ECO4170A Game Theory with Applications in Corporate Finance
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Assignment 2: Solution
16 March 2016

8U2. Consider again the case of the 2011 Citrus. Almost all cars depreciate over time, and so it is with the Citrus. Every month that passes, all sellers of Citrus - regardless of type - are willing to accept $100 less than they were the month before. Also, with every passing month, buyers are maximally willing to pay $400 less for an orange than they were the previous month and $200 less for a lemon. Assume that the example in the text takes place in month 0. Eighty percent of the Citruses are oranges, and this proportion never changes.

(a) Fill out three versions of the following table for month 1, month 2, and month 3.

	Willingness to accept of sellers	Willingness to pay of buyers
Oranges		
Lemons		

(b) Graph the willingness to accept of the sellers of oranges over the next 12 months. On the same figure, graph the price that buyers are willing to pay for a Citrus of unknown type (given that the proportion of oranges is 0.8). (Hint: Make the vertical axis range fro 10 000 to 14 000.)
(c) Is there a market for oranges in month 3? Why or why not?
(d) In what month does the market for oranges collapse?
(e) If owners of lemons never experienced depreciation, i.e., they were never willing to accept anything less than $3000, would this affect the timing of the collapse of the market for oranges? Why or why not? In what month does the market for oranges collapse in this case?
(f) If buyers experienced no depreciation for a lemon, i.e., they weree always willing to pay up to $6000 for a lemon, would this affect the timing of the collapse of the market for oranges? Why or why not? In what month does the market for oranges collapse in this case?

(a) Fill out three versions of the following table for month 1, month 2, and month 3.

Month 0

	Willingness to accept of sellers	*Willingness to pay of buyers*
Oranges	*$12,500*	*$16,000*
Lemons	*$3,000*	*$6,000*

Month 1

	Willingness to accept of sellers	*Willingness to pay of buyers*
Oranges	*$12,400*	*$15,600*
Lemons	*$2,900*	*$5,800*

Month 2

	Willingness to accept of sellers	*Willingness to pay of buyers*
Oranges	*$12300*	*$15,200*
Lemons	*$2,800*	*$5,600*

Month 3\.10

	Willingness to accept of sellers	*Willingness to pay of buyers*
Oranges	*$12,200*	*$14,800*
Lemons	*$2,700*	*$5,400*

(b) Graph the willingness to accept of the sellers of oranges over the next 12 months. On the same figure, graph the price that buyers are willing to pay for a Citrus of unknown type (given that the proportion of oranges is 0.8). (Hint: Make the vertical axis range fro 10 000 to 14 000.).

Let t denote the month. For t = 0, 1, ..., 12, the willingness to accept of the seller of an orange is

```
In[1]:= a[t_] := 12 500 - 100 t
```

For month 0, the willingness to accept of the seller of an orange is

```
In[2]:= a[0]
Out[2]= 12 500
```

For month 1, the willingness to accept of the seller of an orange is

```
In[3]:= a[1]
Out[3]= 12 400
```

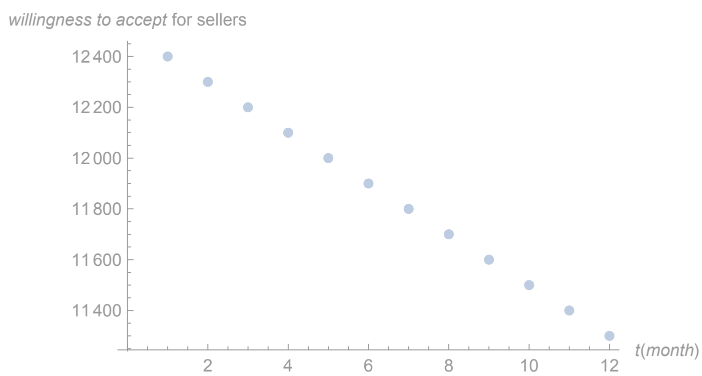
For month 2, the willingness to accept of the seller of an orange is

```
In[4]:= a[2]
Out[4]= 12 300
```

The willingness to accept of the seller of an orange for the next 12 months is

```
In[5]:= aa = Table[a[i], {i, 1, 12}]
Out[5]= {12 400, 12 300, 12 200, 12 100, 12 000,
  11 900, 11 800, 11 700, 11 600, 11 500, 11 400, 11 300}

g[1] = ListPlot[aa, PlotStyle → PointSize[0.02],
  AxesLabel → {"t(month)", "willingness to accept for sellers"}]
```

Out[6]=



For a buyer, the willingness to pay for an orange in month t is

```
In[7]:= b[1, t_] := 16 000 - 400 t
```

For a buyer, the willingness to pay for an orange in month 0 is

```
In[8]:= b[1, 0]
Out[8]= 16 000
```

For a buyer, the willingness to pay for an orange in month 1 is

In[9]:=
```
b[1, 1]
```
Out[9]= 15 600

For a buyer, the willingness to pay for an orange in month 2 is

In[10]:=
```
b[1, 2]
```
Out[10]= 15 200

For a buyer, the willingness to pay for a lemon in month t is

In[11]:=
```
b[2, t_] := 6000 - 200 t
```

For a buyer, the willingness to pay for a lemon in month 0 is

In[12]:=
```
b[2, 0]
```
Out[12]= 6000

For a buyer, the willingness to pay for a lemon in month 1 is

In[13]:=
```
b[2, 1]
```
Out[13]= 5800

For a buyer, the willingness to pay for a lemon in month 2 is

In[14]:=
```
b[2, 2]
```
Out[14]= 5600

For a buyer, who does not know the type of car he buys, the probability of obtaining an orange on the market is

In[15]:=
```
f = 0.8
```
Out[15]= 0.8

If he buys a car of unknown type in month t, then the expected value of the car he buys is

In[16]:=
```
c[t_] := f b[1, t] + (1 - f) b[2, t]
```

In[17]:=
```
c[t]
```
Out[17]= 0.8 (16 000 - 400 t) + 0.2 (6000 - 200 t)

If he buys a car of unknown type in month 0, then the expected value of the car he buys is

In[18]:=
```
c[0]
```
Out[18]= 14 000.

If he buys a car of unknown type in month 1 then the expected value of the car he buys is

In[19]:=
```
c[1]
```
Out[19]= 13 640.

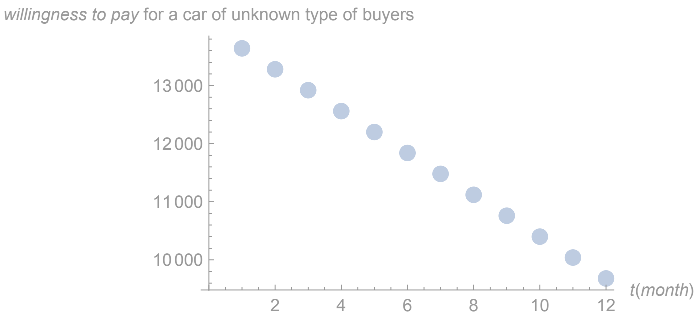
The price that buyers are willing to pay for a car of unknown type in the next 12 months is given by

In[20]:=
```
cc = Table[c[t], {t, 1, 12}]
```

Out[20]= {13 640., 13 280., 12 920., 12 560., 12 200., 11 840., 11 480., 11 120., 10 760., 10 400., 10 040., 9680.}

```
g[2] = ListPlot[cc, PlotStyle → PointSize[0.04], AxesLabel →
   {"t(month)", "willingness to pay for a car of unknown type of buyers"}]
```

Out[21]=



The willingness to accept for a seller of oranges and the willingness to pay for a car of unknown type in the next 12 months are shown together in the same figure

```
g[3] = Show[g[1], g[2], AxesLabel → {"t(month)", "willingness to accept for sellers
willingness to pay for a car of unknown type of buyers"},
  PlotRange → {{0, 12}, {10 000, 14 000}}, AspectRatio → 1]
```

Out[22]=

Note that in the preceding figure the thicker dots represent the willngness to pay for a car of unknown type of a buyer.

(c) In month 3, the willingness to accept for a seller of oranges is

In[23]:=
```
a[3]
```

Out[23]= 12 200

while the willingness to pay for a buyer of a car of unknown type is

In[24]:=
```
c[3]
```

Out[24]= 12 920.

Because, c[3] = 12 920. > a[3] = 11 700, i.e., because the willingness to pay for a car of unknown type exceeds the willingness to accept for a seller of an orange, any price of a car of unknow type, say p, *between 11700 and 12900 will induce the owner of an orange to offer his car for sale and a buyer of a car of unknown type to buy a car of unknown type on the market. Thus, there is a market for oranges in month 3.*

Note that the willingness to accept of the owner of a lemon is

In[25]:=
```
b[2, 3]
```
Out[25]= 5400

which is lower than p, and thus lemons will also be offered for sale on the market.

The graph depicting the willingness to accept for sellers of oranges and the willingness to pay for a car of unknown type indicates that the former dips below the latter in month 6. Thus, the market for oranges will collapse in month 6.

Indeed, the willingness to accept for sellers of oranges and the willingness to pay for a car of unknown type iin month 5 are, respectively,

In[26]:=
```
a[5]
```
Out[26]= 12 000

In[27]:=
```
c[5]
```
Out[27]= 12 200.

while the willingness to accept for sellers of oranges and the willingness to pay for a car of unknown type iin month 6 are, respectively,

In[28]:=
```
a[6]
```
Out[28]= 11 900

In[29]:=
```
c[6]
```
Out[29]= 11 840.

and $a[6] = 11\,900 > c[6] = 11\,840$.

(e) If owners of lemons never experienced depreciation, i.e., they were never willing to accept anything less than $3000, would this affect the timing of the collapse of the market for oranges? Why or why not? In what month does the market for oranges collapse in this case?

No, this will not affect the timing of the market for oranges. The reason is simple. The willingness to pay for a car of unknown type for the next 12 months remains the same. The willingness to accept for owners of oranges for the next 12 months also remains the same. All of this means that the preceding figure still remains the same, i.e., the market for oranges still collapses in month 6.

(f) If buyers experienced no depreciation for a lemon, i.e., they weree always willing to pay up to $6000 for a lemon, the expected value of a car of unknown type in month t is

In[30]:=
```
d[t_] := f b[1, t] + (1 - f) 6000
```

In[31]:=
```
d[t]
```
Out[31]= 1200. + 0.8 (16 000 - 400 t)

The expected value of a car of unknown type in the next 12 months is

In[32]:=
```
dd = Table[d[t], {t, 1, 12}]
```

Out[32]=
```
{13 680., 13 360., 13 040., 12 720., 12 400.,
 12 080., 11 760., 11 440., 11 120., 10 800., 10 480., 10 160.}
```

Recall that the willingness to accept for the owner of an orange in the next 12 months is

In[33]:=
```
aa
```

Out[33]=
```
{12 400, 12 300, 12 200, 12 100, 12 000,
 11 900, 11 800, 11 700, 11 600, 11 500, 11 400, 11 300}
```

The difference between the willingness to pay for a car of unknown type and the willingness to accept for the owner of an orange in the next 12 months is

In[34]:=
```
dd - aa
```

Out[34]=
```
{1280., 1060., 840., 620., 400., 180., -40., -260., -480., -700., -920., -1140.}
```

This difference is negative (-40), starting from month 7. That is, the market for oranges collapses in month 7 in this case.

In[35]:=
```
ClearAll[a, b, c, d, aa, cc, dd, g, f]
```

8U3. An economy has two types of jobs, Good and Bad, and and two types of workers, Qualified and Unqualified. The population consists of 60% Qualified and 40% Unqualified. In a Bad job either type of worker produces 10 units of outputs. In a Good job, a Qualified worker produces 100 units of output and an Unqualified worker produces 0. There is enough demand for worker that for each type of job, companies must pay what they expect the appointee to produce.

Companies must hire each worker without observing his type and pay him before knowing his actual output. But Qualified worker can signal their qualification by getting educated. For a Qualified worker, the cost of getting educated to level n is $\frac{n^2}{2}$, whereas for an Unqualified worker, it is n^2. These costs are measured in the same unit as output, and n must be an integer.

(a) What is the minimum level of n that will achieve separation?
(b) Now suppose that the signal is made unavailable. Which kind of job will be filled by which types of workers and at what wage? Who will gain and who will lose from this change?

(a) If a Qualified worker gets educated to level $n = 1$, and if this signal is accepted by the employers as the his quality, then his net wage is $100 - \frac{1}{2} n^2 = 100 - \frac{1}{2} = 99.5$. For an Unqualified worker, if he also tries to get educated to levle 1, then the employer will offfer him a wage of 100, believing wrongly that they are hiring Qualified worker. The Unqualified worker, who is thus hired, will be put in a Good job. He will produce 0 unit of output while receiving a wage of 100. For such a worker, her net wage is $100 - n^2 = 100 - 1 = 99$. Thus, an Unqualified worker will imitate the signal of a Qualified worker in this case, and $n = 1$ is not a sufficiently high level of education to separate the two types of workers.

If a Qualified worker chooses $n = 10$, then her net wage is $100 = \frac{1}{2} n^2 = 100 - \frac{1}{2} 10^2 = 100 - 50 = 50$. An Unqualified worker who tries to imitate a Qualified worker by choosing the level of education of $n = 10$, will obtain a net wage of $100 - n^2 = 100 - 10^2 = 0$.

If a Qualified worker chooses $n = 11$, then her net wage is $100 = \frac{1}{2} n^2 = 100 - \frac{1}{2} 11^2 = 100 - \frac{121}{2} = 39.5$. An Unqualified worker who tries to imitate a Qualified worker by choosing the level of education of

$n = 11$, will obtain a net wage of $100 - n^2 = 100 - 11^2 = 100 - 121 = -21 < 0$. Thus, it does not pay for an Unqualified worker to get educated to level $n = 11$, i.e., the minimum level of n that will achieve separation is $n = 11$.

(b) Suppose now that the signal is made unavailable. Under this scenario, there exists only a single wage rate, say ω, for a worker of unknown type.That is, on the labor market a worker who can be Qualified (with probabilit 0.6) or Unqualified (with probability 0.4) is hired at the wage rate ω.

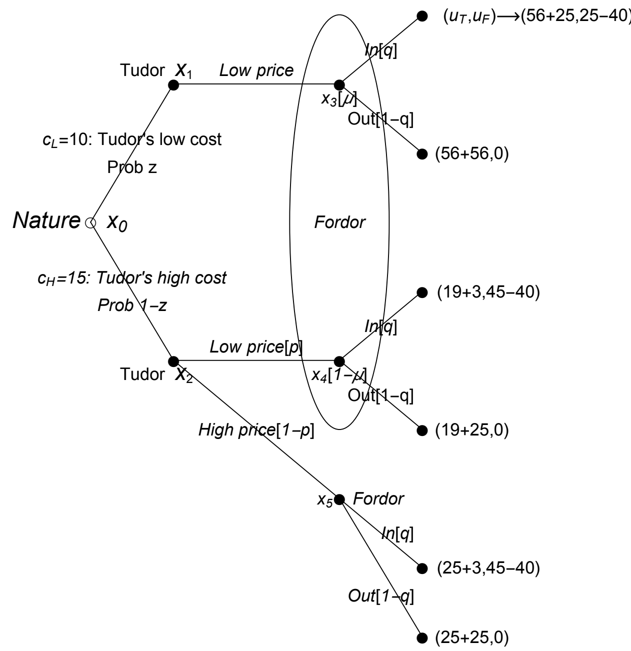
For a firm that hires a worker, the problem is where to put the worker: in a Good job or in a Bad job. If the worker is piut in a Good job, and it turns out that he is (with probability 0.6) a Qualified worker, the realized output is 100. On the other hand, if the worker turns out to be an Unqualified worker, then the output is 0. Thus, the expected output obtained by putting the worker in a Good job is $100 \times 0.6 = 60$. If the worker is put in a Bad job, then the firm obtains a guaranteed output of 10, regardless of the type of the worker. Thus, the firm will put a hired worker in a Good job and obtains an expected output of 60. The wage rate paid to a worker of unknown type is $\omega = 60$, which is the output that a firm can expect.

The answer to question (b) is: "Workers of unknown type are hired at the wage rate of $\omega = 60$, and are then put in a Good job. A Qualified worker produces 100 units of output, and receives a compensation of 60. Relative to the situation he can use the education level as a signal, a Qualified worker is better-off because with signaling his net payoff is only 39.5. As for an Unqualified worker. he definitely gains: he receives a wage of 60 without incurring any cost (with signaling his net payoff is 0.

8U5. Return to the Tudor-Fordor problem of Section 6.C, when Tudor's low cost is $c_L = 10$. Let z be the probability that Tudor actually has a low unit cost.

(a) Rewrite the table in Figure 8.10 in terms of z.
(b) How many pure-strategy equilibria are there when $z = 0$? What type of equilibrium (separating, pooling, or semi-separating) occurs when $z = 0$? Explain.
(c) How many pure-strategy equilibria are there wnen $z = 1$? What type of equilibrium (separating, pooling, or semi-separating) occurs when $z = 1$? Explain.
(c) What is the lowest value of z such that there is a pooling equilibrium?
(d) Explain intuitively why the pooling equilibrium cannot occur when the value of z is too low.

The following figure depicts the game tree.



First, note that Tudor has two information sets: $\{x_1\}$ and $\{x_2\}$. At node x_1, Tudor can either sets low price or high price in period 1. However, we are told that low price is the monopoly price, and thus Tudor always chooses L (Low Price) at node x_1. At node x_2, Tudor can choose L (Low Price) or H (High Price). The following Table depicts the possible pure strategies of Tudor

At x_1	At x_2
L	L
	H

Thus, Tudor has 2 possible pure strategies:

(L, L) : Play L at x_1 and L at x_2,

(L, H) : Play L at x_1 and H at x_2.

Next, note that Fordor has two information sets. The first information sets consists of two nodes x_3 and x_4, while the second information set consists of the single node x_5.

At the second information set, choosing In gives Fordor a payoff of 5, while chousing Out gives Fordor zero payoff. Thus, at node x_5 Fordor will choose Out.

The information set $\{x_3, x_4\}$ represents the event that Tudor has made its move and has chosen <<Low Price>>. Under this situation, it is the turn of Fordor to move, and Fordor can choose In or Out, without knowing whether the cost of Tudor is low or high. As depicted, in the figure, the information set $\{x_3, x_4\}$ is enclosed by an ellipse, and the beliefs μ represents the probability in Fordor's mind that the game has evolved to node x_3, and $1-\mu$ represents the probability in Fordor's mind that the game has evolved to node x_4.

The following Table depicts the possible pure strategies of Fordor

At $\{x_3, x_4\}$	At x_5
In	In
Out	

Thus, Fordor has 2 possible pure strategies:

(In, In) : Play In at $\{x_3, x_4\}$ and In at x_5

(Out, In) : Play Out at $\{x_3, x_4\}$ and In at x_5.

(a) Rewrite the table in Figure 8.10 in terms of z.

The following payoff matrix represents the normal form of the game

		Fordor	
		(In,In)	(Out, In)
Tudor	(L, L)	$(81z+22(1-z), -15z+5(1-z))$	$(112z+44(1-z), 0)$
	(L, H)	$(81z+28(1-z), -15z+5(1-z))$	$(112z+28(1-z), 0\times z+5(1-z))$

(b) How many pure-strategy equilibria are there when $z=0$? What type of equilibrium (separating, pooling, or semi-separating) occurs when $z=0$? Explain.

When $z=0$, the payoff matrix for the normal gform of the game is reduced to

		Fordor	
		(In,In)	(Out, In)
Tudor	(L, L)	(22, 5)	(44, 0)
	(L, H)	(28, 5)	(28, 5)

The only Nash equilibrium is ((L, H), (In, In)). This is an equilibrium in pure strategies under which Tudor sets a Low Price in period 1 if its cost is low and a High Price when its cost is high. As for Fordor, it enters the market when Tudor sets a Low Price and when Tudor sets a High Price. This is a separating equilibrium.

(c) How many pure-strategy equilibria are there wnen $z=1$? What type of equilibrium (separating, pooling, or semi-separating) occurs when $z=1$? Explain.

When $z=1$, the following payoff matrix represents the normal form of the game

		Fordor	
		(In,In)	(Out, In)
Tudor	(L, L)	(81, −15)	(112, 0)
	(L, H)	(81, −15)	(112, 0)

There are two Nash equilibria in pure strategies. The first one is ((L, L), (Out, In)) and the second one is ((L, H), (Out, In)).

Under the first Nash equilibrium in pure strategy, Tudor sets a Low Price when its cost is low (this event has probability $z=1$ in the present case) and a Low Price when its cost is high (this event has zero probability). As for Fordor, it chooses to stay out if Tudor sets a low price and to enter if Tudor sets a

High Price. The equilibrium is a pooling equilibrium although it will never happen.

Under the second Nash equilibrium in pure strategies, Tudor sets a Low Price if its cost is low and a High Price if its cost is high. As for Fordor, it chooses to stay out if Tudor sets a low price and to enter if Tudor sets a High Price.

(d) We are told that Tudor always sets a Low Price when its cost is low. When its cost is high, Tudor can either set a Low Price or a High Price. Under a pooling equilibrium, Tudor sets a Low Price whether its cost is low or high. That is, it will set a Low Price when its cost is high.

To find a pooling equilibrium, let p denote the probability that Tudor sets a Low Price at node x_2 *and q denote the probability that Fordor chooses to enter the market when Tudor sets a Low Price in period 1. The probability that the games evolves to node* x_3 *is*

(1) $\quad z \times 1 = z.$

The probability that the game evolves to node x_4 *is*

(2) $\quad (1 - z)\, p.$

The conditional probability that the game evolves to node x_3*, given that a Low Price set by Tudor has been observed is then given by*

(3) $\quad \mu = \frac{z}{z+p(1-z)}.$

The conditional probability that the game evolves to node x_4*, given that a Low Price set by Tudor has been observed is then given by*

(4) $\quad 1 - \mu = 1 - \frac{z}{z+p(1-z)}.$

Now when Tudor plays Low Price at node x_2 *against* $(q,\, 1 - q)$*, then its expected payoff is*

$$22\, q + 44\,(1 - q) = 44 - 22\, q.$$

On the other hand, if Tudor plays High Price at x_2 *against* $(q,\, 1 - q)$*, then its expected payoff is 28. Thus, playing Low Price at* x_2 *against* $(q,\, 1 - q)$ *is best when*

$$44 - 22\, q \geq 28,$$

i.e., when

(4) $\quad q \leq \frac{16}{22} = \frac{8}{11}.$

Now given its belief μ*, the expected payoff for Fordor if it plays ou is 0. On the other hand, if Fprdor plays In, then its expected payoff is* $-15\,\mu + 5\,(1 - \mu)$*. Under a pooling equilibrium,* $(q,\, 1 - q)$ *is best against Low Price, and Low Price at* x_2 *is best against* $(q,\, 1 - q)$*. That is, under a pooling equilibrium, we have* $p = 1$ *and inequality (4) is satisfied. Furthermore, when* $0 < q \leq \frac{8}{11}$*, Fordor plays a mixed strategy at the information set* $\{x_3,\, x_4\}$*. Also, when* $q = 0$ *in (4), i.e., Fordor plays In with probability 0, we must have*

(5) $\quad -15\,\mu + 5\,(1 - \mu) \leq 0,$

i.e.,

(6) $\quad -20\,\mu + 5 \leq 0 \Rightarrow \mu \geq \frac{5}{20} = 0.25.$

Because $\mu = \frac{z}{z+p(1-z)}$ *and* $p = 1$ *under a pooling equilibrium, we must have*

(7) $\quad \mu = z \geq 0.25.$

That is, the lowest value of z for which a pooling equilibrium exists is 0.25.

(e) Explain intuitively why the pooling equilibrium cannot occur when the value of z is too low.

When z is too low, it means that Tudor most likely has a high cost, and if it bluffs by setting a low price, the action is not credible, and Fordor will certainly enter the market if it observes a low price. The rea-

son is simple: When z is low, the belief μ is also low, i.e., under a pooling equilibrium, the game is most likely have arrived at node x_4 if Tudor has set a low price. Under this belief, Fordor will choose In. Knowing this, Tudor will choose a high price when it s cost is high, contradicting the assumption that the equilibrium in question is a pooling equilibrium.

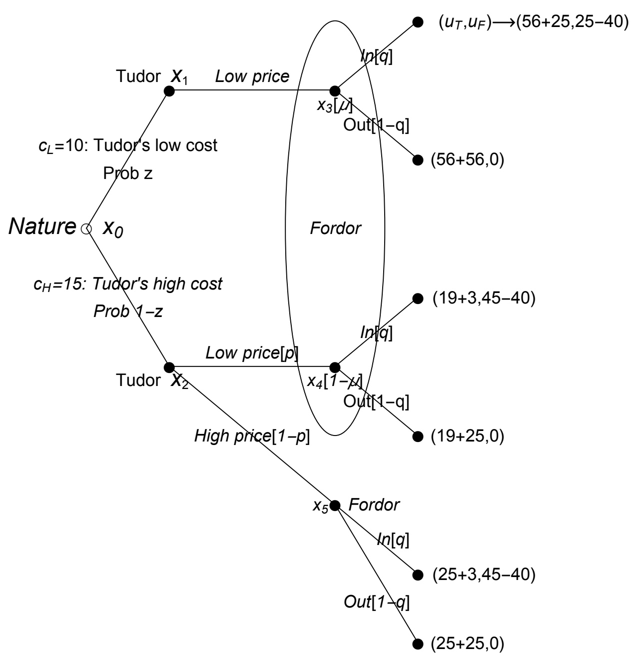
The *Mathematica* codes to draw the game tree

```
In[36]:= g[0] = Graphics[Line[{{0, 0}, {1, 1}, {3, 1}, {4, 1.5}}]];

In[37]:= g[1] = Graphics[Line[{{3, 1}, {4, 0.5}}]];

In[38]:= g[2] = Graphics[Text["○", {0, 0}]];

         g[3] = Graphics[Text["Nature    x0", {-0.25, 0}]];

In[40]:= g[4] = Graphics[Text["●", {1, 1}]];

In[41]:= g[5] = Graphics[Text["cL=10: Tudor's low cost", {0.5, 0.6}]];

In[42]:= g[6] = Graphics[Text["Prob z", {0.5, 0.4}]];

         g[7] = Graphics[Text["Tudor  x1", {0.8, 1.1}]];

         g[8] = Graphics[Text["Low price", {2, 1.1}]];

In[45]:= g[9] = Graphics[Text["●", {3, 1}]];

         g[10] = Graphics[Text["In[q]", {3.5, 1.25}]];

In[47]:= g[11] = Graphics[Text["●", {4, 1.5}]];

In[48]:= g[12] = Graphics[Text["(uT,uF)⟶(56+25,25-40)", {5.4, 1.5}]];

In[49]:= g[13] = Graphics[Text["Out[1-q]", {3.5, 0.75}]];

In[50]:= g[14] = Graphics[Text["●", {4, 0.5}]];

In[51]:= g[15] = Graphics[Text["(56+56,0)", {4.6, 0.5}]];

In[52]:= g[16] = Graphics[Line[{{0, 0}, {1, -1}, {3, -1}, {4, -0.5}}]];

In[53]:= g[17] = Graphics[Text["●", {1, -1}]];

         g[18] = Graphics[Text["Tudor  x2", {0.8, -1.1}]];

         g[19] = Graphics[Text["Low price[p]", {2, -1 + 0.1}]];

In[56]:= g[20] = Graphics[Text["●", {3, -1}]];

         g[21] = Graphics[Text["In[q]", {3.5, 1.25 - 2}]];

In[58]:= g[22] = Graphics[Text["●", {4, 1.5 - 2}]];

In[59]:= g[23] = Graphics[Text["(19+3,45-40)", {4.8, 1.5 - 2}]];

In[60]:= g[24] = Graphics[Text["Out[1-q]", {3.5, 0.75 - 2}]];

In[61]:= g[25] = Graphics[Text["●", {4, 0.5 - 2}]];

In[62]:= g[26] = Graphics[Text["(19+25,0)", {4.6, 0.5 - 2}]];
```

```
In[63]:= g[27] = Graphics[Line[{{3, -1}, {4, -1.5}}]];

In[64]:= g[28] = Graphics[Circle[{3, 0}, {0.6, 1.5}]];

         g[29] = Graphics[Text["Fordor", {3, 0}]];

         g[30] = Graphics[Text["x3[μ]", {3, 1 - 0.1}]];

         g[31] = Graphics[Text["x4[1-μ]", {3, 1 - 0.1 - 2}]];

In[68]:= g[32] = Graphics[Line[{{1, -1}, {3, -2}, {4, -2 - 0.5}}]];

In[69]:= g[33] = Graphics[Text["●", {3, -2}]];

         g[34] = Graphics[Text["x5    Fordor", {3.25, -2}]];

In[71]:= g[35] = Graphics[Text["●", {4, -2.5}]];

         g[36] = Graphics[Text["In[q]", {3.5 + 0.2, -2.25}]];

In[73]:= g[37] = Graphics[Text["(25+3,45-40)", {4.8, 1.5 - 2 - 2}]];

In[74]:= g[38] = Graphics[Line[{{3, -2}, {4, -2 - 1}}]];

         g[39] = Graphics[Text["Out[1-q]", {3.5, -2. - 0.5 - 0.2}]];

In[76]:= g[40] = Graphics[Text["(25+25,0)", {4.6, 1.5 - 2 - 2 - 0.5}]];

In[77]:= g[41] = Graphics[Text["●", {4, -3}]];

         g[42] = Graphics[Text["High price[1-p]", {2, -1.5}]];

         g[43] = Graphics[Text["cH=15: Tudor's high cost", {0.5, 0.6 - 1}]];

         g[44] = Graphics[Text["Prob 1-z", {0.5, 0.4 - 1}]];
```

In[81]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10],
 g[11], g[12], g[13], g[14], g[15], g[16], g[17], g[18], g[19], g[20],
 g[21], g[22], g[23], g[24], g[25], g[26], g[27], g[28], g[29], g[30],
 g[31], g[32], g[33], g[34], g[35], g[36], g[37], g[38], g[39], g[40], g[41],
 g[42], g[43], g[44], AspectRatio → 1.25, PlotRange → {{-1, 7}, {-4, 2}}]
```

Out[81]=



In[82]:=
```
ClearAll[g]
```

8U11. Consider Spence’s job-market signalling model with the following specifications. There are two types of workers, 1 and 2. The productivities of the two types, as functions of the level of education, are

$W_1[E] = E$ and $W_2[E] = 1.5\,E$.

The cost of education of the two types, as functions of the level of educations, are

$C_1[E] = \frac{E^2}{2}$ and $C_2[E] = \frac{E^2}{3}$.

Each worker’s utility is equal to his or her income minus the cost of education. Companies that seek to hire these workers are perfectly competitive in the labor market.

(a) If types are public information (observable and verifiable), find expressions for the levels of education, income, and utilities of the two types of workers.

Now suppose that each worker’s types is his or her private information.
(b) Verify that if the contracts of part (a) are attempted in this situation of information asymmetry, then type 2 does not want to take up the contract of type 1, but type 1 does want to take up the contract intended for type 2, so “natural” separation cannot prevai.
(c) If we leave the contract of type 1 as in part (a), what is the range of contracts (education-wage pairs) for type 2 that can achieve separation?

(d) Of the separating contracts, which one do you expect to prevail? Give a verbal but not a formal explanation for your answer.
(e) Who gains or loses from the information asymmetry? How much?

(a) If types are public information, then a worker will be paid his marginal product. That is, a worker of type 1 will be paid a wage of $W_1[E] = E$ *and and a worker of type 2 a wage of* $W_2[E] = 1.5\,E$. *The net payoff for a worker of type 1 is then*

$$U_1[E] = W_1[E] - C_1[E] = E - \frac{E^2}{2},$$

while that for a worker of type 2 is

$$U_2[E] = W_2[E] - C_2[E] = 1.5\,E - \frac{E^2}{3}.$$

A worker of type 1 will choose to invest in education at the level E that maximizes $U_1[E]$, *and a worker of type 2 will choose to invest in education at the level E that maximizes* $U_2[E]$.

The wage earned by a worker of type 1, say w[1], as a function of her education level e, is

```
In[83]:= w[1] = e
Out[83]= e
```

The cost of attaining this level of education is

In[84]:= $c[1] = \frac{e^2}{2}$

Out[84]= $\frac{e^2}{2}$

The utility obtained by a worker of type 1, say u[1], as a function of her education level e, is

```
In[85]:= u[1] = w[1] - c[1]
```

Out[85]= $e - \frac{e^2}{2}$

The following first-order condition characterizes the optimal levelm of education for a worker of type 1

```
In[86]:= foc[1] = ∂e u[1] == 0
Out[86]= 1 - e == 0
```

The optimal levelm of education for a worker of type 1 is

```
In[87]:= s[1] = Solve[foc[1], e] // Flatten
Out[87]= {e → 1}
```

The wage paid to a worker of type 1 is

```
In[88]:= w[1] /. s[1]
Out[88]= 1
```

The utility for a worker of type 1 is

```
In[89]:= u[1] /. s[1]
```

Out[89]= $\frac{1}{2}$

The contract for a worker of type 1 is $(e_1, w_1) = (1, 1)$.

The wage earned by a worker of type 2, say w[2], as a function of her education level e, is

In[90]:= `w[2] = 1.5 e`

Out[90]= $1.5\,e$

The cost incurred by a worker of type 2 to attain the education level e ,is

In[91]:= $c[2] = \frac{e^2}{3}$

Out[91]= $\frac{e^2}{3}$

The utility obtained by a worker of type 2, say u[2], as a function of her education level e, is

In[92]:= `u[2] = w[2] - c[2]`

Out[92]= $1.5\,e - \frac{e^2}{3}$

The following first-order condition characterizes the optimal levelm of education for a worker of type 2

In[93]:= $foc[2] = \partial_e u[2] == 0$

Out[93]= $1.5 - \frac{2\,e}{3} == 0$

The optimal levelm of education for a worker of type 2 is

In[94]:= `s[2] = Solve[foc[2], e] // Flatten`

Out[94]= $\{e \to 2.25\}$

The wage paid to a worker of type 2 is

In[95]:= `w[2] /. s[2]`

Out[95]= 3.375

The utility for a worker of type 2 is

In[96]:= `u[2] /. s[2]`

Out[96]= 1.6875

The contract for a worker of type 2 is $(e_2, w_2) = (2.25, 3.375)$.

(b) Now suppose that each worker's types is his or her private information and that the contracts of part (a) are attempted in this situation of information asymmetry. That is two contracts are offered on the labor maket: One contract is $(e_1, w_1) = (1, 1)$*, which requires education level 1 and pays a wage of 1. The other contract is (2.25,3.375), which requires education level 2.25 and pays a wage of 3.375.*

For a worker of type 1, choosing the contract $(e_1, w_1) = (1, 1)$ *gives him a utility of* $\frac{1}{2}$*, while choosing the contract* $(e_2, w_2) = (2.25, 3.375)$ *gives him a utility of*

In[97]:= $3.375 - \frac{2.25^2}{2}$

Out[97]= 0.84375

which is higher than $\frac{1}{2}$. Thus, a worker of type 1 will choose the contract $(e_2, w_2) = (2.25, 3.375)$, which is the contract intended for a worker of type 2.

For a worker of type 2, choosing the contract $(e_2, w_2) = (2.25, 3.375)$ gives him a utility of 1.6875, while choosing the contract $(e_1, w_1) = (1, 1)$ gives him a utility of

In[98]:= $1 - \frac{1^2}{3.0}$

Out[98]= 0.666667

which is lower than 1.6875. Thus, a worker of type 2 will not choose the contract intended for a worker of type 1.

(c) Suppose that we now have two contracts (e_1, w_1) and (e_2, w_2), with $(e_1, w_1) = (1, 1)$, the contract intended for workers of type 1 in part (a), while (e_2, w_2) is structured so that only workers of type 2 are willing to sign it. Because a worker of type 2 is paid his marginal product, we must have $w_2 = 1.5\, e_2$; that is, $(e_2, w_2) = (e_2, 1.5\, e_2)$.

Now if a worker of type 1 chooses the contract (e_2, w_2), then he has to obtain the education level e_2 at cost $\frac{e_2^2}{2}$ in order to receive the wage w_2. His net payoff is then given by

$$w_2 - \frac{e_2^2}{2} = 1.5\, e_2 - \frac{e_2^2}{2}.$$

Furthermore, recall that the utility obtained by a worker of type 1 from the contract $(e_1, w_1) = (1, 1)$ is $\frac{1}{2}$. Thus, a worker of type 1 will not select the contract $(e_2, w_2) = (e_2, 1.5\, e_2)$ if the following inequality is satisfied

$$1.5\, e_2 - \frac{e_2^2}{2} \leq \frac{1}{2},$$

i.e., if

(1) $$-1 + 3\, e_2 - e_2^2 \leq 0.$$

Next, note that if a worker of type 2 chooses the contract $(e_2, w_2) = (e_2, 1.5\, e_2)$, then his net payoff is $1.5\, e_2 - \frac{e_2^2}{3}$. On the other hand, if he chooses $(e_1, w_1) = (1, 1)$, the contract intended for workers of type 1, then his net payoff is $w_1 - \frac{e_1^2}{3} = 1 - \frac{1^2}{3} = \frac{2}{3}$. Thus, a worker of type 2 will choose the contract $(e_2, w_2) = (e_2, 1.5\, e_2)$ if the following inequality is satisfied:

$$1.5\, e_2 - \frac{e_2^2}{3} \geq \frac{2}{3},$$

i.e., if

(2) $$-2 + 4.5\, e_2 - e_2^2 \geq 0.$$

Thus, in order to separate the two types of workers, the education level e_2 in the contract $(e_2, w_2) = (e_2, 1.5\, e_2)$ must satisfy simultaneously inequality (1) and inequality (2). We now analyze the range of e_2 in which these two inequalities are simultaneously satisfied.

The following figure depicts the curve

$$e_2 \to -1 + 3\, e_2 - e_2^2.$$

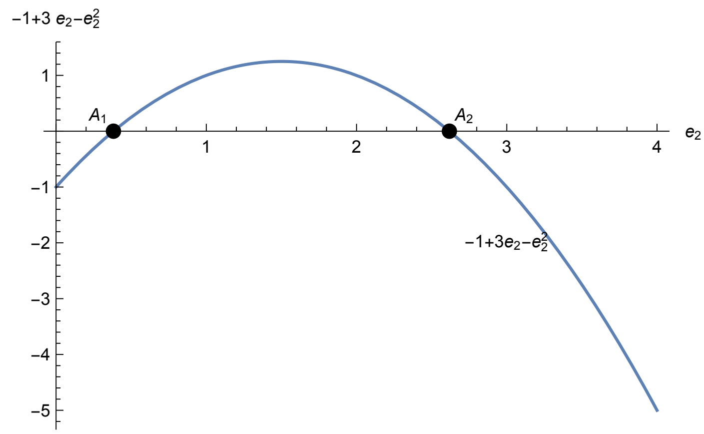


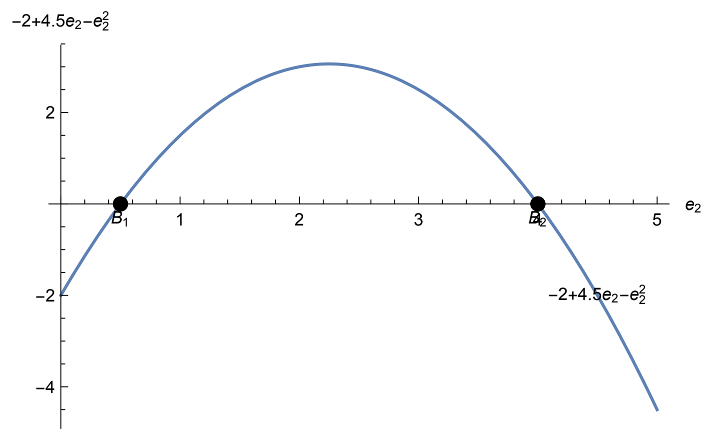
Figure 1.- Inequality (1)

Note that this curves crosses the horizontal axis at two points: $A_1 = 0.381966$, $A_2 = 2.61803$, and inequality (1) is satisfied for the levels of education

(3) $e_2 \leq 0.381966 = A_1$ and $e_2 \geq 2.61803 = A_2$.

The following figure depicts the curve

$$e_2 \to -2 + 4.5\, e_2 - e_2^2.$$



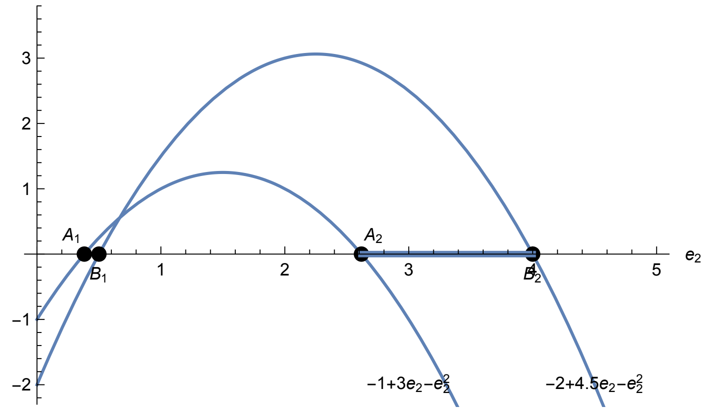
Note that this curves crosses the horizontal axis at two points: $B_1 = 0.5$, $B_2 = 4$, and inequality (2) is satisfied for the levels of education

(4) $B_1 = 0.5 \leq e_2 \leq 4 = B_2$.

Therefore, the levels of education that satisfy both (1) and (2) are given by

(5) $A_2 = 2.61803 \leq e_2 \leq 4 = B_2$.

Inequality (5) is depicted by the thick line segment $A_2 B_2$ in the following figure



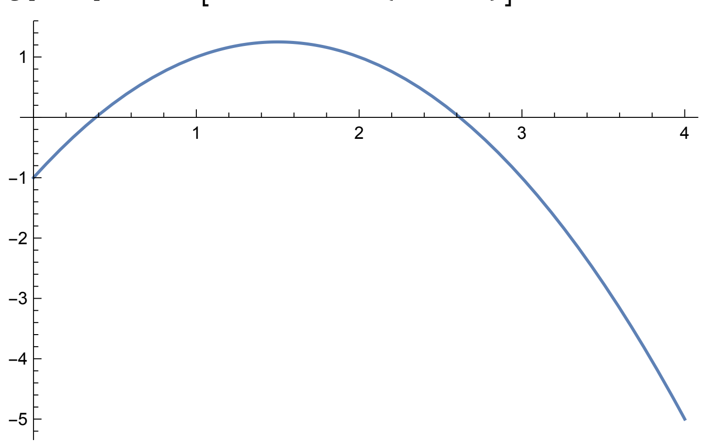
Thus, if the contract $(e_1, w_1) = (1, 1)$, which is intended for workers of type 1, is offered together with the contract $(e_2, w_2) = (e_2, 1.5\, e_2)$, $A_2 \le e_2 \le B_2$, which is intended for workers of type 2, there will be separation of types.

(d) First, recall that when there is full information, the optimal level of education for a worker of type 2 is $e_2 = 2.25$. Next, note that according to the answer to part (c), the lowest level of education for all the contracts that are intended for workers of type 2 and that make separation of types possible is $e_2 = A_2 = 2.61803 > 2.25$. That is, relative to the scenario of full information, a worker of type 2 must overinvest in education in order to distinguish himself from a worker of type 1. Furthermore, the utility of a worker of type 2, namely $u[e_2] = w_2 - \frac{e_2^2}{3} = 1.5\, e_2 - \frac{e_2^2}{3}$, achives a global maximum at $e_2 = 2.25$, and is strictly decreasing for $e_2 > 2.25$. Thus, the contract $(e_2, 1.5\, e_2)$ that yields the highetst utility for a worker of type 2 among the ones that allow for the separation of types is $(e_2, 1.5\, e_2)$, with $e_2 = A_2 = 2.61803$, and this is the contract we can expect to prevail. The end result is a Pareto optimum subject to the constraint of type separation.

(e) Workers of type 1 obtain the same contract under full information and under asymmetric information, and thus neither win nor lose with asymmetric information. As for workers of type 2, they lose under asymmeytric information because they are forced to overinvest in education. Under full information, we recall, the utility for a worker of type 2 is $u_2 = 1.6875$. Under asymmetric information, the utility of a worker of type 2 is $u_2 = 1.5 \times 2.61803 - \frac{2.61803^2}{3} = 1.64235 < 1.6875$. That is, a worker of type 2 loses under asymmetric information, and the loss in utility is $\Delta u_2 = 1.64235 - 1.6875 = -0.04515$.

In[99]:= **g[0, 0] = Plot[-1 + 3 e - e², {e, 0, 4}]**

Out[99]=



In[100]:=
```
a[1] = Solve[-1 + 3 e - e^2 == 0, e] // Flatten
```

Out[100]= $\left\{e \to \frac{1}{2}\left(3-\sqrt{5}\right),\ e \to \frac{1}{2}\left(3+\sqrt{5}\right)\right\}$

In[101]:=
```
{x[1], x[2]} = {e /. a[1][[1]], e /. a[1][[2]]}
```

Out[101]= $\left\{\frac{1}{2}\left(3-\sqrt{5}\right),\ \frac{1}{2}\left(3+\sqrt{5}\right)\right\}$

In[102]:=
```
{x[1], x[2]} = {x[1], x[2]} // N
```

Out[102]= {0.381966, 2.61803}

In[103]:=
```
g[0, 1] = Graphics[{AbsolutePointSize[8], Point[{x[1], 0}]}];
```

In[104]:=
```
g[0, 2] = Graphics[{AbsolutePointSize[8], Point[{x[2], 0}]}];
```

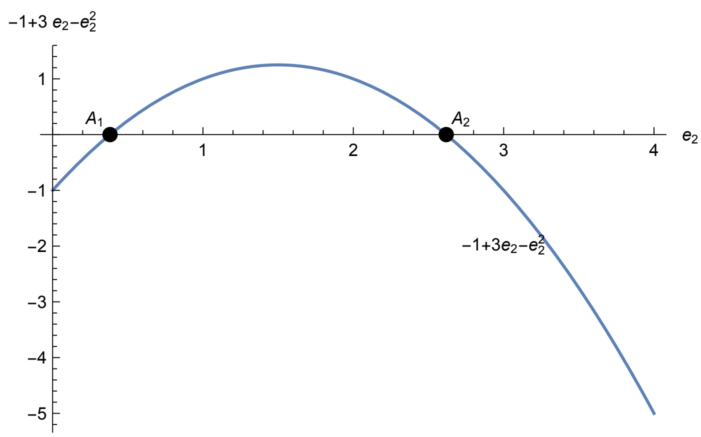
In[105]:=
```
g[0, 3] = Graphics[Text["A1", {x[1] - 0.1, 0.3}]];
```

In[106]:=
```
g[0, 4] = Graphics[Text["A2", {x[2] + 0.1, 0.3}]];
```

In[107]:=
```
g[0, 5] = Graphics[Text["-1+3e2-e2^2", {3, -2}]];
```

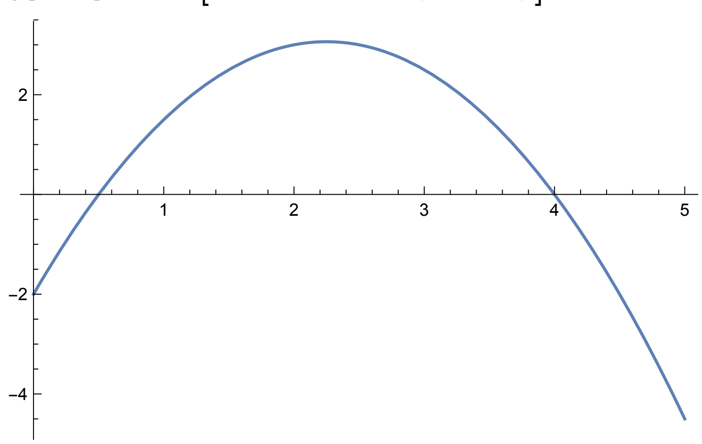
In[108]:=
```
Show[g[0, 0], g[0, 1], g[0, 2], g[0, 3],
 g[0, 4], g[0, 5], AxesLabel → {"e2", "-1+3 e2-e2^2"}]
```

Out[108]=



In[109]:=
```
g[1, 0] = Plot[-2 + 4.5 e - e^2, {e, 0, 5}]
```

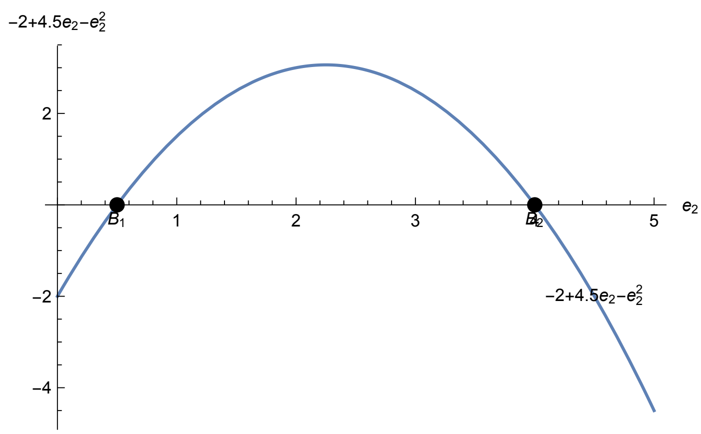
Out[109]=



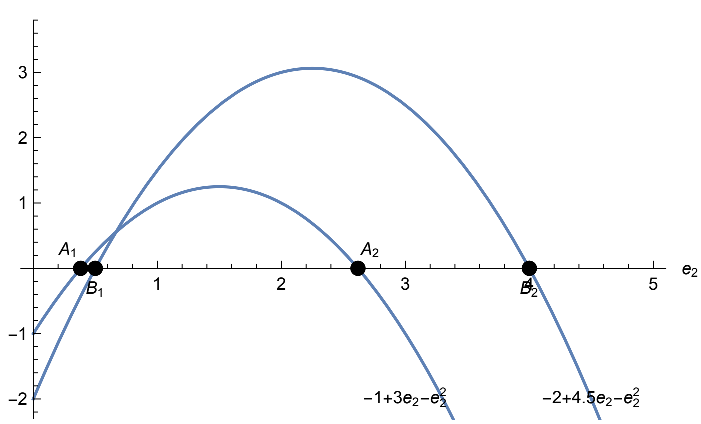
In[110]:=
```
b[1] = Solve[-2 + 4.5 e - e^2 == 0, e] // Flatten
```
Out[110]= {e → 0.5, e → 4.}

In[111]:=
```
{y[1], y[2]} = {e /. b[1][[1]], e /. b[1][[2]]}
```
Out[111]= {0.5, 4.}

In[112]:=
```
g[1, 1] = Graphics[{AbsolutePointSize[8], Point[{y[1], 0}]}];
```
In[113]:=
```
g[1, 2] = Graphics[{AbsolutePointSize[8], Point[{y[2], 0}]}];
```
In[114]:=
```
g[1, 3] = Graphics[Text["B₁", {y[1], -0.3}]];
```
In[115]:=
```
g[1, 4] = Graphics[Text["B₂", {y[2], -0.3}]];
```
In[116]:=
```
g[1, 5] = Graphics[Text["-2+4.5e₂-e₂²", {4.5, -2}]];
```
In[117]:=
```
Show[g[1, 0], g[1, 1], g[1, 2], g[1, 3],
 g[1, 4], g[1, 5], AxesLabel → {"e₂", "-2+4.5e₂-e₂²"}]
```

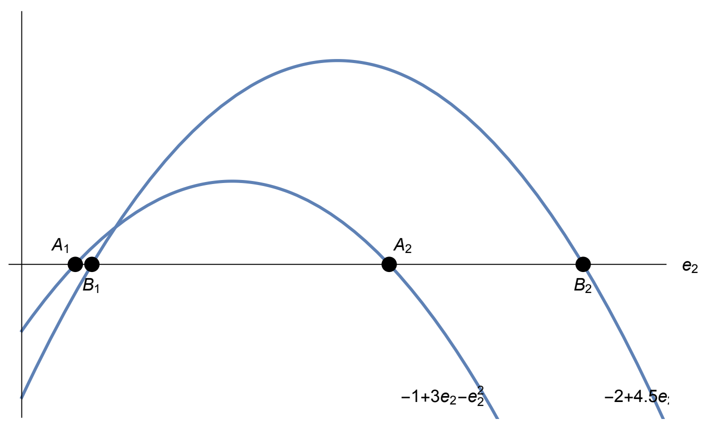
Out[117]=


In[118]:=
```
Show[g[0, 0], g[0, 1], g[0, 2], g[0, 3], g[0, 4], g[0, 5], g[1, 0], g[1, 1], g[1, 2],
 g[1, 3], g[1, 4], g[1, 5], PlotRange → {{0, 5}, {-2, 3.5}}, AxesLabel → {"e₂", ""}]
```

Out[118]=


In[119]:=
```
Show[g[0, 0], g[0, 1], g[0, 2], g[0, 3], g[0, 4],
 g[0, 5], g[1, 0], g[1, 1], g[1, 2], g[1, 3], g[1, 4], g[1, 5],
 PlotRange → {{0, 4.5}, {-2, 3.5}}, AxesLabel → {"e₂", ""}, Ticks → False]
```

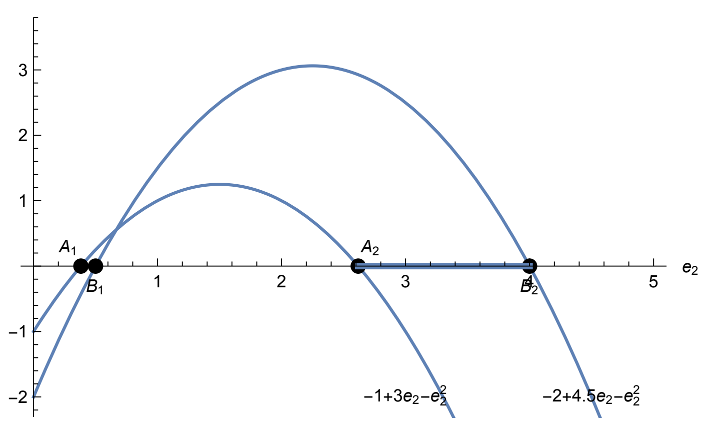
Out[119]=



In[120]:=
```
g[2] = Plot[0, {m, x[2], y[2]}, PlotStyle → Thickness[0.01]];
```

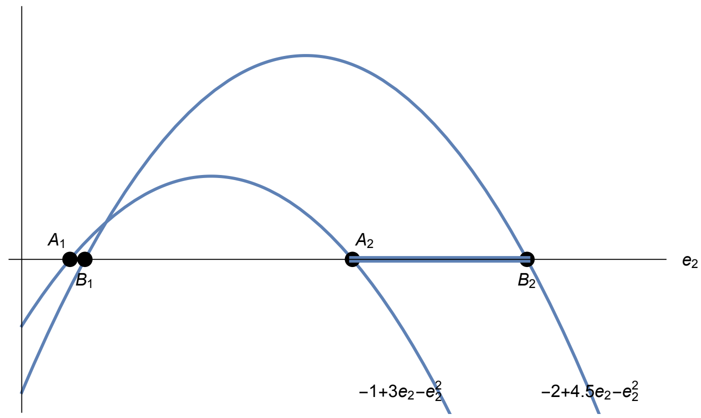
In[121]:=
```
Show[g[0, 0], g[0, 1], g[0, 2], g[0, 3], g[0, 4],
 g[0, 5], g[1, 0], g[1, 1], g[1, 2], g[1, 3], g[1, 4], g[1, 5],
 g[2], PlotRange → {{0, 5}, {-2, 3.5}}, AxesLabel → {"e₂", ""}]
```

Out[121]=



In[122]:=
```
Show[g[0, 0], g[0, 1], g[0, 2], g[0, 3], g[0, 4], g[0, 5],
 g[1, 0], g[1, 1], g[1, 2], g[1, 3], g[1, 4], g[1, 5], g[2],
 PlotRange → {{0, 5}, {-2, 3.5}}, AxesLabel → {"e₂", ""}, Ticks → False]
```

Out[122]=



In[123]:= `u[2]`

Out[123]= $1.5\,e - \frac{e^2}{3}$

In[124]:= `x[2]`

Out[124]= 2.61803

In[125]:= `u[2] /. e → x[2]`

Out[125]= 1.64235

In[126]:= `ClearAll[u, a, b, x, y, g, w, s, foc]`

8S10. Wanda works as a waitress and consequently has the opportunity to earn cash tips that are not reported by her employer to the Internal Revenu Service. Her tip income is rather variable. In a good year (G), she earns a high income, so her tax liability to the IRS is $5000. In a bad year (B), she earns a low income, and her tax liability to the IRS is $0. The IRS knows that the probability of her having a good year is 0.6, and the probability of her having a bad year is 0.4, but it does not know for sure which outcome has resulted for her this tax year.

In this game, first Wanda decides how much income to report to the IRS. If she reports high income (H), she pays the IRS $5000. If she reports low income (L), she pays the IRS $0. The the IRS has to decide whether to audit Wanda. If she reports high income, they do not audit because they automatically know they've receiving the tax payment Wanda owes. If she reports low income, then the IRS can either audit (A) or not audit (N). When the IRS audits, it costs the IRS $1000 in administrative costs, and also costs Wanda $1000 in the opportunity cost of the time spent gathering bank record and meeting with the auditor. If the IRS audits Wanda in a bad year (B), then she owes nothing to the IRS, although she and the IRS have each incurred the $1000 auditing cost. If the IRS audits Wanda in a good year (G), then she has to pay the $5000 she owes to the IRS, in addition to her and the IRS each incurring the cost of auditing.

(a) Suppose Wanda has a good year (G), but she reports low income (L). Suppose that the IRS then audits her (A). What is the total payoff to Wanda, and what is the total payoff to the IRS.

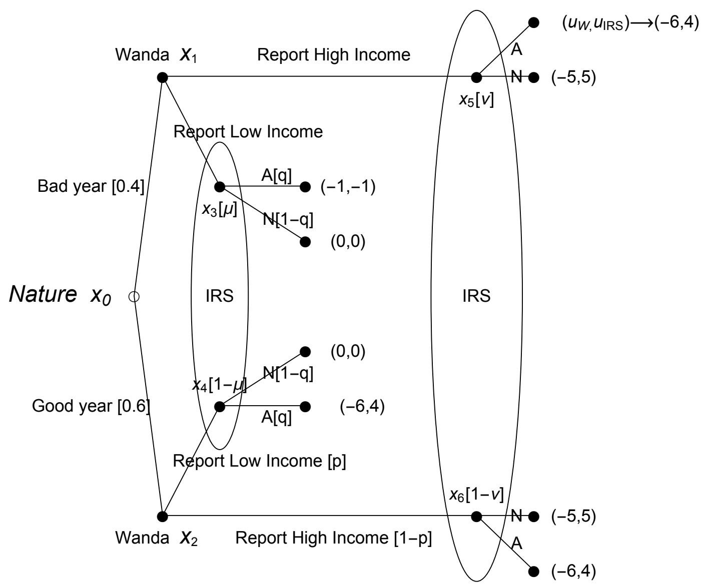
Wanda then has to pay to the IRS $5000, the tax liability. She also has to incur the audit cost of $1000. Her net payoff is -$6000. As for the IRS, it receives $5000 from Wanda, but has to incur the audit cost of $1000. Its net payoff is $4000.

(b) Which of the two player has an incentive to bluff (that is to give a false signal) in this game? What would bluffing consist of?

Wanda has an incentive to bluff by reporting a low income when in fact she had a high income.

(c) Show this game in extensive form.

The following figure depicts the game tree. In the figure, payoffs are expressed in thousands of dollars.



(d) How many pure strategies does each player have in this game? Explain your reasoning.

Wanda moves at node x_1 and at node x_2. At node x_1, she has two choices: L (report low income) and H (report high income). Depending on whether the IRS plays A (audit Wanda) or N (not audit Wanda), her payoff obtained from playing L (report low income) is -1 or 0. On the other hand, depending on whether the IRS plays A (audit Wanda) or N (not audit Wanda), her payoff obtained from playing H (report low income) is -6 or -5. Thus, at x_1 playing H is strictly dominated by playing L, i.e., Wanda always plays L at x_1. At x_2, *Wanda can play either L or H. Thus, Wanda has only two pure strategies: LL (play L at x_1* and play L at x_2) and LH (play L at x_1 and play H at x_2).

As for the IRS, it has two information sets: $\{x_3, x_4\}$ and $\{x_5, x_6\}$. The first information set represents the event that Wanda reports a low income, while the second information set represents the event that Wanda reports a high income. At each information set, the IRS has two possible moves: Audit (A) and not audit (N). Note that at the information set $\{x_5, x_6\}$ A is strictly dominated by N, and thus the IRS will not audit Wanda if she reports a high income. At the information set $\{x_3, x_4\}$, the IRS has two choices: A and N. Thus, the IRS has two pure strategies: AN (audit at $\{x_3, x_4\}$, *but not audit at $\{x_5, x_6\}$*) *and NN (not audit at* $\{x_3, x_4\}$, not audit at $\{x_5, x_6\}$).

(e) Write down the strategic-form game matrix for this game. Find all the Nash equilibria to this game. Identify whether the equlibrium you find are separating, pooling, or semi-separating.

Internal Revenu Service

		AN[q]	*NN[1-q]*
	LL[p]	(−4, 2)	(0, 0)
Wanda	*LH[1-p]*	(−3.4, 2.6)	(-3,3)

As can be seen from the payoff matrix, there is no Nash equilibrium in pure strategies. That is, the normal form of the game has no Nash equilibrium in pure strategies. We look for a Nash equilibrium in mixed strategies.

Let q be the probability that the IRS plays A and 1 − q be the probability that the IRS plays N. If Wanda plays LL against (q, 1 − q), then her expected payoff is

$-4q + 0(1-q) = -4q.$

If Wanda plays LH against (q, 1 − q), then her expected payoff is

$-3.4q - 3(1-q) = -3 - 0.4q.$

When Wanda plays a mixed strategy against (q, 1 − q), we must have

$-4q = -3 - 0.4q \Rightarrow q = \frac{5}{6}.$

Let p be the probability that Wanda plays LL and (1 − p) be the probability that Wanda plays LH. If the IRS plays A against (p, 1 − p), then its expected payoff is

$2p + 2.6(1-p) = 2.6 - 0.6p$

If the IRS plays N against (p, 1 − p), then its expected payoff is

$0p + 3(1-p) = 3 - 3p.$

If the IRS plays a mixed strategy against (p, 1 − p), then we must have

$2.6 - 0.6p = 3 - 3p \Rightarrow p = \frac{1}{6}.$

The Nash equilibrium in mixed strategies for the normal form of the game is $(p, q) = \left(\frac{1}{6}, \frac{5}{6}\right)$. For the extensive form of the game, this equilibrium corresponds to a semi-separating equilibrium under which Wanda bluffs (reporting a low income when she had a good year) is $p = \frac{1}{6}$. As for the IRS, it audits Wanda with prrobability $\frac{1}{6}$ if she reports a low income. It will not audit Wanda when she reports a high income.

(f) Let x be the probability that Wanda has a good year. In the original version of this problem, we had $x = 0.6$. Find a value of x such that Wanda always reports low income in equilibrium.

(g) Find the full range of the value of x for which Wanda always reports low income in equilibrium.

We have already argued that Wanda will report a low income at x_1. At x_2 (a good year), if Wanda reports a high income, then her payoff is -5. If she reports a low income at x_2, then her expected pyoff is $-6q + 0 \times (1-q) = -6q$. Thus, Wanda will report a low income at x_2 if

(1) $-6q \geq -5 \Rightarrow q \leq \frac{5}{6}.$

Next, note that if x is the probability that Wanda has a good year and that she reports low income when the year was bad, then the probability that the game evolves to node x_2 is x. Also, the probability that *the game evolves to x_1 is $(1-x)$. Thus, the probability that the game evolves to node x_1, given that Wanda has reported a low income is*

(2) $\mu = \frac{1-x}{(1-x)+x} = 1 - x.$

With μ representing the belief that the game has evolved to x_1 under a pooling equilibrium, the payoff for the IRS if it plays A (audit) is

(3) $-1\,\mu + 4\,(1-\mu) = 4 - 5\,\mu = 4 - 5\,(1-x).$

On the other hand, play N yields a payoff of

(4) 0

for the IRS.

Thus when $0 < q \leq \frac{5}{6}$, we the payoffs in (3) and (4) must be equal, i.e.,

(5) $4 - 5\,(1-x) = 0.$

On the other hand, when $q = 0$, the payoff obtained from playing A must not exceed that obtained from playing N, i.e.,

(6) $4 - 5\,(1-x) \leq 0.$

We have just shown that if $0 \leq q \leq \frac{5}{6}$, then (6) must hold, and this last result implies

(7) $4 - 5 + 5\,x \leq 0 \Rightarrow x \leq \frac{1}{5} = 0.2.$

Therefore, when $0 \leq x \leq 0.2$, Wanda always reports a low income. Furthermore, $q = 0$ if $x < 0.2$ and $0 < q \leq \frac{5}{6}$ if $x = 0.2$.

The *Mathematica* codes to draw the game tree

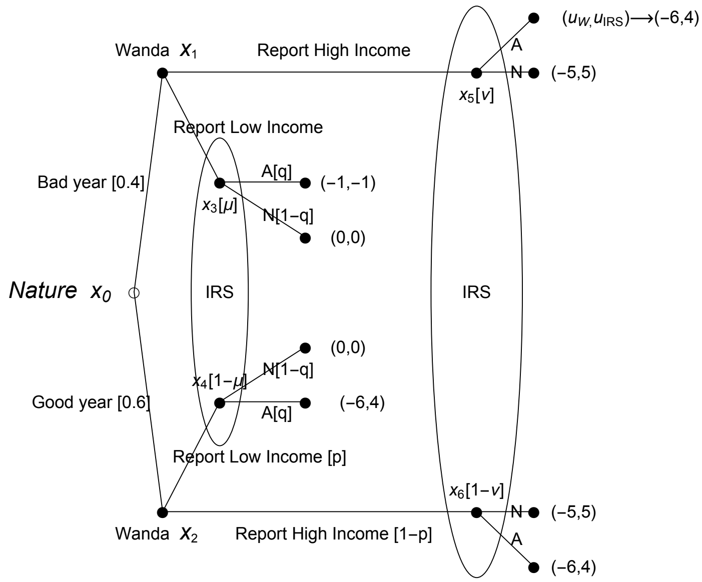
```
In[127]:= g[0] = Graphics[Line[{{0, 0}, {1, 2}, {12, 2}, {14, 2.5}}]];

In[128]:= g[1] = Graphics[Line[{{1, 2}, {3, 1}}]];

In[129]:= g[2] = Graphics[Line[{{0, 0}, {1, -2}, {12, -2}, {14, -2.5}}]];

In[130]:= g[3] = Graphics[Line[{{1, -2}, {3, -1}}]];

In[131]:= g[4] = Graphics[Text["○", {0, 0}]];

          g[5] = Graphics[Text["Nature  x0", {-2.6, 0}]];

In[133]:= g[6] = Graphics[Text["●", {1, 2}]];

In[134]:= g[7] = Graphics[Text["●", {1, -2}]];

In[135]:= g[8] = Graphics[Text["●", {3, 1}]];

In[136]:= g[9] = Graphics[Text["●", {3, -1}]];

In[137]:= g[10] = Graphics[Text["Bad year [0.4]", {-1.5, 1}]];

In[138]:= g[11] = Graphics[Text["Good year [0.6]", {-1.5, -1}]];
```

```
g[12] = Graphics[Text["Wanda  x1", {0.8, 2.2}]];

g[13] = Graphics[Text["Wanda  x2", {0.8, -2.2}]];

In[141]:= g[14] = Graphics[Text["Report Low Income", {4, 1.5}]];

In[142]:= g[15] = Graphics[Text["Report Low Income [p]", {4.4, -1.5}]];

In[143]:= g[15, 1] = Graphics[Text["Report Low Income [p=1/6]", {4.4, -1.5}]];

In[144]:= g[16] = Graphics[Circle[{3, 0}, {1, 1.4}]];

In[145]:= g[17] = Graphics[Text["IRS", {3, 0}]];

In[146]:= g[18] = Graphics[Text["x3[μ]", {3, 0.8}]];

In[147]:= g[19] = Graphics[Text["x4[1-μ]", {3, -0.8}]];

In[148]:= g[20] = Graphics[Line[{{3, 1}, {6, 1}}]];

In[149]:= g[21] = Graphics[Line[{{3, -1}, {6, -1}}]];

In[150]:= g[22] = Graphics[Line[{{3, 1}, {6, 0.5}}]];

In[151]:= g[23] = Graphics[Line[{{3, -1}, {6, -0.5}}]];

In[152]:= g[24] = Graphics[Text["●", {6, 1}]];

In[153]:= g[25] = Graphics[Text["●", {6, -1}]];

In[154]:= g[26] = Graphics[Text["●", {6, 0.5}]];

In[155]:= g[27] = Graphics[Text["●", {6, -0.5}]];

In[156]:= g[28] = Graphics[Text["A[q]", {5, 1.1}]];

In[157]:= g[28, 1] = Graphics[Text["A[q=5/6]", {5, 1.1}]];

In[158]:= g[29] = Graphics[Text["A[q]", {5, -1.1}]];

In[159]:= g[30] = Graphics[Text["N[1-q]", {5.4, 1.2 - 0.5}]];

In[160]:= g[31] = Graphics[Text["N[1-q]", {5.4, -1.2 + 0.5}]];

In[161]:= g[32] = Graphics[Text["(-1,-1)", {7.5, 1}]];

In[162]:= g[33] = Graphics[Text["(-6,4)", {8, -1}]];

In[163]:= g[34] = Graphics[Text["(0,0)", {7.5, 0.5}]];

In[164]:= g[35] = Graphics[Text["(0,0)", {7.5, -0.5}]];

In[165]:= g[36] = Graphics[Text["●", {12, 2}]];

In[166]:= g[37] = Graphics[Text["●", {12, -2}]];

In[167]:= g[38] = Graphics[Circle[{12, 0}, {1.6, 2.6}]];

In[168]:= g[39] = Graphics[Text["Report High Income", {7, 2.2}]];
```

```
In[169]:= g[40] = Graphics[Text["Report High Income [1-p]", {7, -2.2}]];
In[170]:= g[41] = Graphics[Text["x5[ν]", {12, 2 - 0.2}]];
In[171]:= g[42] = Graphics[Text["x6[1-ν]", {12, -2 + 0.2}]];
In[172]:= g[43] = Graphics[Text["IRS", {12, 0}]];
In[173]:= g[44] = Graphics[Text["●", {14, 2.5}]];
In[174]:= g[45] = Graphics[Line[{{12, 2}, {14, 2}}]];
In[175]:= g[46] = Graphics[Text["●", {14, 2}]];
In[176]:= g[47] = Graphics[Text["(uW,uIRS)⟶(-6,4)", {17.4, 2.5}]];
In[177]:= g[48] = Graphics[Text["(-5,5)", {15.4, 2}]];
In[178]:= g[49] = Graphics[Text["A", {13.4, 2.25}]];
In[179]:= g[50] = Graphics[Text["N", {13.4, 2}]];
In[180]:= g[51] = Graphics[Line[{{12, -2}, {14, -2}}]];
In[181]:= g[52] = Graphics[Text["●", {14, -2}]];
In[182]:= g[53] = Graphics[Text["●", {14, -2.5}]];
In[183]:= g[54] = Graphics[Text["A", {13.4, -2.25}]];
In[184]:= g[55] = Graphics[Text["N", {13.4, -2}]];
In[185]:= g[56] = Graphics[Text["(-5,5)", {15.4, -2}]];
In[186]:= g[57] = Graphics[Text["(-6,4)", {15.4, -2.5}]];
```

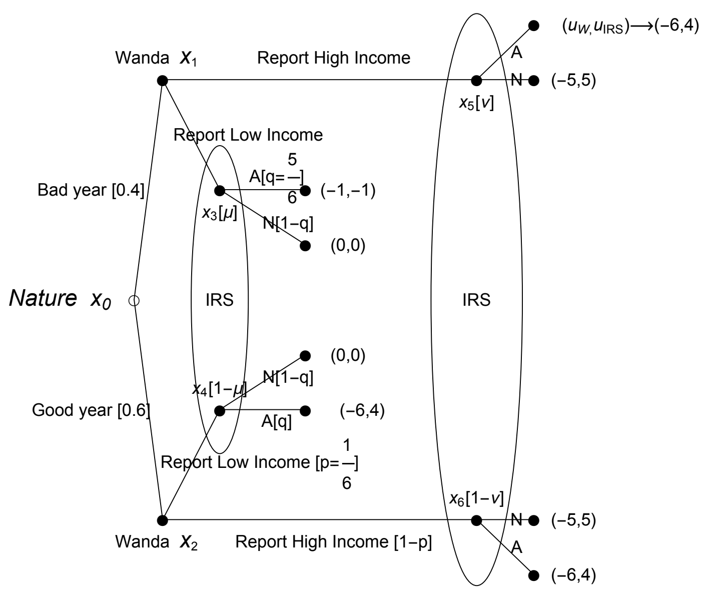
In[187]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10], g[11], g[12],
 g[13], g[14], g[15], g[16], g[17], g[18], g[19], g[20], g[21], g[22], g[23], g[24],
 g[25], g[26], g[27], g[28], g[29], g[30], g[31], g[32], g[33], g[34], g[35], g[36],
 g[37], g[38], g[39], g[40], g[41], g[42], g[43], g[44], g[45], g[46], g[47], g[48],
 g[49], g[50], g[51], g[52], g[53], g[54], g[55], g[56], g[57], AspectRatio → 01]
```

Out[187]=



In[188]:=
```
Show[g[0], g[1], g[2], g[3], g[4], g[5], g[6], g[7], g[8], g[9], g[10],
 g[11], g[12], g[13], g[14], g[15, 1], g[16], g[17], g[18], g[19], g[20],
 g[21], g[22], g[23], g[24], g[25], g[26], g[27], g[28, 1], g[29], g[30],
 g[31], g[32], g[33], g[34], g[35], g[36], g[37], g[38], g[39], g[40],
 g[41], g[42], g[43], g[44], g[45], g[46], g[47], g[48], g[49], g[50],
 g[51], g[52], g[53], g[54], g[55], g[56], g[57], AspectRatio → 01]
```

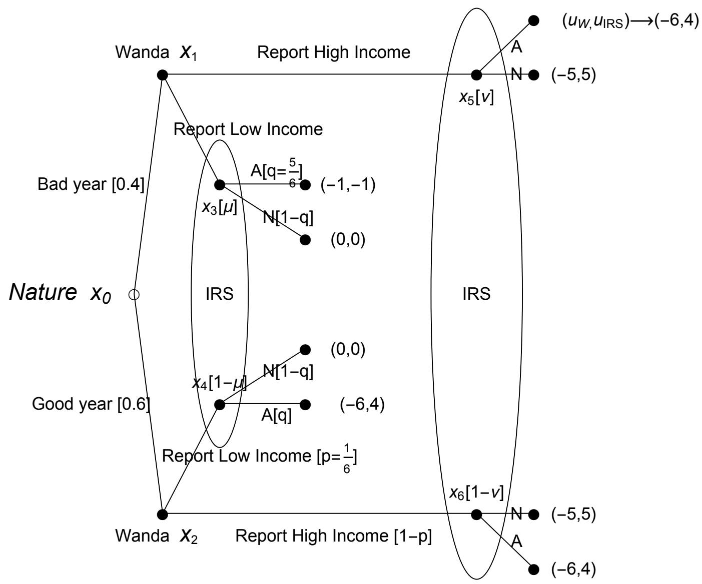
Out[188]=



8U13. An auditor for the IRS is reviewing Wanda's latest tax return (see Exercise S10), on which she reprts having had a bad year. Assume that Wanda is playing according her equilibrium strategy and that the auditor knows this.

(a) Using Bayes' rule, find the probability that Wanda had a good year, given that she reports having had a bad year.
(b) Explain why the answer to part (a) is more or less than the baseline probability of having a good year, 0.6.

The semiseparating equilibrium is depicted in the following game tree. The game tree shows $p = \frac{1}{6}$ *as the probability that Wanda reports a low income when the year was yood. As for the IRS, at the information set* $\{x_3, x_4\}$*, it plays A with probability* $\frac{5}{6}$ *and N with probability* $\frac{1}{6}$*.*



The probability that the game arrives at x_3 *is 0.4, and the probability that the game arrives at* x_4 *is* $0.6\,p = 0.6 \times \frac{1}{6} = 0.1.8U13$*. The probability that the game arrives at the information set* $\{x_3, x_4\}$ *is* $0.4 + 0.1 = 0.5$*.*

(a) According to Bayes' rule, the probability that Wanda had a good year, given that she reports having had a bad year is

$$\frac{Prob[\text{Wanda had a good year and Wandas reports a low income}]}{Prob[\text{the game arrives at the information set } \{x_3, x_4\}]} = \frac{0.1}{0.1+0.4} = 0.2.$$

(b) The answer to part (a) is 0.2, which is less than 0.6, the baseline probability of having a good year, 0.6. Before Wanda reorts her income the probability of a good year was 0.6. After she has reported a low income, we can expect that the revised (posterior) probability that she had a good year will fall, and the computation using Bayes' rule confirms this intuition.