First Name: __________ Last Name: __________

Student-No: __________ Section: __________

Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes.

   (a) Evaluate $\tan\left(\frac{\pi}{3}\right)$.

   Answer: $\sqrt{3}$

   Solution:

   $$\sin \pi/3 = \frac{\sqrt{3}}{2} \qquad \cos \pi/3 = \frac{1}{2} \qquad \text{so } \tan \pi/3 = \sqrt{3}$$

   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   (b) Compute $\lim_{t\to -1}\left(\frac{t-2}{t+3}\right)$.

   Answer: $-3/2$

   Solution:

   $$\lim_{t\to -1}\left(\frac{t-2}{t+3}\right) = \frac{\lim_{t\to -1}(t-2)}{\lim_{t\to -1}(t+3)} \qquad = -3/2.$$

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find all solutions to $x^3 - 3x^2 - x + 3 = 0$

   Solution:

   $$x^3 - 3x^2 - x + 3 = x^2(x-3) - (x-3) = (x^2-1)(x-3) = (x-1)(x+1)(x-3)$$

   So solutions are $x = -1, 1, 3$.

   (b) Compute the limit $\lim_{x\to 2}\frac{x-2}{x^2-4}$

   Solution: If try naively then we get 0/0, so we simplify first:

   $$\frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}$$

   Hence the limit is $\lim_{x\to 2}\frac{1}{x+2} = 1/4$.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x\to 1} \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.

$$\begin{aligned}\frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} &= \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} \cdot \frac{\sqrt{x+2}+\sqrt{4-x}}{\sqrt{x+2}+\sqrt{4-x}} \\ &= \frac{(x+2)-(4-x)}{(x-1)(\sqrt{x+2}+\sqrt{4-x})} \\ &= \frac{2x-2}{(x-1)(\sqrt{x+2}+\sqrt{4-x})} \\ &= \frac{2}{\sqrt{x+2}+\sqrt{4-x}}\end{aligned}$$

So the limit is

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{x+2}-\sqrt{4-x}}{x-1} &= \lim_{x\to 1} \frac{2}{\sqrt{x+2}+\sqrt{4-x}} \\ &= \frac{2}{\sqrt{3}+\sqrt{3}} \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

First Name: ____________ Last Name: ____________

Student-No: ____________ Section: ____________

Very short answer questions

1. [2 marks] Each part is worth 1 marks. Please write your answers in the boxes.

   (a) Compute $\tan\left(\frac{\pi}{6}\right)$.

   Answer: $\frac{1}{\sqrt{3}}$

   Solution:

   $$\cos \pi/6 = \frac{\sqrt{3}}{2} \qquad \sin \pi/6 = \frac{1}{2} \qquad \text{so } \tan \pi/3 = \frac{1}{\sqrt{3}}$$

   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   (b) Compute $\lim_{t\to -2}\left(\frac{t-5}{t+4}\right)$.

   Answer: $-7/2$

   Solution:

   $$\lim_{t\to -2}\left(\frac{t-5}{t+4}\right) = \frac{\lim_{t\to -2}(t-5)}{\lim_{t\to -2}(t+4)} \qquad = -7/2.$$

Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

   (a) Find all solutions to $x^3 - x^2 - 4x + 4 = 0$

   Solution:

   $$x^3 - x^2 - 4x + 4 = x^2(x-1) - 4(x-1) = (x^2-4)(x-1) = (x-2)(x+2)(x-1)$$

   So solutions are $x = 2, -2, 1$.

   (b) Compute the limit $\lim_{x\to 3} \frac{x-3}{x^2-9}$

   Solution: If try naively then we get 0/0, so we simplify first:

   $$\frac{x-3}{x^2-9} = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3}$$

   Hence the limit is $\lim_{x\to 3} \frac{1}{x+3} = 1/6$.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x\to 3} \frac{\sqrt{x-2}-\sqrt{4-x}}{x-3}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.

$$\begin{aligned}
\frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} &= \frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} \cdot \frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}} \\
&= \frac{(x-2)-(4-x)}{(x-3)(\sqrt{x-2}+\sqrt{4-x})} \\
&= \frac{2x-6}{(x-3)(\sqrt{x-2}+\sqrt{4-x})} \\
&= \frac{2}{\sqrt{x-2}+\sqrt{4-x}}
\end{aligned}$$

So the limit is

$$\begin{aligned}
\lim_{x\to 3} \frac{\sqrt{x-2}-\sqrt{4-x}}{x-3} &= \lim_{x\to 3} \frac{2}{\sqrt{x-2}+\sqrt{4-x}} \\
&= \frac{2}{1+1} \\
&= 1.
\end{aligned}$$

First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes.

   (a) Evaluate $\csc\left(\frac{\pi}{3}\right)$.

   Answer: $\frac{2}{\sqrt{3}}$

   Solution:

   $$\sin \pi/3 = \frac{\sqrt{3}}{2} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \text{so } \csc\pi/3 = \frac{2}{\sqrt{3}}$$

   Else draw the appropriate $2 : 1 : \sqrt{3}$ triangle.

   (b) Compute $\lim_{t\to-1}\left(\frac{t^2}{t-1}\right)$.

   Answer: $-1/2$

   Solution:

   $$\lim_{t\to-1}\left(\frac{t^2}{t-1}\right) = \frac{\lim_{t\to-1}(t^2)}{\lim_{t\to-1}(t-1)} \qquad = -1/2.$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Let $f(x) = 3x^2-7x-3$ and $g(x) = 2x^2-6x+3$. Find all values of x for which $f(x) = g(x)$.

   Solution:

   $$3x^2 - 7x - 3 = 2x^2 - 6x + 3 \quad \Leftrightarrow x^2 - x - 6 = 0 \quad \Leftrightarrow (x-3)(x+2) = 0$$

   So solutions are $x = 3, -2$.

   (b) Compute the limit $\lim_{x\to-2}\frac{x+2}{x^2-4}$

   Solution: If try naively then we get $0/0$, so we simplify first:

   $$\frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x-2}$$

   Hence the limit is $\lim_{x\to-2}\frac{1}{x-2} = -1/4$.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x\to 1} \dfrac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.

$$\begin{aligned}\frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1} &= \frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1}\cdot\frac{\sqrt{3x+5}+\sqrt{2x+6}}{\sqrt{3x+5}+\sqrt{2x+6}}\\ &= \frac{(3x+5)-(2x+6)}{(x-1)(\sqrt{3x+5}+\sqrt{2x+6})}\\ &= \frac{x-1}{(x-1)(\sqrt{3x+5}+\sqrt{2x+6})}\\ &= \frac{1}{\sqrt{3x+5}+\sqrt{2x+6}}\end{aligned}$$

So the limit is

$$\begin{aligned}\lim_{x\to 1}\frac{\sqrt{3x+5}-\sqrt{2x+6}}{x-1} &= \lim_{x\to 1}\frac{1}{\sqrt{3x+5}+\sqrt{2x+6}}\\ &= \frac{1}{\sqrt{8}+\sqrt{8}}\\ &= \frac{1}{2\sqrt{8}} = \frac{1}{4\sqrt{2}}\end{aligned}$$

First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes.

   (a) Evaluate $\tan\left(\frac{3\pi}{4}\right)$.

   Answer: -1

   Solution:

   $$\sin(3\pi/4) = \frac{1}{\sqrt{2}} \qquad \cos(3\pi/4) = -\frac{1}{\sqrt{2}} \qquad \text{so} \qquad \tan(3\pi/4) = -1$$

   (b) Compute $\lim_{t\to 2^+} \sqrt{2t^3 - 16}$.

   Answer: 0

   Solution:

   $$\lim_{t\to 2^+} \sqrt{2t^3 - 16} = \sqrt{\lim_{t\to 2^+} 2t^3 - 16} = \sqrt{16 - 16} = 0$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Find all x such that $x^2 + 5x + 6 > 0$.

   Solution:

   $$x^2 + 5x + 6 = (x+2)(x+3)$$

   So the expression is positive for $x < -3$ or $x > -2$.

   (b) Compute the limit $\lim_{x\to -7} \frac{2x + 14}{x^2 - 49}$

   Solution: If try naively then we get 0/0, so we simplify first:

   $$\frac{2x+14}{x^2-49} = \frac{2(x+7)}{(x+7)(x-7)} = \frac{2}{x-7}$$

   Hence the limit is $\lim_{x\to -7} \frac{2}{x-7} = -\frac{2}{14} = -\frac{1}{7}$.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x \to -1} \frac{x+1}{\sqrt{x^2+15}-4}$.

Solution: If we try to do the limit naively we get 0/0. Hence we must simplify.

$$\begin{aligned}
\frac{x+1}{\sqrt{x^2+15}-4} &= \frac{x+1}{\sqrt{x^2+15}-4} \cdot \frac{\sqrt{x^2+15}+4}{\sqrt{x^2+15}+4} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x^2+15)-4^2} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x^2-1)} \\
&= \frac{(x+1)(\sqrt{x^2+15}+4)}{(x+1)(x-1)} \\
&= \frac{\sqrt{x^2+15}+4}{(x-1)}
\end{aligned}$$

So the limit is

$$\begin{aligned}
\lim_{x \to -1} \frac{x+1}{\sqrt{x^2+15}-4} &= \lim_{x \to -1} \frac{\sqrt{x^2+15}+4}{(x-1)} \\
&= \frac{8}{-2} \\
&= -4
\end{aligned}$$