related to *y*, then *y* is also related to *x*; otherwise it is *nonsymmetric*
- *transitive* if and only if, for all objects *x*, *y*, and *z*, if *x* is related to *y* and *y* is related to *z*, then *x* is also related to *z*; otherwise it is *nontransitive*

Reflexivity, Symmetry & Transitivity (149)

A relation that holds between two (possibly distinct) objects is

- *reflexive* if and only if, for all objects *x*, *x* is related to itself; otherwise it is *nonreflexive*

- *symmetric* if and only if, for all objects *x* and *y*, if *x* is related to *y*, then *y* is also related to *x*; otherwise it is *nonsymmetric*

- *transitive* if and only if, for all objects *x*, *y*, and *z*, if *x* is related to *y* and *y* is related to *z*, then *x* is also related to *z*; otherwise it is *nontransitive*

A relation that is reflexive, symmetric and transitive is an *equivalence relation*

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- *x* is the father of *y*

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- *x* is the father of *y*
- *x* is to the right of *y*

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- *x* is the father of *y*
- *x* is to the right of *y*
- *x* is identical to *y*

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- x is the father of y
- x is to the right of y
- x is identical to y
- x is distinct from y

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- x is the father of y
- x is to the right of y
- x is identical to y
- x is distinct from y
- x is a square root of y

Example — Reflexivity, Symmetry, Transitivity

Are the following relations reflexive? Are they symmetric? Are they transitive?

- *x* is the father of *y*
- *x* is to the right of *y*
- *x* is identical to *y*
- *x* is distinct from *y*
- *x* is a square root of *y*
- *x* is implied by *y*

Example — Reflexivity, Symmetry, Transitivity >

Are the following relations reflexive? Are they symmetric? Are they transitive?

- *x* is the father of *y*
- *x* is to the right of *y*
- *x* is identical to *y*
- *x* is distinct from *y*
- *x* is a square root of *y*
- *x* is implied by *y*
- *x* is the same height as *y*

Example — Reflexivity, Symmetry, Transitivity

Are the following relations reflexive? Are they symmetric? Are they transitive?

- x is the father of y
- x is to the right of y
- x is identical to y
- x is distinct from y
- x is a square root of y
- x is implied by y
- x is the same height as y
- x is a different height than y

System *RP*

System *RP*

- has the same propositional constants and grouping indicators as system *P*

System *RP*

System *RP*

- has the same propositional constants and grouping indicators as system *P*
- uses new propositional connectives, including $\rightarrow$ (the arrow) instead of $\supset$ (the hook)

System *RP*

(148) >

System *RP*

- has the same propositional constants and grouping indicators as system *P*
- uses new propositional connectives, including → (the arrow) instead of ⊃ (the hook)
- uses connectives that depend on whether propositions share a common subject matter

System *RP* (148)

System *RP*

- has the same propositional constants and grouping indicators as system *P*
- uses new propositional connectives, including → (the arrow) instead of ⊃ (the hook)
- uses connectives that depend on whether propositions share a common subject matter
- helps avoid the fallacy of *ignoratio elenchi* in a way that system *P* does not

Relation *R*

(148-149) >

Relation *R*

- is written "$R(p, q)$" and is read "*p* is related to *q* "

Relation *R*

(148-149) >

Relation *R*

- is written "*R*(*p, q*)" and is read "*p* is related to *q* "
- holds between propositions that share a common subject matter

Relation *R*

(148-149) >

Relation *R*

- is written "$R(p, q)$" and is read "p is related to q "
- holds between propositions that share a common subject matter
- is reflexive, symmetric and non-transitive

Relation *R* (148-149) >

Relation *R*

- is written "$R(p, q)$" and is read "p is related to q "
- holds between propositions that share a common subject matter
- is reflexive, symmetric and non-transitive

Example

- Let p be "Socrates lived in Athens"

Relation *R* (148-149) >

Relation *R*

- is written "$R(p, q)$" and is read "*p* is related to *q* "
- holds between propositions that share a common subject matter
- is reflexive, symmetric and non-transitive

Example

- Let *p* be "Socrates lived in Athens"
- Let *q* be "Athens is in Greece"

Relation *R* (148-149)

Relation *R*

- is written "$R(p, q)$" and is read "p is related to q "
- holds between propositions that share a common subject matter
- is reflexive, symmetric and non-transitive

Example

- Let p be "Socrates lived in Athens"
- Let q be "Athens is in Greece"
- Then $R(p, q)$ is true since p and q share the common subject matter, "Athens"

R-Conditional

(150-151) >

p	q	$R(p, q)$	$p \rightarrow q$
T	T		
T	F		
F	T		
F	F		

R-Conditional

(150-151) >

p	q	$R(p, q)$	$p \rightarrow q$
T	T	T	
T	F	T	
F	T	T	
F	F	T	
		F	
		F	
		F	
		F	

R-Conditional

(150-151) >

p	q	$R(p, q)$	$p \rightarrow q$
T	T	T	
T	F	T	
F	T	T	
F	F	T	
T	T	F	
T	F	F	
F	T	F	
F	F	F	

R-Conditional

(150-151) >

p	q	$R(p, q)$	$p \rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	T	T
T	T	F	F
T	F	F	F
F	T	F	F
F	F	F	F

R-Conditional (150-151)

p	q	$R(p, q)$	$p \rightarrow q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	T	T
T	T	F	F
T	F	F	F
F	T	F	F
F	F	F	F

- An *R-conditional* (→) is false iff its antecedent is true and its consequent is false or they are unrelated

R-Negation

(150) >

p	$R(p, p)$	$\neg p$
T		
F		

R-Negation

(150) >

p	$R(p, p)$	$\neg p$
T	T	
F	T	

R-Negation

p	$R(p, p)$	$\neg p$
T	T	F
F	T	T

R-Negation (150)

p	$R(p, p)$	$\neg p$
T	T	F
F	T	T

- An *R-negation* (¬) has the opposite truth value of the proposition negated

R-Conjunction & *R*-Disjunction

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T			
T	F			
F	T			
F	F			
T	T			
T	F			
F	T			
F	F			

R-Conjunction & *R*-Disjunction

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T	T		
T	F	T		
F	T	T		
F	F	T		
T	T	F		
T	F	F		
F	T	F		
F	F	F		

R-Conjunction & *R*-Disjunction

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T	T	T	
T	F	T	F	
F	T	T	F	
F	F	T	F	
T	T	F	T	
T	F	F	F	
F	T	F	F	
F	F	F	F	

R-Conjunction & *R*-Disjunction

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	F
T	T	F	T	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

R-Conjunction & *R*-Disjunction

(151) >

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	F
T	T	F	T	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

- An *R-conjunction* ($\wedge$) is true iff both of its conjuncts are true

R-Conjunction & *R*-Disjunction (151)

p	q	$R(p, q)$	$p \wedge q$	$p \vee q$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	F
T	T	F	T	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

- An *R-conjunction* ($\wedge$) is true iff both of its conjuncts are true
- An *R-disjunction* ($\vee$) is true iff its two disjuncts are related and at least one of its disjuncts is true

***R*-Biconditional** (150-151) >

p	*q*	*R*(*p*, *q*)	$p \leftrightarrow q$
T	T		
T	F		
F	T		
F	F		
T	T		
T	F		
F	T		
F	F		

R-Biconditional

(150-151) >

p	q	$R(p, q)$	$p \leftrightarrow q$
T	T	T	
T	F	T	
F	T	T	
F	F	T	
T	T	F	
T	F	F	
F	T	F	
F	F	F	

R-Biconditional

(150-151) >

p	q	$R(p, q)$	$p \leftrightarrow q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	T	T
T	T	F	F
T	F	F	F
F	T	F	F
F	F	F	F

R-Biconditional (150-151)

p	q	$R(p, q)$	$p \leftrightarrow q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	T	T
T	T	F	F
T	F	F	F
F	T	F	F
F	F	F	F

- An *R-biconditional* ($\leftrightarrow$) is true iff its its antecedent and consequent are related and they share the same truth value

Example — Truth Tables in *P* and *RP* (153-154) >

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

Example — Truth Tables in *P* and *RP* (153-154) >

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

	A	B	$R(A, B)$	$B \rightarrow A$	$R(A, B \rightarrow A)$	$A \rightarrow (B \rightarrow A)$
(1)	T	T				
(2)	T	F				
(3)	F	T				
(4)	F	F				
(5)	T	T				
(6)	T	F				
(7)	F	T				
(8)	F	F				

Example — Truth Tables in *P* and *RP*

(153-154) >

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

	A	B	$R(A, B)$	$B \rightarrow A$	$R(A, B \rightarrow A)$	$A \rightarrow (B \rightarrow A)$
(1)	T	T	T			
(2)	T	F	T			
(3)	F	T	T			
(4)	F	F	T			
(5)	T	T	F			
(6)	T	F	F			
(7)	F	T	F			
(8)	F	F	F			

Example — Truth Tables in *P* and *RP* (153-154) >

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

	A	B	$R(A, B)$	$B \rightarrow A$	$R(A, B \rightarrow A)$	$A \rightarrow (B \rightarrow A)$
(1)	T	T	T	T		
(2)	T	F	T	T		
(3)	F	T	T	F		
(4)	F	F	T	T		
(5)	T	T	F	F		
(6)	T	F	F	F		
(7)	F	T	F	F		
(8)	F	F	F	F		

Example — Truth Tables in *P* and *RP* (153-154) >

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

	A	B	$R(A, B)$	$B \rightarrow A$	$R(A, B \rightarrow A)$	$A \rightarrow (B \rightarrow A)$
(1)	T	T	T	T	T	
(2)	T	F	T	T	T	
(3)	F	T	T	F	T	
(4)	F	F	T	T	T	
(5)	T	T	F	F	T	
(6)	T	F	F	F	T	
(7)	F	T	F	F	T	
(8)	F	F	F	F	T	

Example — Truth Tables in *P* and *RP* (153-154)

	A	B	$B \supset A$	$A \supset (B \supset A)$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	F	T
(4)	F	F	T	T

	A	B	$R(A, B)$	$B \rightarrow A$	$R(A, B \rightarrow A)$	$A \rightarrow (B \rightarrow A)$
(1)	T	T	T	T	T	T
(2)	T	F	T	T	T	T
(3)	F	T	T	F	T	T
(4)	F	F	T	T	T	T
(5)	T	T	F	F	T	F
(6)	T	F	F	F	T	F
(7)	F	T	F	F	T	T
(8)	F	F	F	F	T	T

Modus Ponens in System *RP* (154-155) >

$p \rightarrow q$
p

$\therefore q$

Modus Ponens **in System *RP*** (154-155) >

$p \rightarrow q$
p

$\therefore q$

	p	q	$R(p, q)$	$p \rightarrow q$
(1)	T	T		
(2)	T	F		
(3)	F	T		
(4)	F	F		
(5)	T	T		
(6)	T	F		
(7)	F	T		
(8)	F	F		

Modus Ponens* in System *RP

(154-155) >

$p \rightarrow q$
p

$\therefore q$

	p	q	$R(p, q)$	$p \rightarrow q$
(1)	T	T	T	
(2)	T	F	T	
(3)	F	T	T	
(4)	F	F	T	
(5)	T	T	F	
(6)	T	F	F	
(7)	F	T	F	
(8)	F	F	F	

Modus Ponens* in System *RP (154-155) >

$p \rightarrow q$
p

$\therefore q$

	p	q	$R(p, q)$	$p \rightarrow q$
(1)	T	T	T	T
(2)	T	F	T	F
(3)	F	T	T	T
(4)	F	F	T	T
(5)	T	T	F	F
(6)	T	F	F	F
(7)	F	T	F	F
(8)	F	F	F	F

Modus Ponens* in System *RP (154-155)

$p \rightarrow q$
p

$\therefore q$

	p	q	$R(p, q)$	$p \rightarrow q$
(1)	T	T	T	T
(2)	T	F	T	F
(3)	F	T	T	T
(4)	F	F	T	T
(5)	T	T	F	F
(6)	T	F	F	F
(7)	F	T	F	F
(8)	F	F	F	F

- *Modus Ponens* is deductively valid in *RP*

Addition in System *RP*

$$\frac{p}{\therefore\ p \vee q}$$

Addition in System *RP*

(155) >

p

$\therefore\ p \vee q$

	p	q	$R(p, q)$	$p \vee q$
(1)	T	T		
(2)	T	F		
(3)	F	T		
(4)	F	F		
(5)	T	T		
(6)	T	F		
(7)	F	T		
(8)	F	F		

Addition in System *RP*

(155) >

p

$\therefore\ p \vee q$

	p	q	$R(p, q)$	$p \vee q$
(1)	T	T	T	
(2)	T	F	T	
(3)	F	T	T	
(4)	F	F	T	
(5)	T	T	F	
(6)	T	F	F	
(7)	F	T	F	
(8)	F	F	F	

Addition in System *RP*

(155) >

p

$\therefore\ p \vee q$

	p	q	$R(p, q)$	$p \vee q$
(1)	T	T	T	T
(2)	T	F	T	T
(3)	F	T	T	T
(4)	F	F	T	F
(5)	T	T	F	F
(6)	T	F	F	F
(7)	F	T	F	F
(8)	F	F	F	F

Addition in System *RP*

p

$\therefore\ p \vee q$

		p	q	$R(p, q)$	$p \vee q$
	(1)	T	T	T	T
	(2)	T	F	T	T
	(3)	F	T	T	T
	(4)	F	F	T	F
X	(5)	T	T	F	F
X	(6)	T	F	F	F
	(7)	F	T	F	F
	(8)	F	F	F	F

- Addition is not deductively valid in *RP*

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*

- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*

- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Example

	A	$R(A, \neg A)$	$\neg A$	$A \vee \neg A$	$\neg(A \vee \neg A)$
(1)	T				
(2)	F				

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*

- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Example

	A	$R(A, \neg A)$	$\neg A$	$A \vee \neg A$	$\neg(A \vee \neg A)$
(1)	T	T			
(2)	F	T			

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*
- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*
- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Example

	A	$R(A, \neg A)$	$\neg A$	$A \vee \neg A$	$\neg(A \vee \neg A)$
(1)	T	T	F		
(2)	F	T	T		

Tautologies, Contradictions & Contingencies (153) >

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*
- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*
- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Example

	A	$R(A, \neg A)$	$\neg A$	$A \vee \neg A$	$\neg(A \vee \neg A)$
(1)	T	T	F	T	
(2)	F	T	T	T	

Tautologies, Contradictions & Contingencies (153)

- A *relatedness tautology* (or a *relatedness necessity*) is a wff that is necessarily true as a result of its connectives in system *RP*

- A *relatedness contradiction* (or a *relatedness impossibility*) is a wff that is necessarily false as a result of its connectives in system *RP*

- A *relatedness contingency* is a wff that is neither a relatedness tautology nor a relatedness contradiction

Example

	A	$R(A, \neg A)$	$\neg A$	$A \vee \neg A$	$\neg(A \vee \neg A)$
(1)	T	T	F	T	F
(2)	F	T	T	T	F

Examples

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

$$(A \leftrightarrow \neg A)$$

$$(A \rightarrow \neg A)$$

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T			
(2)	F			

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T			
(2)	F			

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T		
(2)	F	T		

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T			
(2)	F			

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T	F	
(2)	F	T	T	

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T			
(2)	F			

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T	F	F
(2)	F	T	T	F

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T			
(2)	F			

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T	F	F
(2)	F	T	T	F

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T	T		
(2)	F	T		

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T	F	F
(2)	F	T	T	F

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T	T	F	
(2)	F	T	T	

Tautologies

- Are the following propositions *relatedness tautologies, relatedness contradictions* or *relatedness contingencies?*

	A	$R(A, \neg A)$	$\neg A$	$(A \leftrightarrow \neg A)$
(1)	T	T	F	F
(2)	F	T	T	F

	A	$R(A, \neg A)$	$\neg A$	$(A \rightarrow \neg A)$
(1)	T	T	F	F
(2)	F	T	T	T

Modal Propositional Logic

(158) >

Modal propositional logic involves two new propositional connectives

Modal Propositional Logic

Modal propositional logic involves two new propositional connectives

- ◊, for "It is possible that" or "Possibly"

Modal Propositional Logic

Modal propositional logic involves two new propositional connectives

- ◊, for "It is possible that" or "Possibly"
- □, for "It is necessary that" or "Necessarily"

Modal Propositional Logic

Modal propositional logic involves two new propositional connectives

- ◊, for "It is possible that" or "Possibly"
- ☐, for "It is necessary that" or "Necessarily"

Example

Let

p = The square root of 1,136,356 is 1,066

Modal Propositional Logic

Modal propositional logic involves two new propositional connectives

- ◊, for "It is possible that" or "Possibly"
- □, for "It is necessary that" or "Necessarily"

Example

Let

p = The square root of 1,136,356 is 1,066

Then

◊p = It is possible that the square root of 1,136,356 is 1,066

Modal Propositional Logic

Modal propositional logic involves two new propositional connectives

- ◊, for "It is possible that" or "Possibly"
- □, for "It is necessary that" or "Necessarily"

Example

Let

p = The square root of 1,136,356 is 1,066

Then

◊p = It is possible that the square root of 1,136,356 is 1,066

□p = It is necessary that the square root of 1,136,356 is 1,066

Possibility & Necessity

It is possible that ...

p	$\Diamond p$
T	
F	

It is necessary that ...

p	$\Box p$
T	
F	

Possibility & Necessity

It is possible that ...

p	$\Diamond p$
T	T
F	

It is necessary that ...

p	$\Box p$
T	
F	

Possibility & Necessity

It is possible that ...

p	◊*p*
T	T
F	?

It is necessary that ...

p	□*p*
T	
F	

Possibility & Necessity

It is possible that ...

p	$\Diamond p$
T	T
F	?

It is necessary that ...

p	$\Box p$
T	
F	F

Possibility & Necessity

It is possible that ...

p	$\Diamond p$
T	T
F	?

It is necessary that ...

p	$\Box p$
T	?
F	F

Possibility & Necessity

It is possible that ...

p	$\Diamond p$
T	T
F	?

It is necessary that ...

p	$\Box p$
T	?
F	F

- Both of these connectives are only *partially truth-functional*

Valid Modal Inferences

- Bill is in Banff

  ∴ It is possible that Bill is in Banff

Valid Modal Inferences

• Bill is in Banff	p
∴ It is possible that Bill is in Banff	∴ $\Diamond p$

Valid Modal Inferences

- Bill is in Banff

  ∴ It is possible that Bill is in Banff

  p

  $\therefore \Diamond p$

- It is not possible that Bill is in Banff

  ∴ It is necessary that Bill is not in Banff

Valid Modal Inferences (158)

- Bill is in Banff — p

  ∴ It is possible that Bill is in Banff — $\therefore \Diamond p$

- It is not possible that Bill is in Banff — $\sim\Diamond p$

  ∴ It is necessary that Bill is not in Banff — $\therefore \Box \sim p$

Epistemic Propositional Logic

Epistemic logic involves two new propositional connectives

Epistemic Propositional Logic

(165) >

Epistemic logic involves two new propositional connectives

- **B**, for "It is believed that"

Epistemic Propositional Logic

Epistemic logic involves two new propositional connectives

- **B**, for "It is believed that"
- **K**, for "It is known that"

Epistemic Propositional Logic

(165) >

Epistemic logic involves two new propositional connectives

- **B**, for "It is believed that"
- **K**, for "It is known that"

Example

Let

p = There is an inactive volcano on the dark side of the moon

Epistemic Propositional Logic

Epistemic logic involves two new propositional connectives

- **B**, for "It is believed that"
- **K**, for "It is known that"

Example

Let

p = There is an inactive volcano on the dark side of the moon

Then

Bp = It is believed that there is an inactive volcano on the dark side of the moon

Epistemic Propositional Logic (165)

Epistemic logic involves two new propositional connectives

- **B**, for "It is believed that"
- **K**, for "It is known that"

Example

Let

p = There is an inactive volcano on the dark side of the moon

Then

Bp = It is believed that there is an inactive volcano on the dark side of the moon

Kp = It is known that there is an inactive volcano on the dark side of the moon

Belief & Knowledge

(159) >

It is believed that ...

p	$\mathbf{B}p$
T	
F	

It is known that ...

p	$\mathbf{K}p$
T	
F	

Belief & Knowledge

It is believed that ...

p	$\mathbf{B}p$
T	?
F	

It is known that ...

p	$\mathbf{K}p$
T	
F	

Belief & Knowledge

It is believed that ...

p	**B***p*
T	?
F	?

It is known that ...

p	**K***p*
T	
F	

Belief & Knowledge

It is believed that ...

p	**B**p
T	?
F	?

It is known that ...

p	**K**p
T	
F	F

Belief & Knowledge

It is believed that ...

p	**B***p*
T	?
F	?

It is known that ...

p	**K***p*
T	?
F	F

Belief & Knowledge

It is believed that ...

p	**B**p
T	?
F	?

It is known that ...

p	**K**p
T	?
F	F

- Neither of these connectives is fully *truth-functional*

Valid Epistemic Inferences

- It is known that Bill is in Palo Alto

  ∴ It is believed that Bill is in Palo Alto

Valid Epistemic Inferences

(164) >

- It is known that Bill is in Palo Alto — **K***p*

  ∴ It is believed that Bill is in Palo Alto — ∴ **B***p*

Valid Epistemic Inferences

- It is known that Bill is in Palo Alto — $\mathbf{K}p$

  ∴ It is believed that Bill is in Palo Alto — ∴ $\mathbf{B}p$

- It is known that Bill is in Palo Alto

  ∴ Bill is in Palo Alto

Valid Epistemic Inferences (164)

• It is known that Bill is in Palo Alto	**K**p
$\therefore$ It is believed that Bill is in Palo Alto	$\therefore$ **B**p

• It is known that Bill is in Palo Alto	**K**p
$\therefore$ Bill is in Palo Alto	$\therefore$ p

Deontic Propositional Logic

Deontic logic involves two new propositional connectives

Deontic Propositional Logic

Deontic logic involves two new propositional connectives

- **P**, for "It may be the case that" or "It is permissible that"

Deontic Propositional Logic

Deontic logic involves two new propositional connectives

- **P**, for "It may be the case that" or "It is permissible that"
- **O**, for "It ought to be the case that" or "It is obligatory that"

Deontic Propositional Logic

Deontic logic involves two new propositional connectives

- **P**, for "It may be the case that" or "It is permissible that"
- **O**, for "It ought to be the case that" or "It is obligatory that"

Example

Let

p = Sue is in Whistler

Deontic Propositional Logic

Deontic logic involves two new propositional connectives

- **P**, for "It may be the case that" or "It is permissible that"
- **O**, for "It ought to be the case that" or "It is obligatory that"

Example

Let

p = Sue is in Whistler

Then

Pp = It is permissible that Sue is in Whistler

Deontic Propositional Logic (165)

Deontic logic involves two new propositional connectives

- **P**, for "It may be the case that" or "It is permissible that"
- **O**, for "It ought to be the case that" or "It is obligatory that"

Example

Let

p = Sue is in Whistler

Then

Pp = It is permissible that Sue is in Whistler

Op = It is obligatory that Sue is in Whistler

Permission & Obligation

(167-168) >

It is permissible that ...

p	$\mathbf{P}p$
T	
F	

It is obligatory that ...

p	$\mathbf{O}p$
T	
F	

Permission & Obligation

(167-168) >

It is permissible that ...

p	$\mathbf{P}p$
T	?
F	

It is obligatory that ...

p	$\mathbf{O}p$
T	
F	

Permission & Obligation

(167-168) >

It is permissible that ...

p	**P**p
T	?
F	?

It is obligatory that ...

p	**O**p
T	
F	

Permission & Obligation

(167-168) >

It is permissible that ...

p	**P***p*
T	?
F	?

It is obligatory that ...

p	**O***p*
T	?
F	

Permission & Obligation

(167-168) >

It is permissible that ...

p	$\mathbf{P}p$
T	?
F	?

It is obligatory that ...

p	$\mathbf{O}p$
T	?
F	?

Permission & Obligation (167-168)

It is permissible that ...

p	$\mathbf{P}p$
T	?
F	?

It is obligatory that ...

p	$\mathbf{O}p$
T	?
F	?

- Neither of these connectives is *truth-functional*

Valid Deontic Inferences

(167-168) >

- It is obligatory that Bill is in Montreal

$\therefore$ It is permissible that Bill is in Montreal

Valid Deontic Inferences

(167-168) >

- It is obligatory that Bill is in Montreal — **O***p*

  ∴ It is permissible that Bill is in Montreal — ∴ **P***p*

Valid Deontic Inferences

(167-168) >

- It is obligatory that Bill is in Montreal

  ∴ It is permissible that Bill is in Montreal

  $\mathbf{O}p$

  $\therefore \mathbf{P}p$

- It is obligatory that Bill is in Montreal

  ∴ It is not permissible that Bill not be in Montreal

Valid Deontic Inferences (167-168)

• It is obligatory that Bill is in Montreal	**O***p*
∴ It is permissible that Bill is in Montreal	∴ **P***p*
• It is obligatory that Bill is in Montreal	**O***p*
∴ It is not permissible that Bill not be in Montreal	∴ ~**P**~*p*

Systems of Modal Logic

(161-162) >

D1. $\Box p =_{df} \sim\Diamond\sim p$

Systems of Modal Logic

(161-162) >

D1. $\Box\, p =_{df} {\sim}\Diamond{\sim} p$

D2. $(p \Rightarrow q) =_{df} \Box\, (p \supset q)$

Systems of Modal Logic

(161-162) >

D1. $\Box p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box (p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

Systems of Modal Logic (161-162) >

D1. $\Box\, p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box\, (p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

System *S1*

- A1. $(p \wedge q) \Rightarrow (q \wedge p)$
- A2. $(p \wedge q) \Rightarrow p$
- A3. $p \Rightarrow (p \wedge p)$
- A4. $[(p \wedge q) \wedge r] \Rightarrow [p \wedge (q \wedge r)]$
- A5. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
- A6. $[p \wedge (p \Rightarrow q)] \Rightarrow q$

Systems of Modal Logic

(161-162) >

D1. $\Box\, p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box\,(p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

System *S1*

A1. $(p \wedge q) \Rightarrow (q \wedge p)$

A2. $(p \wedge q) \Rightarrow p$

A3. $p \Rightarrow (p \wedge p)$

A4. $[(p \wedge q) \wedge r] \Rightarrow [p \wedge (q \wedge r)]$

A5. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

A6. $[p \wedge (p \Rightarrow q)] \Rightarrow q$

System *S2*

Axioms A1 to A6 plus

A7. $\Diamond(p \wedge q) \Rightarrow \Diamond p$

Systems of Modal Logic

(161-162) >

D1. $\Box\, p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box\,(p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

System *S1*

A1. $(p \wedge q) \Rightarrow (q \wedge p)$

A2. $(p \wedge q) \Rightarrow p$

A3. $p \Rightarrow (p \wedge p)$

A4. $[(p \wedge q) \wedge r] \Rightarrow [p \wedge (q \wedge r)]$

A5. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

A6. $[p \wedge (p \Rightarrow q)] \Rightarrow q$

System *S2*

Axioms A1 to A6 plus

A7. $\Diamond(p \wedge q) \Rightarrow \Diamond p$

System *S3*

Axioms A1 to A6 plus

A8. $(p \Rightarrow q) \Rightarrow (\sim\Diamond q \Rightarrow \sim\Diamond p)$

Systems of Modal Logic

(161-162) >

D1. $\Box\, p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box\,(p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

System *S1*

A1. $(p \wedge q) \Rightarrow (q \wedge p)$

A2. $(p \wedge q) \Rightarrow p$

A3. $p \Rightarrow (p \wedge p)$

A4. $[(p \wedge q) \wedge r] \Rightarrow [p \wedge (q \wedge r)]$

A5. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

A6. $[p \wedge (p \Rightarrow q)] \Rightarrow q$

System *S2*

Axioms A1 to A6 plus

A7. $\Diamond(p \wedge q) \Rightarrow \Diamond p$

System *S3*

Axioms A1 to A6 plus

A8. $(p \Rightarrow q) \Rightarrow (\sim\Diamond q \Rightarrow \sim\Diamond p)$

System *S4*

Axioms A1 to A6 plus

A9. $\Box p \Rightarrow \Box\,\Box p$

Systems of Modal Logic (161-162)

D1. $\Box\, p =_{df} \sim\Diamond\sim p$

D2. $(p \Rightarrow q) =_{df} \Box\, (p \supset q)$

D3. $(p \Leftrightarrow q) =_{df} [(p \Rightarrow q) \wedge (q \Rightarrow p)]$

System *S1*

A1. $(p \wedge q) \Rightarrow (q \wedge p)$

A2. $(p \wedge q) \Rightarrow p$

A3. $p \Rightarrow (p \wedge p)$

A4. $[(p \wedge q) \wedge r] \Rightarrow [p \wedge (q \wedge r)]$

A5. $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

A6. $[p \wedge (p \Rightarrow q)] \Rightarrow q$

System *S2*

Axioms A1 to A6 plus

A7. $\Diamond(p \wedge q) \Rightarrow \Diamond p$

System *S3*

Axioms A1 to A6 plus

A8. $(p \Rightarrow q) \Rightarrow (\sim\Diamond q \Rightarrow \sim\Diamond p)$

System *S4*

Axioms A1 to A6 plus

A9. $\Box p \Rightarrow \Box\, \Box p$

System *S5*

Axioms A1 to A6 plus

A10. $\Diamond p \Rightarrow \Box\, \Diamond p$

Example — Proof in Modal Logic

To Prove:

$A \Rightarrow \sim\sim A$ in system S1

Example — Proof in Modal Logic

(162) >

To Prove:

$A \Rightarrow \sim\sim A$ in system S1

Proof:

1. $A \Rightarrow (A \wedge A)$ A3 Sub

Example — Proof in Modal Logic

To Prove:

$A \Rightarrow \sim\sim A$ in system S1

Proof:

1. $A \Rightarrow (A \wedge A)$ A3 Sub
2. $(A \wedge A) \Rightarrow A$ A2 Sub

Example — Proof in Modal Logic

To Prove:

$A \Rightarrow \sim\sim A$ in system S1

Proof:

1. $A \Rightarrow (A \wedge A)$	A3 Sub
2. $(A \wedge A) \Rightarrow A$	A2 Sub
3. $A \Rightarrow A$	1, 2 HS

Example — Proof in Modal Logic (162)

To Prove:

$A \Rightarrow \sim\sim A$ in system S1

Proof:

1. $A \Rightarrow (A \wedge A)$	A3 Sub
2. $(A \wedge A) \Rightarrow A$	A2 Sub
3. $A \Rightarrow A$	1, 2 HS
4. $A \Rightarrow \sim\sim A$	3 DN

Example — Proof in Modal Logic

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Example — Proof in Modal Logic

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Proof:

1. $[A \wedge (A \Rightarrow B)] \Rightarrow B$ A6 Sub

Example — Proof in Modal Logic

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Proof:

1. $[A \wedge (A \Rightarrow B)] \Rightarrow B$ A6 Sub
2. $([A \wedge (A \Rightarrow B)] \Rightarrow B) \Rightarrow (\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)])$ A8 Sub

Example — Proof in Modal Logic

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Proof:

1.	$[A \wedge (A \Rightarrow B)] \Rightarrow B$	A6 Sub
2.	$([A \wedge (A \Rightarrow B)] \Rightarrow B) \Rightarrow (\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)])$	A8 Sub
3.	$\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)]$	2, 1 MP

Example — Proof in Modal Logic

(162) >

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Proof:

1. $[A \wedge (A \Rightarrow B)] \Rightarrow B$	A6 Sub
2. $([A \wedge (A \Rightarrow B)] \Rightarrow B) \Rightarrow (\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)])$	A8 Sub
3. $\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)]$	2, 1 MP
4. $\sim\sim\Diamond B \vee \sim\Diamond[A \wedge (A \Rightarrow B)]$	3 Imp

Example — Proof in Modal Logic (162)

To Prove:

$\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$ in system S3

Proof:

1. $[A \wedge (A \Rightarrow B)] \Rightarrow B$	A6 Sub
2. $([A \wedge (A \Rightarrow B)] \Rightarrow B) \Rightarrow (\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)])$	A8 Sub
3. $\sim\Diamond B \Rightarrow \sim\Diamond[A \wedge (A \Rightarrow B)]$	2, 1 MP
4. $\sim\sim\Diamond B \vee \sim\Diamond[A \wedge (A \Rightarrow B)]$	3 Imp
5. $\sim(\sim\Diamond B \wedge \Diamond[A \wedge (A \Rightarrow B)])$	4 DeM

Are Valid Arguments always Reliable?

(167) >

The argument form *Modus Ponens* is valid

- $p \supset q$

  p

  $\therefore q$

Are Valid Arguments always Reliable? (167)

The argument form *Modus Ponens* is valid

- $p \supset q$

  p

  $\therefore q$

But does this guarantee the reliability of the rule of inference

- $B(p \supset q)$

  $B(p)$

  $\therefore B(q)$

Example — Valid Argument

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

Example — Valid Argument (167)

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

- Understood as an argument in the narrow sense, clearly this argument is valid

Example — Reliable Inference?

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(P3) It is not the case that the Secretary-General of the UN lives in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

Example — Reliable Inference? (167) >

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(P3) It is not the case that the Secretary-General of the UN lives in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

- Understood as an argument in the broad sense, the addition of (P3) makes this a non-reliable inference

Example — Reliable Inference?

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(P3) It is not the case that the Secretary-General of the UN lives in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

- Understood as an argument in the broad sense, the addition of (P3) makes this a non-reliable inference
- Bill's believing (P1) and (P2) does not reliably lead him to infer (C)

Example — Reliable Inference? (167)

(P1) If the UN General Assembly is located in Geneva, then the Secretary-General of the UN lives in Geneva.

(P2) The UN General Assembly is located in Geneva.

(P3) It is not the case that the Secretary-General of the UN lives in Geneva.

(C) Therefore, the Secretary-General of the UN lives in Geneva.

- Understood as an argument in the broad sense, the addition of (P3) makes this a non-reliable inference
- Bill's believing (P1) and (P2) does not reliably lead him to infer (C)
- Instead, since (P1) through (P3) imply both (C) and not-(C), this leads us to re-evaluate (P1) through (P3)

Multi-valued Propositional Logics (168-169) >

- Classical logic is committed to exactly two truth values: *truth* and *falsehood*

Multi-valued Propositional Logics (168-169) >

- Classical logic is committed to exactly two truth values: *truth* and *falsehood*
- Some arguments support the view that there are more than two truth values

Multi-valued Propositional Logics

(168-169) >

- Classical logic is committed to exactly two truth values: *truth* and *falsehood*
- Some arguments support the view that there are more than two truth values

Example

- Arguments based on Aristotle's problem of future contingents

Multi-valued Propositional Logics (168-169)

- Classical logic is committed to exactly two truth values: *truth* and *falsehood*
- Some arguments support the view that there are more than two truth values

Example

- Arguments based on Aristotle's problem of future contingents
- Arguments based on the paradox of the liar

Aristotle’s Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

Aristotle's Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false

Aristotle's Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false
- If the above proposition is true (now), then it is already determined that Athens will win the sea battle tomorrow

Aristotle's Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false
- If the above proposition is true (now), then it is already determined that Athens will win the sea battle tomorrow
- If the above proposition is false (now) then it is already determined that Athens will not win the sea battle tomorrow

Aristotle’s Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false
- If the above proposition is true (now), then it is already determined that Athens will win the sea battle tomorrow
- If the above proposition is false (now) then it is already determined that Athens will not win the sea battle tomorrow
- Thus, whatever the outcome, it is already determined

Aristotle’s Problem of Future Contingents (169) >

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false
- If the above proposition is true (now), then it is already determined that Athens will win the sea battle tomorrow
- If the above proposition is false (now) then it is already determined that Athens will not win the sea battle tomorrow
- Thus, whatever the outcome, it is already determined
- Since there was nothing special about our original proposition, everything that happens must happen necessarily

Aristotle's Problem of Future Contingents (169)

Consider the proposition "Athens will win the sea battle tomorrow"

- If every proposition is either true or false then the above proposition must be either true or false
- If the above proposition is true (now), then it is already determined that Athens will win the sea battle tomorrow
- If the above proposition is false (now) then it is already determined that Athens will not win the sea battle tomorrow
- Thus, whatever the outcome, it is already determined
- Since there was nothing special about our original proposition, everything that happens must happen necessarily
- But because we have free will, we know that not everything happens necessarily

Lukasiewicz's Three-valued Logic

Conjunction

$p \wedge q$		q: T	I	F
	T			
p	I			
	F			

Lukasiewicz's Three-valued Logic

Conjunction

		q		
	$p \wedge q$	T	I	F
	T	T		F
p	I			
	F	F		F

Lukasiewicz's Three-valued Logic

Conjunction

$p \wedge q$		q: T	I	F
	T	T	I	F
p	I	I	I	F
	F	F	F	F

Lukasiewicz's Three-valued Logic

(170) >

Conjunction

		q		
	$p \wedge q$	T	I	F
	T	T	I	F
p	I	I	I	F
	F	F	F	F

Disjunction

		q		
	$p \vee q$	T	I	F
	T	T	T	T
p	I	T	I	I
	F	T	I	F

Lukasiewicz's Three-valued Logic

Conjunction

	$p \wedge q$	q: T	I	F
	T	T	I	F
p	I	I	I	F
	F	F	F	F

Disjunction

	$p \vee q$	q: T	I	F
	T	T	T	T
p	I	T	I	I
	F	T	I	F

Negation

$\sim p$	p
F	T
I	I
T	F

Lukasiewicz's Three-valued Logic

Conjunction

			q	
	p ∧ q	T	I	F
	T	T	I	F
p	I	I	I	F
	F	F	F	F

Disjunction

			q	
	p ∨ q	T	I	F
	T	T	T	T
p	I	T	I	I
	F	T	I	F

Negation

~ p	**p**
F	T
I	I
T	F

Conditional

			q	
	p ⊃ q	T	I	F
	T	T	I	F
p	I	T	T	I
	F	T	T	T

Lukasiewicz's Three-valued Logic

Conjunction

	$p \wedge q$	q: T	I	F
	T	T	I	F
p	I	I	I	F
	F	F	F	F

Disjunction

	$p \vee q$	q: T	I	F
	T	T	T	T
p	I	T	I	I
	F	T	I	F

Negation

$\sim p$	p
F	T
I	I
T	F

Conditional

	$p \supset q$	q: T	I	F
	T	T	I	F
p	I	T	T	I
	F	T	T	T

Biconditional

	$p \equiv q$	q: T	I	F
	T	T	I	F
p	I	I	T	I
	F	F	I	T

Bochvar's Three-valued Logic

Conjunction

			q	
	p ∧ ***q***	T	I	F
	T	T		F
p	I			
	F	F		F

Bochvar's Three-valued Logic

Conjunction

$p \wedge q$	q: T	I	F
p T	T	I	F
I	I	I	I
F	F	I	F

Bochvar's Three-valued Logic

Conjunction

$p \wedge q$	q: T	I	F
p: T	T	I	F
I	I	I	I
F	F	I	F

Disjunction

$p \vee q$	q: T	I	F
p: T	T	I	T
I	I	I	I
F	T	I	F

Bochvar's Three-valued Logic

(171) >

Conjunction

		q		
p* ∧ *q		T	I	F
	T	T	I	F
p	I	I	I	I
	F	F	I	F

Disjunction

		q		
p* ∨ *q		T	I	F
	T	T	I	T
p	I	I	I	I
	F	T	I	F

Negation

~ *p*	***p***
F	T
I	I
T	F

Bochvar's Three-valued Logic

(171) >

Conjunction

	$p \wedge q$	q: T	I	F
p	T	T	I	F
	I	I	I	I
	F	F	I	F

Disjunction

	$p \vee q$	q: T	I	F
p	T	T	I	T
	I	I	I	I
	F	T	I	F

Negation

$\sim p$	p
F	T
I	I
T	F

Conditional

	$p \supset q$	q: T	I	F
p	T	T	I	F
	I	I	I	I
	F	T	I	T

Bochvar's Three-valued Logic

Conjunction

$p \wedge q$	q: T	I	F
p: T	T	I	F
I	I	I	I
F	F	I	F

Disjunction

$p \vee q$	q: T	I	F
p: T	T	I	T
I	I	I	I
F	T	I	F

Negation

$\sim p$	p
F	T
I	I
T	F

Conditional

$p \supset q$	q: T	I	F
p: T	T	I	F
I	I	I	I
F	T	I	T

Biconditional

$p \equiv q$	q: T	I	F
p: T	T	I	F
I	I	I	I
F	F	I	T

Law of Excluded Middle

(171) >

	p	$\sim p$	$p \vee \sim p$
(1)	T	F	
(2)	I	I	
(3)	F	T	

Law of Excluded Middle

	p	$\sim p$	$p \vee \sim p$
(1)	T	F	T
(2)	I	I	I
(3)	F	T	T

Law of Excluded Middle (171)

	p	$\sim p$	$p \vee \sim p$
(1)	T	F	T
(2)	I	I	I
(3)	F	T	T

- The Law of Excluded Middle is not a tautology in either Lukasiewicz's logic or Bochvar's logic

Law Non-contradiction

	p	$\sim p$	$p \land \sim p$	$\sim(p \land \sim p)$
(1)	T	F		
(2)	I	I		
(3)	F	T		

Law Non-contradiction

	p	$\sim p$	$p \wedge \sim p$	$\sim(p \wedge \sim p)$
(1)	T	F	F	
(2)	I	I	I	
(3)	F	T	F	

Law Non-contradiction

	p	$\sim p$	$p \wedge \sim p$	$\sim(p \wedge \sim p)$
(1)	T	F	F	T
(2)	I	I	I	I
(3)	F	T	F	T

Law Non-contradiction (171)

	p	$\sim p$	$p \wedge \sim p$	$\sim(p \wedge \sim p)$
(1)	T	F	F	T
(2)	I	I	I	I
(3)	F	T	F	T

- The Law of Non-contradiction is not a tautology in either Lukasiewicz's logic or Bochvar's logic

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T					
(2)	T	I					
(3)	T	F					
(4)	I	T					
(5)	I	I					
(6)	I	F					
(7)	F	T					
(8)	F	I					
(9)	F	F					

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T				
(2)	T	I	I				
(3)	T	F	F				
(4)	I	T	T				
(5)	I	I	T				
(6)	I	F	I				
(7)	F	T	T				
(8)	F	I	T				
(9)	F	F	T				

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T			
(2)	T	I	I	T			
(3)	T	F	F	T			
(4)	I	T	T	I			
(5)	I	I	T	T			
(6)	I	F	I	T			
(7)	F	T	T	F			
(8)	F	I	T	I			
(9)	F	F	T	T			

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T		
(2)	T	I	I	T	I		
(3)	T	F	F	T	F		
(4)	I	T	T	I	I		
(5)	I	I	T	T	T		
(6)	I	F	I	T	I		
(7)	F	T	T	F	F		
(8)	F	I	T	I	I		
(9)	F	F	T	T	T		

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	
(2)	T	I	I	T	I	I	
(3)	T	F	F	T	F	F	
(4)	I	T	T	I	I	I	
(5)	I	I	T	T	T	T	
(6)	I	F	I	T	I	I	
(7)	F	T	T	F	F	F	
(8)	F	I	T	I	I	I	
(9)	F	F	T	T	T	T	

Lukasiewicz's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	T
(2)	T	I	I	T	I	I	T
(3)	T	F	F	T	F	F	T
(4)	I	T	T	I	I	I	T
(5)	I	I	T	T	T	T	T
(6)	I	F	I	T	I	I	T
(7)	F	T	T	F	F	F	T
(8)	F	I	T	I	I	I	T
(9)	F	F	T	T	T	T	T

Lukasiewicz's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	T
(2)	T	I	I	T	I	I	T
(3)	T	F	F	T	F	F	T
(4)	I	T	T	I	I	I	T
(5)	I	I	T	T	T	T	T
(6)	I	F	I	T	I	I	T
(7)	F	T	T	F	F	F	T
(8)	F	I	T	I	I	I	T
(9)	F	F	T	T	T	T	T

- The Law of Equivalence is a tautology in Lukasiewicz's logic

Bochvar's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T					
(2)	T	I					
(3)	T	F					
(4)	I	T					
(5)	I	I					
(6)	I	F					
(7)	F	T					
(8)	F	I					
(9)	F	F					

Bochvar's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T				
(2)	T	I	I				
(3)	T	F	F				
(4)	I	T	I				
(5)	I	I	I				
(6)	I	F	I				
(7)	F	T	T				
(8)	F	I	I				
(9)	F	F	T				

Bochvar's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T			
(2)	T	I	I	I			
(3)	T	F	F	T			
(4)	I	T	I	I			
(5)	I	I	I	I			
(6)	I	F	I	I			
(7)	F	T	T	F			
(8)	F	I	I	I			
(9)	F	F	T	T			

Bochvar's Law of Equivalence

(172) >

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T		
(2)	T	I	I	I	I		
(3)	T	F	F	T	F		
(4)	I	T	I	I	I		
(5)	I	I	I	I	I		
(6)	I	F	I	I	I		
(7)	F	T	T	F	F		
(8)	F	I	I	I	I		
(9)	F	F	T	T	T		

Bochvar's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	
(2)	T	I	I	I	I	I	
(3)	T	F	F	T	F	F	
(4)	I	T	I	I	I	I	
(5)	I	I	I	I	I	I	
(6)	I	F	I	I	I	I	
(7)	F	T	T	F	F	F	
(8)	F	I	I	I	I	I	
(9)	F	F	T	T	T	T	

Bochvar's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	T
(2)	T	I	I	I	I	I	I
(3)	T	F	F	T	F	F	T
(4)	I	T	I	I	I	I	I
(5)	I	I	I	I	I	I	I
(6)	I	F	I	I	I	I	I
(7)	F	T	T	F	F	F	T
(8)	F	I	I	I	I	I	I
(9)	F	F	T	T	T	T	T

Bochvar's Law of Equivalence

	A	B	$A \supset B$	$B \supset A$	$(A \supset B) \wedge (B \supset A)$	$A \equiv B$	$[(A \supset B) \wedge (B \supset A)] \equiv (A \equiv B)$
(1)	T	T	T	T	T	T	T
(2)	T	I	I	I	I	I	I
(3)	T	F	F	T	F	F	T
(4)	I	T	I	I	I	I	I
(5)	I	I	I	I	I	I	I
(6)	I	F	I	I	I	I	I
(7)	F	T	T	F	F	F	T
(8)	F	I	I	I	I	I	I
(9)	F	F	T	T	T	T	T

- The Law of Equivalence is not a tautology in Bochvar's logic

Recall the Paradox of the Liar (117)

Is the following proposition true or false?

This proposition is false

- If every proposition is either true or false then this proposition will be either true or false
- If it is true, then it is true that it is false; so it must be both true and false
- If it is false, then it is false that it is false; so it must be true; so it must be both true and false
- So in both cases it is both true and false, which is impossible

Paradox of the Strengthened Liar

(173) >

Consider the proposition "This proposition is not true"

Paradox of the Strengthened Liar

Consider the proposition "This proposition is not true"

- If every proposition is true, false, or neither, then this proposition must be true, false, or neither

Paradox of the Strengthened Liar

Consider the proposition "This proposition is not true"

- If every proposition is true, false, or neither, then this proposition must be true, false, or neither
- If it is true, then it is true that it is not true; so it must be both true and not true

Paradox of the Strengthened Liar

(173) >

Consider the proposition "This proposition is not true"

- If every proposition is true, false, or neither, then this proposition must be true, false, or neither
- If it is true, then it is true that it is not true; so it must be both true and not true
- If it is false, then it is not true (since being false is one way of not being true), and since it is not true, and it says that it is not true, it must be true; so it must be both true and not true

Paradox of the Strengthened Liar

Consider the proposition "This proposition is not true"

- If every proposition is true, false, or neither, then this proposition must be true, false, or neither
- If it is true, then it is true that it is not true; so it must be both true and not true
- If it is false, then it is not true (since being false is one way of not being true), and since it is not true, and it says that it is not true, it must be true; so it must be both true and not true
- If it is neither true nor false, then it is not true (since being neither true nor false is one way of not being true), and since it is not true, and it says that it is not true, it must be true; so it must be both true and not true

Paradox of the Strengthened Liar

Consider the proposition "This proposition is not true"

- If every proposition is true, false, or neither, then this proposition must be true, false, or neither
- If it is true, then it is true that it is not true; so it must be both true and not true
- If it is false, then it is not true (since being false is one way of not being true), and since it is not true, and it says that it is not true, it must be true; so it must be both true and not true
- If it is neither true nor false, then it is not true (since being neither true nor false is one way of not being true), and since it is not true, and it says that it is not true, it must be true; so it must be both true and not true
- So in all three cases the proposition is both true and not true

FALSE

true