ECO4170A Game Theory with Applications in Corporate Finance
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Assignment 3: Solution
6 April 2016

13U1. What problems of moral hasard and adverse selection arise in your dealings with each of the following? In each case, outline some appropriate incentive schemes and/or signaling and screening strategies to cope with these problems. No mathematical analysis is expected, but you should state clearly the economic reasoning of why and how your suggested methods work.

(a) Your financial advisor tells you what stocks to buy or sell.
(b) You consult a realtor when you are selling your house.
(c) You visist your doctor, for routine check-ups or treatments.

(a) There is asymmetry of information: the financial advisor knows more about the stocks than you do. Furthermore, his objective is to make as much money (commissions) as possible from telling you what stocks to buy and what stocks to sell. The problem here is moral hasard: because you do not know as much as the financial advisor about the financial market, he might take advantage of your lack of knowledge to tell you to buy or sell stocks that are not in your best interest. A solution to this moral hasard problem is to condition the commission on the profits he earns for you. In this manner, the financial advisor makes more money for himself if he makes more profits for you, and thus has an incentive to give you the best advice.

(b) Because the commission earned by the realtor is a fixed percentage of the price of the house you wants him to sell, he has an incentive to search out the buyer who is willing to offer the highest price for the house. Furthermore, as an expert in the filed, he has more knowledge about the housing market than you do. Thus, his interest and your interest align with each other.

(c) Your doctor knows more about medicine than you do: this is a problem of asymmetric information. He might order more routine check-ups or treatments than necessary to earn more income. The problem here is that of moral hasard, and one solution is to have a second opinion by consulting another doctor about a treatment proposed by your doctor or the optimal number of routine check-ups.

13U2. MicroStuff is a software company that sells two popular applications, WordStuff and ExcelStuff. It does not cost anything for MicroSoft to make each additional copy of its applications. MicroStuff has three types of potential customers, representing by Ingrid, Javiera, and Kathy. There are 100 million potential customers of each type, whose valuations for each application are as follows:

	WordStuff	ExcelStuff
Ingrid	100	20
Javiera	30	100
Kathy	80	0

(a) If MicroStuff sets separate prices for WordStuff and ExcelStuff, what price should it set for each application to maximize profits? How much profit does MicroStuff earn with these prices?

Let p denote the price that MicroStuff sets for WordStuff.

If $p \leq 30$, then Ingrid, Javiera, and Kathy all buy the application, and the profit made by MicroStuff is then given by

(1) $1\,000\,000\,(100\,p + 30\,p + 80\,p)$, $p \leq 30$.

For the range of p that applies to (1), the value of p that maximizes (1) is $p = 30$ in which case the profit earned by the application WordStuff is

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In[1]:= m[1] = 100 000 000 (p + p + p) /. p → 30
Out[1]= 9 000 000 000
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For $30 < p \leq 80$, only Ingrid and Kathy are willing to buy this application, and the profit earned by Microstuff is given by

(2) $1\,000\,000\,(p + p)$, $30 < p \leq 80$.

For the range of p that applies to (2), the value of p that maximizes (2) is $p = 80$, in which case the profit earned by the application WordStuff is

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In[2]:= m[2] = 100 000 000 (p + p) /. p → 80
Out[2]= 16 000 000 000
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For $80 < p \leq 100$, only Ingrid is willing to buy this application, and the profit earned by Microstuff is given by

(3) $100\,000\,000\,(p)$, $80 < p \leq 100$.

For the range of p that applies to (3), the value of p that maximizes (3) is $p = 100$, in which case the profit earned by the application WordStuff is

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In[3]:= m[3] = 100 000 000 (p) /. p → 100
Out[3]= 10 000 000 000
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In[4]:= Max[{m[1], m[2], m[3]}]
Out[4]= 16 000 000 000
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MicroStuff maximizes the profit from the WordStuff application by setting the price of this application to $p = 80$, which yield a profit of 16 000 000 000. At this price, Ingrid and Kathy buy WordStuff.

Let q denote the price that MicroStiuff sets for ExcelStuff.

If $0 < q \leq 20$, only Ingrid and Javiera buy the application, and the profit made by MicroStuff is then given by

(4) $100\,000\,000\,(q + q)$, $0 < q \leq 20$.

For the range of q that applies to (4), the value of q that maximizes (4) is $q = 20$ in which case the profit earned by the application ExcelStuff is

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In[5]:= m[4] = 100 000 000 (q + q) /. q → 20
Out[5]= 4 000 000 000
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If $20 < q \leq 100$, only Javiera buys the application, and the profit made by MicroStuff is then given by

(5) $100\,000\,000\,(q)$, $20 < q \leq 100$.

For the range of q that applies to (5), the value of q that maximizes (5) is $q = 100$, in which case the profit earned by the application ExcelStuff is

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In[6]:= m[5] = 100 000 000 ( q) /. q → 100
Out[6]= 10 000 000 000
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In[7]:= **Max[{m[4], m[5]}]**

Out[7]= 10 000 000 000

MicroStuff maximizes the profit from the ExcelStuff application by setting the price of the application at $q = 100$, which yield a profit of 10 000 000 000.At this price, only Javiera buys ExcelStuff.

(b) What does each type of customer (Ingrid, Javier, and Kathy) buy when MicroStuff sets profit-maximizing separate prices for WordStuff and ExcelStuff?

The following table summarizes the results for the pricing scheme under which the two applicatins are sold separately as two products.

Table 1: MicroStuff sets separate prices for the two applications

	WordStuff ($p = 80$)	ExcelStuff ($q = 100$)	Net Payoff
Ingrid	Yes	No	$100 - 80 = 20$
Javiera	No	Yes	$100 - 100 = 0$
Kathy	Yes	No	$80 - 80 = 0$
MicroStuff	16 000 000 000	10 000 000 000	26 000 000 000

(c) Instead of selling the applications separately, MicroStuff decides always to sell WordStuff and ExcelStuff in a bundle, charging a single price for both. What single price for the bundle would maximize its profit? How much profit does MicroStuff make selling its software only in bundles?

The following table summarizes the utility obtained by each type of customer for the bundle

	Bundle
Ingrid	$100 + 20 = 120$
Javiera	$30 + 100 = 130$
Kathy	$80 + 0 = 80$

If MicroStuff sets the price of the bundle at $b \leq 80$, all the three types of customers will buy the bundle, and MicroStuff maximizes profit in this case by setting $b = 80$ as the price of the bundle to obtain the following profit:

In[8]:= **m[6] = 100 000 000 (3 b) /. b → 80**

Out[8]= 24 000 000 000

If MicroStuff set the price of the bundle at $80 < b \leq 120$, only Ingrid and Javiera will buy the bundle, and MicroStuff maximizes profit in this case by setting $b = 120$ as the price of the bundle to obtain the following profit:

In[9]:= **m[7] = 100 000 000 (b + b) /. b → 120**

Out[9]= 24 000 000 000

If MicroStuff set the price of the bundle $120 < b \leq 130$, only Javiera will buy the bundle, and MicroStuff maximizes profit in this case by setting $b = 130$ as the price of the bundle to obtain the following profit:

In[10]:= **m[8] = 100 000 000 (b) /. b → 130**

Out[10]= 13 000 000 000

The price of the bundle that maximizes MicroStuff's profit is $b = 80$ or $b = 120$, which yield a profit of 24000000000. Although MicroStuff obtains the same profit of 24 000 000 000 with either price, the *lower price* ($b = 80$) i*s, however, better for consumers, and thus, we can assume that MicroStuff, if it bundles the two applicatioins, will set the price of the bundle at* $b = 80$. *At this price, all consumers buy the bundle.*

(d) What does each type of customer buy when MicroStuff sets a single profit-maximizing price for a bundle of WirdStuff and ExcellStuff? How does this compare with the answer in Part (b)?

The following table summarizes the results for the pricing scheme under which the two applicatins are sold separately as two products.

Table 2: MicroStuff bundles the two applications

	Bundle ($b = 80$)	*Net Payoff*
Ingrid	*Yes*	$100 + 20 - 80 = 40$
Javiera	*Yes*	$30 + 100 - 80 = 50$
Kathy	*Yes*	$80 - 80 = 0$
MicroStuff		*24 000 000 000*

A comparison of Table 1 and Table 2 reveals that when MicroStuff sets separate prices for the two applications, each type of consumer buys only one application. When MicroStuff bundles the two applications all consumers buy the bundle and use both applications.

(e) Which pricing scheme does each consumer type prefer? Why?

The following table summarizes the payoffs for the three consumer types under both schemes: <<sell the two applications as separate products>> and << bundle the two applications>>.

Table 3: Payoffs under each scheme

	Payoff under separate prices scheme ($p = 80$, $q = 100$)	*Payoff with bundle scheme (b=80)*
Ingrid	*20*	*40*
Javiera	*0*	*50*
Kathy	*0*	*0*

According to Table 3, both Ingrid and Javiera are strictly better-off under the bundle scheme while Kathy is indifferent between the two schemes.

(f) If MicroStuff sold the applications both as a bundle and separately, which products (WordStuff, ExcelStuff, or the bundle) would it want to sell to each customer type? How can MicroStuff make sure that each customer type purchases exactly the product that it intends for this type to purchase.

(g) What prices - for WordStuff, Excel, and the bundle - would MicroStuff set to maximize profit? How much profit does MicroStuff make selling selling the products at hese prices?

Answers to (f) and (g) are given together here.

First, note that Kathy has a valuation of 80 for WordStuff, but 0 for ExcelStuff. Thus, Kathy will not buy ExcelStuff. It is then reasonable to expect that MicroStuff wants to sell WordStuff to Kathy, and it should set the price of WordStuff at $p = 80$.

Second, note that if Ingrid buys WordStuff at the price $p = 80$, her net payoff is $100 - 80 = 20 > 0$. To induce Ingrid to buy ExcelStuff, but not WordStuff, MicroStuff must set the price of ExcelStuff at $q = 0$ to give Ingrid the same payoff as that she obtains from buyng WordStuff. Such an action ($q = 0$) seems unreasonable. So, we expect that MicroStuff will try to sell the bundle to Ingrid. If this choice is taken, the price of the bundle must give Ingrid a net payoff of 20, and this means MicroStuff must set the price of the bundle at $b = 100 - \epsilon$, with ϵ being a very small positive number. What remains is to set the price for ExcelStuff. If the price of ExcelStuff is set at $q = 70 - 2\epsilon$, Javiera will obtain a net payoff of $100 - (70 - 2\epsilon) = 30 + 2\epsilon$ by buying ExcelStuff, and a net payoff of $130 - (100 - \epsilon) = 30 + \epsilon$ by buying the bundle, and we can presume that she will buy ExcelStuff if MicroStuff sets the price of Excell at $q = 70 - 2\epsilon$. The profit made by MicroStuff if it carries out this policy is

$$100\,000\,000\,(80 + 70 + 100) = 25\,000\,000\,000.$$

The following table summarizes the results of the scheme under which WordStuff, ExcelStuff, and the bundle are sold as three distinct products.

Table 4: WorStuff, ExcelStuff, and the bundle as three distinct products

	WordStuff ($p = 80$)	ExcelStuff ($q = 70$)	Bundle ($b = 100$)	Payoff
Ingrid			Yes	100 + 20 − 100 = 20
Javiera		Yes		100 − 70 = 30
Kathy	Yes			80 − 80 = 0
MicroStuff				25 000 000 000

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In[11]:= m[9] = 100000000 (80 + 70 + 100)

Out[11]= 25000000000
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(h) How do the answers to Parts (a), (c), and (g) differ? Explain why.

The profit made by MicroStuff is highest (26 000 000 000) when it sells the two applications as two products. Its profit is 24 000 000 000 when it bundles the two applications. When it sell WordStuff, ExcelStuff, and the bundle as three distinct products, it makes 25 000 000 000 in profit. These results suggest that when MicroStuff sets separate prices for WordStuff and ExcelStuff it is able to exploit all the three types of customers and induce each type to purchase exactly one product. Its profit falls to 24 000 000 000 when it bundles the two applications and must set a low price ($b = 80$) to induce all types into buying the bundle. Bundling the two applications reducs the flexibility of its pricing scheme. When it offers three products: WordStuff, ExcelStuff, and the bundle, it has a more flexible pricing scheme than just offering the bundle, and MicroStuff's profit rises from 24 000 000 000 to 25 000 000 000. However, this last pricing scheme still falls short of the one under which the two applications are priced separately.

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In[12]:= ClearAll[m]
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13U3. Consider a managerial effort example similar to the one in Section 5. The value of a successful project is \$420000; the probability of success are $\frac{1}{2}$ with good supervision and $\frac{1}{4}$ without. The manager is risk neutral, not risk averse as in the text, so his expected utility equals his expected income minus his disutility of effort. He can get other jobs paying \$90000, and his disutility for exerting the extra effort for good supervision on your project is \$100 000.

(a) Show that inducing high effort would require the firm to offer a compensation scheme with negative

base salary; that is, if the project fails, the manager pays the firm an amount stipulated in the scheme.
(b) How might a negative base salary be implemented in reality?
(c) Show that if a negative base salary is not feasible, then the firm does better to settle for the low-pay low-effort situation.

(a) *Let x be the amount that the manager is paid if the project is a success, and y be the amount that the manager is paid if the project is a failure. The expected utility obtained by the manager if he signs then contract, and then exerts the extra effort is*

In[13]:= $u[1] = \frac{1}{2} x + \frac{1}{2} y - 100\,000$

Out[13]= $-100\,000 + \frac{x}{2} + \frac{y}{2}$

In order for the manager to sign the contract, u[1] *must yield at least the reservation utility of 90000. That is, the following participation constraint must hold:*

In[14]:= $c[1] = u[1] - 90\,000 \geq 0$

Out[14]= $-190\,000 + \frac{x}{2} + \frac{y}{2} \geq 0$

If the manager signs the contract, and then shirks, his expected utility is

In[15]:= $u[2] = \frac{1}{4} x + \frac{3}{4} y$

Out[15]= $\frac{x}{4} + \frac{3 y}{4}$

In order for the manager to sign the contract, and then exerts the extra effort, the following incentive compatibility constraint must hold

In[16]:= $c[2] = u[1] - u[2] \geq 0$

Out[16]= $-100\,000 + \frac{x}{4} - \frac{y}{4} \geq 0$

If the two constraints c[1] and *c*[2] *hold, then the manager will sign the contract, and then exerts the extra effort. Your problem is to find (x, y) that satisfies the two constraints to maximize your expected payoff. More specifically, you solve the following constrained maximization problem:*

(1) $\max_{(x,y)} \frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y)$

subject to

(2) $-190\,000 + \frac{x}{2} + \frac{y}{2} \geq 0,$

(3) $-100\,000 + \frac{x}{4} - \frac{y}{4} \geq 0.$

Mathematica gives the following solution of the preceding constrained maximization problem:

In[17]:= $\text{Maximize}\left[\left\{\frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y), -190\,000 + \frac{x}{2} + \frac{y}{2} \geq 0, -100\,000 + \frac{x}{4} - \frac{y}{4} \geq 0\right\}, \{x, y\}\right]$

Out[17]= $\{20\,000, \{x \to 390\,000, y \to -10\,000\}\}$

Observe that the optimal compensation scheme requires the manager to pay the owner $10000 if the project is a failure. Also, note that the expected payoff for the owner is 20 000.

The constrained maximization problem can be solved by hand in the following manner.

Note that the participation constraint (2) is either strict or holds with equality. If (2) is strict, and if (3) is also strict, then the owner can reduce x slightly to increase his expected profit without violating either of the two constraints. Thus, one constraint must hold with equality at the optimum, i.e., either (2) or (3) holds with equality.

Suppose first that (2) holds with equality, i.e.,

(4) $-190\,000 + \frac{x}{2} + \frac{y}{2} = 0 \Rightarrow y = 380\,000 - x.$

Using the expression of $y = 380\,000$ *in (3), we obtain*

(5) $-100\,000 + \frac{x}{4} - \frac{380\,000 - x}{4} \geq 0,$

or

(6) $-400\,000 + x - 380\,000 + x \geq 0 \Rightarrow x \geq 390\,000.$

Using $y = 380\,000 - x$ *in the objective function, we obtain*

(7) $\frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y) = \frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - 380\,000 + x) = 20\,000.$

Thus, the value of x that satisfies the constraint $x \geq 390\,000$ *and that maximizes (7) is* $x = 390\,000$. *The value of y is then given by* $y = 380\,000 - x = 380\,000 - 390\,000 = -10\,000$. *This is the solution given by Mathematica.*

Next, suppose that (3) holds with equality, i.e.,

(8) $-100\,000 + \frac{x}{4} - \frac{y}{4} = 0 \Rightarrow y = -400\,000 + x.$

Using this expression for y in (2), we obtain

(9) $-190\,000 + \frac{x}{2} + \frac{-400\,000 + x}{2} \geq 0 \Rightarrow x \geq 390\,000.$

Using the expression $y = -400\,000 + x$ *in the objective function (1), we obtain*

(10) $\frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y) = \frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 + 400\,000 - x) = 410\,000 - x.$

The value of x that maximizes (10) under the constraint $x \geq 390\,000$ *is x=390000. Using* $x = 390\,000$ *in (8), we obtain* $y = -10\,000$. *We obtain the same solution in this case.*

We have just shown by hand that the solution of the constrained maximization problem for the owner is $x = 390\,000$, $y = -20\,000$, *which is the solution given by Mathematica.*

(b) Accordin to the answer to Part (a), the net payoff for the owner is 20000. Thus, to implement a negative base salary in reality, the owner can sell the project to the manager for a price of 20000. The manager is willing to buy the project, and then exerts the extra effort to carry out the project. His expected payoff is then given by

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In[18]:= -20 000 - 100 000 + 1/2 (420 000) + 1/2 (0)

Out[18]= 90 000
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which is equal to his reservation utility.

(c) Let $y \geq 0$ *be given. In order for the manager to exert the extra effort, x must satisfy the following incentive compatibility constraint:*

$$\frac{1}{2}x + \frac{1}{2}y - 100\,000 \geq \frac{1}{4}x + \frac{3}{4}y$$

$$\frac{1}{4}x \geq 100\,000 + \frac{1}{4}y.$$

(11) $x \geq 400\,000 + y$

Also, in order for the manager to sign the contract, the following participation constraint must be satisfied:

$\frac{1}{2}x + \frac{1}{2}y - 100\,000 \geq 90\,000$

(12) $x \geq 380\,000 - y$

Because we are imposing the constraint $y \geq 0$, the inequality (12) is certainly satisfied if the inequality (11) is satisfied, i.e., (12) is redundant when $y \geq 0$. The expected payoff of the owner is then given by

(13) $\frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y) = 210\,000 - \frac{x}{2} - \frac{y}{2}$.

Observe that (13) is strictly decreasing in x. Thus, for a given $y \geq 0$, the value of $x \geq 400\,000 + y$ that maximizes (13) is subject to the inequality (11) is

(14) $x = 400\,000 + y$.

Using the value of x, given by (14), we can reduce (13) to

(15) $\frac{1}{2}(420\,000 - x) + \frac{1}{2}(0 - y) = 210\,000 - \frac{400\,000+y}{2} - \frac{y}{2} = 10\,000 - y$.

The value of $y \geq 0$ that maximize (15) is then

(16) $y = 0$.

Using (16) in (14), we obtain

(17) $x = 400\,000$.

The maximum expected payoff that the owner can expect if he designs a contract that the manager is willing to sign and thenexerts the extra effort is then given by

(18) $\frac{1}{2}(420\,000 - 400\,000) + \frac{1}{2}(0 - 0) = 10\,000$.

If the owner offers to pay the manager 90000, regardless of the outcome, then the manager will sign the contract and then shirk. The expected payoff for the owner is then given by

(19) $\frac{1}{4}(420\,000) - 90\,000 = 15\,000 > 10\,000$.

That is, the owner is better-off offering the manager a low-effort contract.

In[19]:= $\frac{1}{4}$ **420 000 - 90 000**

Out[19]= 15 000

13U4. Cheapskates is a very minor-league professional hockey team. Its facility is large enough to accommodate all of the 1000 fans who might want to watch its home games. It can provide two types of seats- ordinary and luxury. There are also two sorts of fans: 60% of the fans are blue-collar fans, and the rest are white-collar fans. The costs of providing each type of seat and the fans' willingness to pay for each type of seat are given in the following table (measured in dollars):

			Willingness to Pay	
		Cost	Blue-Collar	White-Collar
Seating	Ordinary	4	12	14
	Luxury	8	15	22

Each fan will buy at most one seat, depending on the consumer surplus he would get (maximum willingness to pay minus the actual price paid) from the two kinds. If the surplus for both is negative, then he won't buy any. If at least one kind gives him non-negative surplus, then he will buy the kind that gives him the larger surplus. If the two kinds give him equal non-negative surplus, then the blue-collar fan will buy the ordinary kind of seat, and the withe-collar fan will buy the luxury kind.

The team owners provide and price their seating to maximize profit., measured in thousands of dollars per game. They set prices for each kind of seats, sell as many tickets as are demanded at these prices and then provide the numbers and types of seats of each kind for tickets have sold.

(a) First, suppose that the team owners can identify the type of each individial fan who arrives at the ticket window (presumably bythe color of his collar) and can offer him just one type of seat at a stated price on a take-it-or-leave-it basis. What is the owners' maximum profit π^* under this system?

(a) For a blue-collar worker, he is willing to pay 12 for an ordinary seat, and thus, the owners can sell an ordinary seat to a blue-collar worker at the price of 12 and make a profit of $12 - 4 = 8$. The owners can also sell a luxury ticket at the price of 15 and make a profit of $15 - 8 = 7 < 8$. Thus, it is better for the owners to sell an ordinary seat to a blue-collar fan and earn a profit of 8.

A white-collar worker is willing to pay 14 for an ordinary seat, and thus the owners can sell an ordinary seat to a white-collar fan to earn a profit of $14 - 4 = 10$. The owners can also sell a luxury seat to a white-collar fan at the price 22 and make a profit of $22 - 8 = 14 > 10$. Thus, the owners will sell a white-collar fan a luxury seat at the price 22.

The owners' maximum profit is

(1) $\pi^* = 0.6 \times 1000 \times (12 - 4) + 0.4 \times 1000 \times (22 - 8) = 4800 + 5600 = 10\,400.$

(b) Now suppose that the owners cannot identify any individual fan, but they still know the proportion of blue-collar fans. Let the price of an ordinary seat be X and the price of a luxury seat be Y. What are the incentive compatibility constraints that will ensure the blue-collar fans buy the ordinary seats and the white-collar fans buy the luxury seats? Graph these constraints on an (X, Y) – coordinate plane.

(b) If a blue-collar fan buys an ordinary seat, his surplus id $12 - X$. If he buys a luxury seat, his surplus is $15 - Y$. Thus, he will buy an ordinary seat if

$$12 - X \geq 15 - Y$$

or

(2) $Y \geq X + 3.$

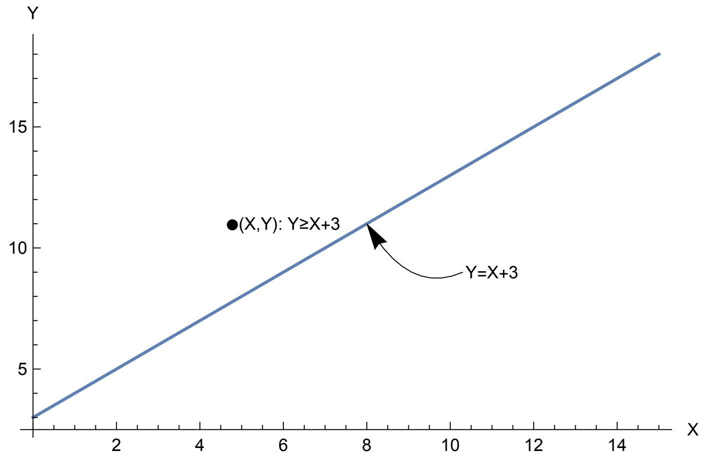
If a white-collar fan buys an ordinary seat, his surplus id $14 - X$. If he buys a luxury seat, his surplus is $22 - Y$. Thus, he will buy an ordinary seat if

$$14 - X \leq 22 - Y$$

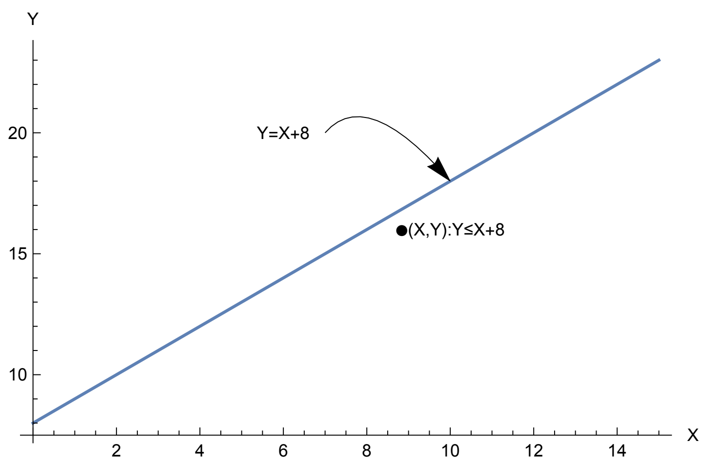
or

(3) $Y \leq X + 8.$

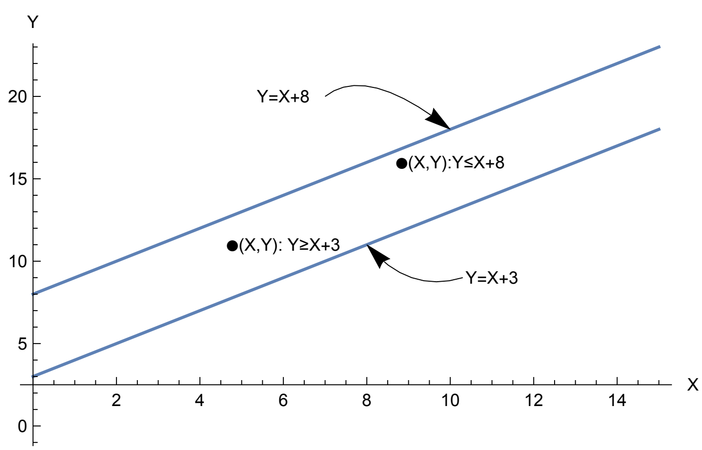
The incentive compatibility constraint (2) is depicted in the following figure. In the figure, the points (X, Y) that satisfy this constraints lie on or above the straight line $Y = X + 3$.



The incentive compatibility constraint (3) is depicted in the following figure. In the figure, the points (X, Y) *that satisfy this constraints lie on or below the straight line* $Y = X + 8$*.*



When the second figure is superposed on the first figure, the region lying above the straight line $Y = X + 3$ *and below the straight line* $Y = X + 8$ *constitutes all the points* (X, Y) *that satisfy the two incentive compatibility constraints (2) and (3).*



(c) What are the fans' participation constraints for the fans' decisions on whether to buy tickets at all? Add these constraints to the grap in part (b).

The participation constraint for a blue-collar fan is

(4) $\quad 12 - X \geq 0$.

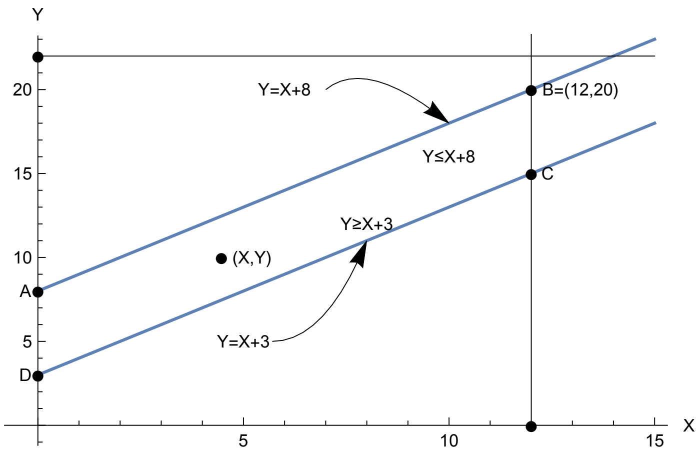
The points (X, Y) *that satisfy the participation constraint (4) are on or to the left of the vertical line with horizontal coordinate* $x = 12$.

The participation constraint for a white-collar fan is

(5) $\quad 22 - Y \geq 0$.

The points (X, Y) *that satisfy the participation constraint (4) are on or below the horizontal line with vertical coordinate* $Y = 22$.

When the two incentive constraints (2) and (3) and the two participation constraints (4) and (5) are represented in the same figure, we obtain



(d) Given the constraints in (b) and (c), what prices X and Y maximize the owners' profit π_2 under this price system? What is the value of π_2?

The points (X, Y) that satisfy the constraints (2)-(5) are inside or on the boundary of the parallelogram ABCD. The owners solve the following constrained profit maximization problem

(6) $\quad \pi_2 = \max_{(X,Y) \in ABCD} 0.6 \times 1000\, X + 0.4 \times 1000\, Y$.

To sove (6) first note that the objective function is increasing in X and Y. Second, note that if a point (X, Y) lies inside or on the boundary of the paralleogram ABCD, we must have (X, Y) ≤ (12, 20), where $B = (12, 20)$ is upper right vertex of the parallelogram ABCD. Thus, profit is maximized at (X, Y) = (12, 20)., and we have

(7) $\quad \pi_2 = 600\,(X - 4) + 400\,(Y - 8) = 600 \times (12 - 4) + 400 \times (20 - 8) = 9600$.

```
In[20]:= 0.6 × 1000 (X - 4) + 0.4 × 1000 (Y - 8) /. {X → 12, Y → 20}

Out[20]= 9600.
```

(e) The owners are considering whether to set prices so that only the white-collar fans will buy tickets. What is theor profit π_w if they decide to cater to only white-collar fans?

If the owners decide to cater only to the white-collar fans, then the hiighest price that they can set for a ordinary ticket is 14. At this price, the blue-collar worker will not buy an ordinary ticket because their valuation of an ordinary seat is only 12. In this case, the profit made by the owners is $400 \times (14 - 4) = 4000$. *On the other hand, if the owners try to cater only to white-collar fans and set the price of a luxury seat at the highest price possible of 22, then no blue-collar fan whos values a luxury seat at 15 will buy such a ticket, and the profit they make is $400 \times (22 - 8) = 5600$.*

Thus, if the owners decide to cater only to white-collar fans, then they shouls only sell these fans luxury seats ant the price of 22 and make a profit of $\pi_w = 5600$.

In[21]:=
```
400 (22 - 8)
```

Out[21]= 5600

(f) Comparing π_2 and π_w, determine the pricing policy that the owners will set. How does their profit achieved from this policy compare with the case of full information, where they earn π^*?

Because $\pi_w = 5600 < \pi_2 = 9600$*, the owners will serve both types of fans and set the price of an ordinary ticket at 12 and the price of a luxury ticket at 22.*

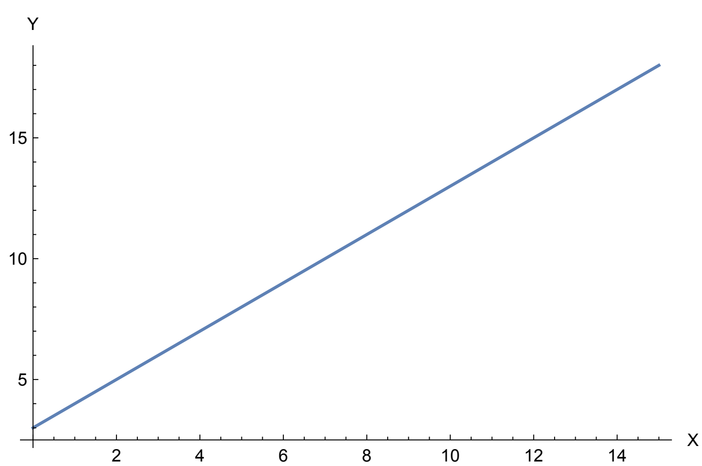
(g) What is the cost of coping with the information asymmetry in part f? Who beats this cost? Why?

Under full information, the profit made by the owners is $\pi^* = 10400$. *Under asymmetric information the profit is* $\pi_2 = 9600$*. Thus, the cost of coping with the information asymmetry for the owners is* $10400 - 9600 = 800$.

Because blue-collar fans pay 12 for an ordinary seat under full information as well as under asymmetric information. They neither win nor lose with asymmetric information. However, for white-collar fans, they pay 22 for a luxury seat under full information, but only 20 under asymmetric information. Thus, white-collar gain with asymmetric information and collectively this grou gains $(22 - 20)\,400 = 800$*, the same loss suffered by the owners. There is a transfer of 800 from the owners to the white-collar fans as a group of 800 due to information asymmetry.*

In[22]:=
```
g[1] = Plot[X + 3, {X, 0, 15}, AxesLabel → {"X", "Y"}]
```

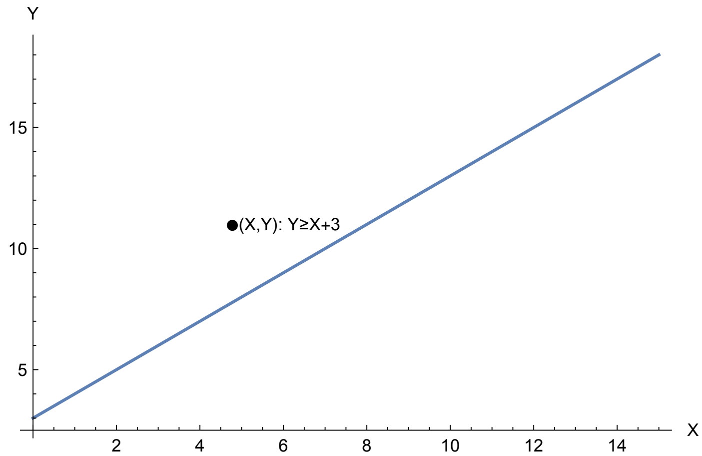
Out[22]=



In[23]:=
```
g[1, 1] = Graphics[Text["●(X,Y): Y≥X+3", {6, 11}]];
```

In[24]:= `Show[g[1], g[1, 1]]`

Out[24]=

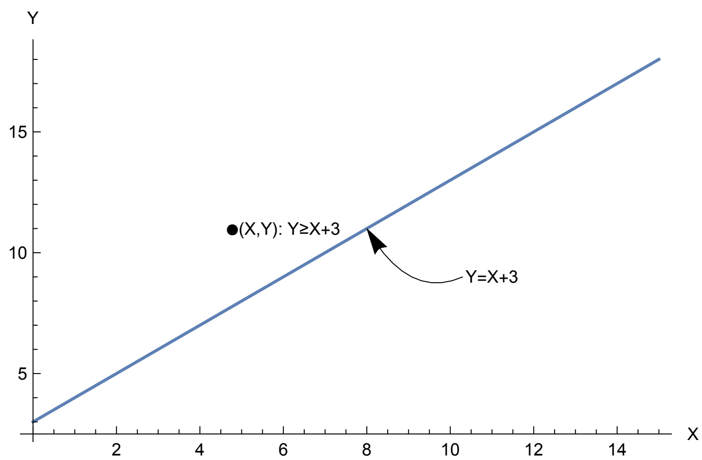


In[25]:= `g[1, 2] = Graphics[Text["Y=X+3", {11, 9}]];`

In[26]:= `g[1, 3] = Graphics[Arrow[BezierCurve[{{10.3, 9}, {9, 8}, {8, 11}}]]];`

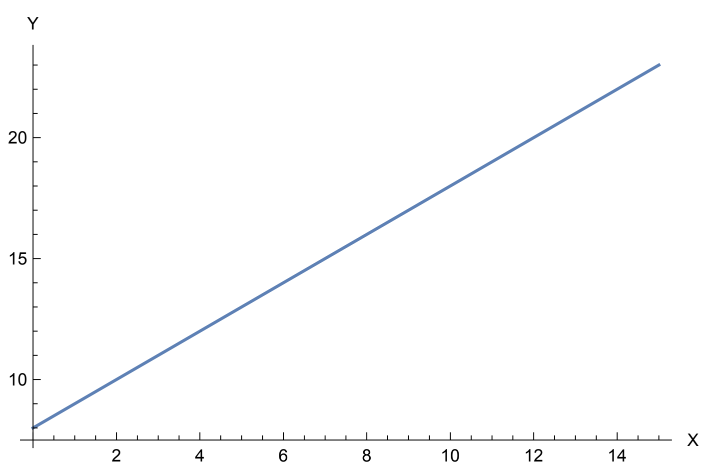
In[27]:= `Show[g[1], g[1, 1], g[1, 2], g[1, 3]]`

Out[27]=



In[28]:= `g[2] = Plot[X + 8, {X, 0, 15}, AxesLabel → {"X", "Y"}]`

Out[28]=



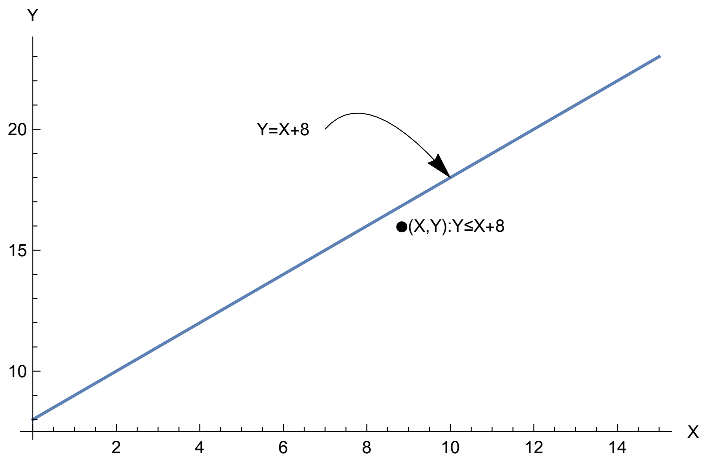
In[29]:=
```
g[2, 1] = Graphics[Text["Y=X+8", {6, 20}]];
```

In[30]:=
```
g[2, 2] = Graphics[Text["●(X,Y):Y≤X+8", {10, 16}]];
```

In[31]:=
```
g[2, 3] = Graphics[Arrow[BezierCurve[{{7, 20}, {8, 22}, {10, 18}}]]];
```

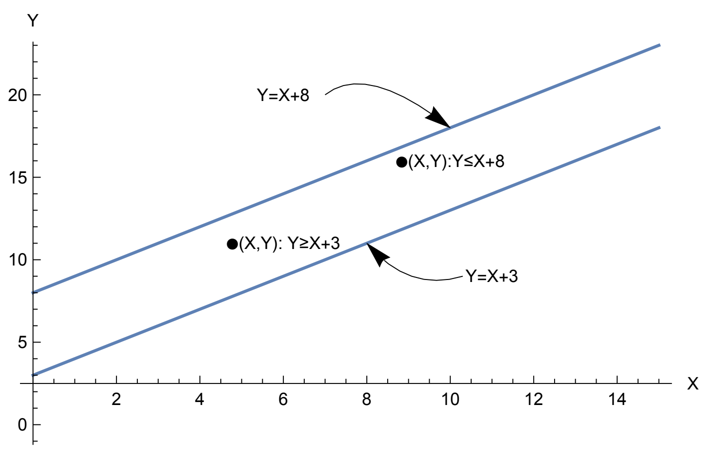
In[32]:=
```
Show[g[2], g[2, 1], g[2, 2], g[2, 3]]
```

Out[32]=



In[33]:=
```
Show[g[1], g[1, 1], g[1, 2], g[1, 3], g[2],
 g[2, 1], g[2, 2], g[2, 3], PlotRange → {{0, 15}, {0, 22}}]
```

Out[33]=

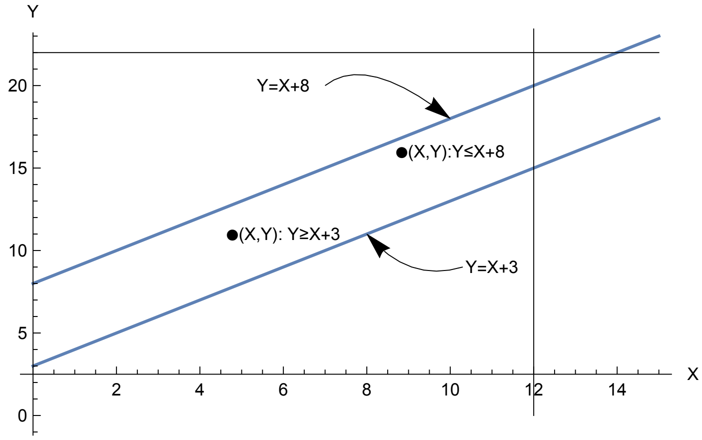


In[34]:=
```
g[3] = Graphics[Line[{{12, 0}, {12, 25}}]];
```

In[35]:=
```
g[4] = Graphics[Line[{{0, 22}, {15, 22}}]];
```

In[36]:=
```
Show[g[1], g[1, 1], g[1, 2], g[1, 3], g[2], g[2, 1],
 g[2, 2], g[2, 3], g[3], g[4], PlotRange → {{0, 15}, {0, 22}}]
```

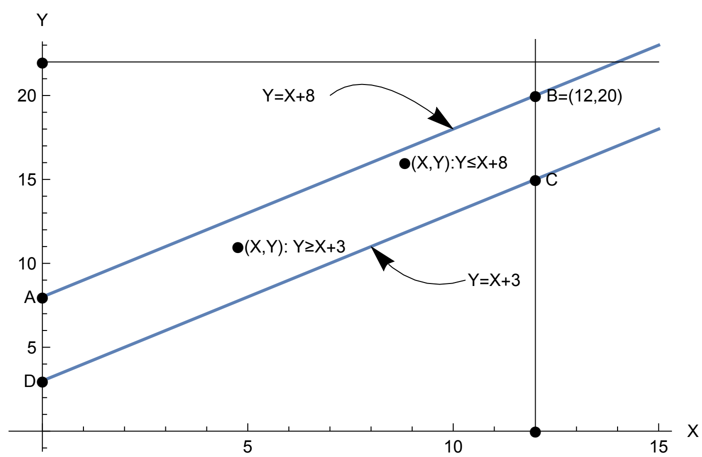
Out[36]=




```
In[37]:= g[5] = Graphics[Text["●", {12, 15}]];
In[38]:= g[6] = Graphics[Text["●", {12, 20}]];
In[39]:= g[7] = Graphics[Text["●", {0, 3}]];
In[40]:= g[8] = Graphics[Text["●", {0, 8}]];
In[41]:= g[9] = Graphics[Text["A", {-0.3, 8}]];
In[42]:= g[10] = Graphics[Text["D", {-0.3, 3}]];
In[43]:= g[11] = Graphics[Text["B=(12,20)", {13.2, 20}]];
In[44]:= g[12] = Graphics[Text["C", {12 + 0.4, 15}]];
In[45]:= g[13] = Graphics[Text["●", {12, 0}]];
In[46]:= g[14] = Graphics[Text["●", {0, 22}]];
```

In[47]:=
```
Show[g[1], g[1, 1], g[1, 2], g[1, 3], g[2], g[2, 1], g[2, 2],
 g[2, 3], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11], g[12],
 g[13], g[14], AxesOrigin → {0, 0}, PlotRange → {{-0.5, 15}, {0, 22}}]
```

Out[47]=

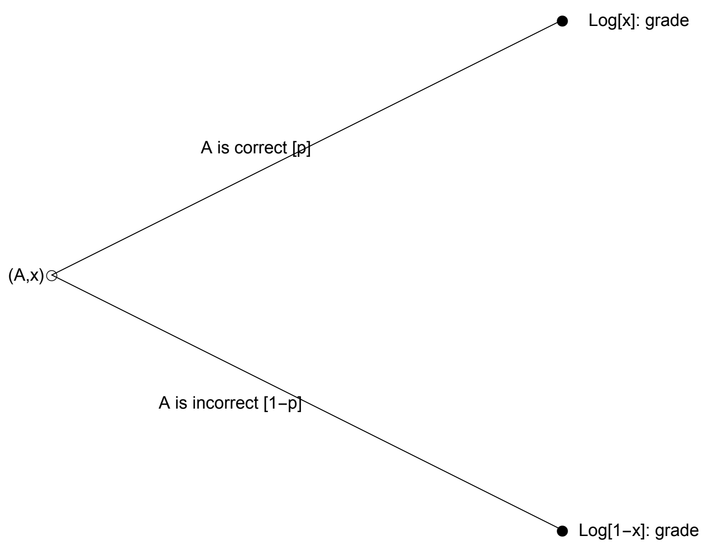


In[48]:=
```
ClearAll[g]
```

13U9. A teacher wants to find out how confident the students are about their own abilities. He proposes the following scheme: <<After you answer this question, state your estimate of the probability that you are right. I will then check your answer to the question. Suppose you have given the probability estimate *x*. If your answer is actually correct, your grade will be Log[x]. If incorrect, it will be Log[1-x].>> Show that this scheme will elicit the students' own truthful estimates - that is if the truth is *p*, show that a student's stated estimate is $x = p$.

Let p denote the probabilty that a student answer the question correctly. The probability p comes from the student's own experience in problem solving, and he knows exactly the percentage of the answers that are correct. However, the teacher does not know the exact value of p. The probability p is a given parameter, and can be interpreted as the type of the students. If another person asks the student to reveal the value of p, the student might exaggerate p to make himself look good. Or the student might understate p to simulate ignorance. We want to show that the scheme designed by the teacher will induce the student, who only cares about his own interest (his own grade in this case) to reveal his type truthfully.

Let A be the answer given by the student and x be the probability that he states at the end of the answer describing how confident he is about the chance that his answer is correct. The ordered pair (A, x) represents the anser given by the student and her stated of probability that A is correct.



Because the answe is correct with the true probability p, and incorrect with probability 1 − p, the expected grade for the student, as a function of x, the probability that he claims the answer A is correct, is given by

(1) $u[x] = p\,Log[x] + (1 - p)\,Log[1 - x]$.

The sudent's objective is to choose x to maximize (1). The following first-order condition characterizes the optimal value of x :

(2) $u'[x] = \frac{p}{x} - \frac{(1-p)}{1-x} = 0.$

The value of x that satisfies the preceding first-order condition is x = p. That is, the student will state truthfully the probability that he answers the question correctly.

In[49]:=
```
g[0] = Graphics[Line[{{2, 1}, {0, 0}, {2, -1}}]];
```

In[50]:=
```
g[1] = Graphics[Text["○", {0, 0}]];
```

```
In[51]:= g[2] = Graphics[Text["●", {2, -1}]];
In[52]:= g[3] = Graphics[Text["●", {2, 1}]];
In[53]:= g[4] = Graphics[Text["(A,x)", {-0.1, 0}]];
In[54]:= g[5] = Graphics[Text["A is correct [p]", {1 - 0.2, 0.5}]];
In[55]:= g[6] = Graphics[Text["A is incorrect [1-p]", {1 - 0.3, -0.5}]];
In[56]:= g[7] = Graphics[Text["Log[x]: grade", {2 + 0.3, 1}]];
In[57]:= g[8] = Graphics[Text["Log[1-x]: grade", {2 + 0.3, -1}]];
In[58]:= Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8]]
```

Out[58]=