Concordia University

Faculty of Engineering and Computer Science

Department of Mechanical and Industrial Engineering

ENGR 311- Midterm Examination

Total Marks - 30; All Questions Carry Equal Marks October 21, 2003

Professors: Dr. P.Gauthier & Dr. A. Kaushal

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1) (a) Given $F(s)=e^{\frac{-\pi}{2}s}\left[\dfrac{s}{s^2+10s+29}\right]$.

- Find the Inverse Laplace Transform, $\mathcal{L}^{-1}\{F(s)\}=f(t)$ (4 marks)
- Draw a rough sketch of f(t) (2 marks)

(b) Solve the following Integral equation:

$$\int_0^t y(x)y(t-x)dx=6t^3$$ (4 marks)

2.) (a) Solve the following Initial Value Problem:

$$y''+4y=u(\overset{x}{\cancel{t}}-\pi)-u(x-2\pi)$$

$$y(0)=1 \quad \text{and} \quad y'(0)=0$$ (7 marks)

(b) Write the following function in terms of a unit step function

$$f(x)=\begin{cases} x & 0\le x<1 \\ 2-x & 1\le x<2 \\ 0 & x>2 \end{cases}$$

(3marks)

3.) Consider the following function

$$f(x) = \begin{cases} -1 & -\pi\le x<0 \\ x & 0<x\le\pi \end{cases}$$

a) Find the Fourier Series representation of $f(x)$. (6 marks)

b) Sketch the Fourier Series representation of $f(x)$ on $[-7\pi,7\pi]$ (2 marks)

c) To what values will the Fourier Series Converge at $x=2\pi,\frac{5\pi}{2},5\pi$ (2 marks)

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Concordia University

Department of Mechanical and Industrial Engineering

Final Examination

Course:	ENGR 311: Laplace Transforms and Partial Differential Equations
Date:	December 10th, 2002.
Given by:	Dr. Pierre Q. Gauthier
Instructions:	Answer all questions, only non-programmable calculators are permitted.

1. Solve the following differential equation using Laplace transforms

$$y'' + 4y' + 29y = t - 4tu(t-3) = t - [4(t-3) + 12]\,u(t-3)$$

$$y(0) = 0 \quad \text{and} \quad y'(0) = 0$$

2. Solve,

a) $\mathcal{L}^{-1}\left\{\ln(1 + \frac{1}{s})\right\}$

b) $\int_0^t y(u)y(t-u)du = \frac{1}{2}[\sin(t) - t\cos(t)]$

3. Consider the following function

$$f(x) = \begin{cases} 0 & , \; -5 \le x \le 0 \\ 2 + 2x & , \; 0 \le x \le 3 \\ 8 & , \; 3 \le x \le 5 \end{cases}$$

a) Find the Fourier Series of $f(x)$ and sketch it on the interval $[-15, 15]$.

b) To what values will this series converge at x = 0, x = 13 and x = -15?

c) Evaluate:

$$\sqrt{\frac{100}{21}\sum_{n=1}^{\infty}\frac{1-\cos(\frac{3n\pi}{5})}{n^2}}$$

Concordia University
Final Examination
EMAT 311

Date : December 2000.
Duration : 3 hours.
Instructors :H.Bouchard
Course Examiner : H.Bouchard.
Directives : Answer all 7 questions. CALCULATOR ALLOWED. Formula sheet is provided.

MARKS

(12) 1. Evaluate.

a) $L\{e^{3t}\cos(t-\pi)\}$ c) $L^{-1}\left\{\frac{s+1}{s(s^2+1)}\right\}$

b) $L\{t^2u(t-2)\}$ d) $L^{-1}\left\{\frac{1}{(s-1)(s-2)(s-3)}\right\}$

(12) 2. Find the Fourier series (period p=2L=2π) for the following functions.

a) $f(x)=|x| \quad (-\pi<x<\pi)$

b) $g(x)=x \quad (-\pi<x<\pi)$

c) $h(x)=\cos^3(x) \quad (-\pi<x<\pi)$

HINT : $\cos(x)=\left(e^{ix}+e^{-ix}\right)/2$

(10) 3. Solve the initial value problem using Laplace transform :

$y''(t)+9y(t)=r(t), \quad y'(0)=y(0)=0;$

$$r(t)=\begin{cases}\pi-t & \text{if } t<\pi,\\ \sin(t) & \text{if } t>\pi.\end{cases}$$

$r(t)=(\pi-t)[u(t-\pi)]+$

$[u(t)-u(t-a)]f(a)$

(8) 4. Solve the integral equation using Laplace transform :

$$y(t)=(t+1)-\int_0^t(\tau-1)y(t-\tau)d\tau.$$

(10) 5. Find the deflection function $u(x,t)$ of a vibrating spring with length L=π and c=1. Suppose that the <u>initial deflection</u> is $u(x,0)=f(x)=\sin(2x)$ and the <u>initial velocity</u> is $u_t(x,0)=g(x)=-\sin(2x)$.

(10) 6. The faces of a thin square plate (side a=10) are perfectly insulated and c=1. The upper side is kept at a temperature of 10°C and the other sides are kept at 0°C. Find the steady state temperature u(x,y) in the plate.

(8) 7. Find non trivial solutions $u(x,y)$ of $xu_x - yu_y = 0$ by separating variables.

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Department of Mathematics and Statistics

Course	**Number**	**Section(s)**	
Engineering Mathematics	311	All	
Examination	**Date**	**Time**	**Pages**
Final	June 23, 1999	3 hours	2

Instructor: Chukova
Special Instructions: Formula Sheet attached
Calculators allowed
Show all your work

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1. Evaluate: [5] a. $\mathcal{L}^{-1}\left\{\arctan\left(\frac{\omega}{s}\right)\right\}$

[5] b. $\mathcal{L}\left\{\int_0^t \frac{\sin u}{u} du\right\}$

[5] c. $\mathcal{L}\left\{t\, e^{-t} \sin t\right\}$

2. Use the Laplace transform to solve:

[10] a. $y'' + 3y' + 2y = r(t);\ r(t) = 4t$ if $0 < t < 1$ and $r(t) = 8$ if $t > 1$.
$y(0) = y'(0) = 0$

[10] b. $y(t) = 1 - \sinh(t) + \int_0^t (1+\tau)y(t-\tau)d\tau$ conv.

3. Let $f(x) = \begin{cases} \sin\frac{\pi x}{L}, & 0 \le x < \frac{L}{2} \\ 0, & \frac{L}{2} < x < L \end{cases}$

[3] a. Graph an odd periodic extension of f(x) on [-3L,3L].

[5] b. Expand f(x) in the Fourier sine series.

[5] c. Find $f\left(\frac{L}{2}\right)$, $f(0)$, $f\left(-\frac{L}{2}\right)$, $f\left(-\frac{5L}{2}\right)$.

[8] 4. Represent the given function by an appropriate cosine or sine Fourier Integral .

$$f(x) = \begin{cases} 0, & |x| < 1 \\ \pi, & 1 < |x| < 2 \\ 0, & |x| > 2 \end{cases}$$

What is the value of this integral at x = 1?

[10] 5. Find the temperature $u(x,t)$ in a bar of length $L = \pi$ and $c = 1$ that is perfectly insulated and whose ends are kept at temperature 0, assuming that the initial temperature is

$$\mathbf{f(x)} = \begin{cases} \mathbf{x}, & 0 \le \mathbf{x} < \frac{\mathbf{L}}{2} \\ \mathbf{L - x}, & \frac{\mathbf{L}}{2} < \mathbf{x} < \mathbf{L} \end{cases}$$

[10] 6. The faces of a thin square plate (side $a = 24$) are perfectly insulated. The **lower** side is kept at temperature 20°C and the other sides are kept at 0°C. Find the steady-state temperature $u(x,y)$ in the plate.

$a = b = 24$ $f(x) = 20°$

Hint: Show that

$$u(x,y) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a} \sinh\frac{n\pi(a-y)}{a}, \quad A_n = \frac{2}{a\sinh(n\pi)}\int_0^a f(x)\sin\frac{n\pi x}{a}\,dx.$$

7. [7] a. Find solution $u(x,y)$ of $\mathbf{u}_x - \mathbf{y}\mathbf{u}_y = \mathbf{0}$ by separating the variables.

[7] b. Solve $\mathbf{x}\mathbf{u}_{xy} = \mathbf{y}\mathbf{u}_{yy} + \mathbf{u}_y$ using the transformation $v = x$, $z = xy$.

[10] 8. Find the deflection of the vibrating string (length $= \pi$, ends fixed, $c = 1$) corresponding to zero initial velocity and initial deflection $\mathbf{f(x)} = \sin \mathbf{x} - \frac{1}{2}\sin 2\mathbf{x}$.

$c = 1$

$L = \pi$

$u(0,t) = u(\pi,t) = 0$

$g(x) = 0$

b_n

Concordia University
Department of Mathematics and Statistics

Course	**Number**	**Section (s)**	
Engineering Mathematics	311/2	all	
Examination	**Date**	**Time**	**Pages**
Final	21 December 1998	3 hours	3
Instructors			
Harnad, Keviczky			

Special Instructions

1 - Calculators allowed.
2 - Laplace Transforms attached.
3 - All problems have equal value.

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[10] **1.** Find the inverse Laplace transform of the following two functions:

(A) $\frac{e^{-4s}}{s} + \frac{2e^{-5s}}{s^2+4}$

(B) $\frac{2}{(s-1)(s+1)}$

[10] **2.** Solve the following ordinary differential equation by means of the Laplace transform:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 2u(t-2) \quad \text{with} \quad y(0) = y'(0) = 0.$$

where $u(t-2)$ is the unit step function

$$u(t-2) = \begin{cases} 0 & \text{if} \quad t < 2 \\ 1 & \text{if} \quad t > 2 \end{cases}$$

[10] **3.** By constructing the odd extension of

$$f(x) = \begin{cases} 0 & \text{if} \quad 0 < x < \frac{\pi}{2} \\ -3 & \text{if} \quad \frac{\pi}{2} < x < \pi \end{cases}$$

on the interval $(-\pi, \pi)$, and its periodic extension of period 2π, find the Fourier sine expansion $\sum_{n=1}^{\infty} b_n \sin nx$ of $f(x)$ and indicate the precise value that it converges to at $x = \frac{\pi}{2}$.

[10] **4.** Represent the function:

$$f(x) = \begin{cases} \sin 3x & \text{if} \quad 0 < x < 10 \\ 0 & \text{if} \quad 10 < x \end{cases}$$

in terms of the Fourier cosine integral and indicate the exact value this Fourier integral assumes at $x = 10$.

[10] **5.** Find the deflection of the vibrating string of length L=π, with ends fixed, and $c^2 = \frac{T}{\rho} = 1$, initial deflection $f(x) = 0.4\, sin\ 5x$ and initial velocity $g(x) = -0.7 \sin\ 12x$.

[10] **6.** Find the temperature $u(x,t)$ in a bar of length L that is perfectly insulated at the ends at $x = 0$, and $x = L$, assuming that

$$u_x(0,t) = 0,\ u_x(L,t) = 0, \quad \text{for all } t,$$
$$u(x,0) = f(x).$$

Show that the method of separation of variables yields the solution

$$u(x,f) = \frac{A_o}{2} + \sum_{n=1}^{\infty} A_n e^{-(cn\pi/L)^2 t}\ cos\left(\frac{n\pi}{L}x\right),$$

where $A_n = \frac{2}{L}\int_0^L f(x)\ cos\left(\frac{n\pi x}{L}\right) dx$ for all integers $n \geq 0$.

[10] **7.** Given the Laplace equation $\nabla^2\ u = 0$ in polar coordinates (r,θ), where

$(x,y) = (rcos\theta, r\ sin\theta)$ and

$\nabla^2\ u = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) + \overset{0}{\cancel{\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}}},$

find the general form the solution u takes, if it is assumed to be independent of θ (a function of r only).

$$0 = \cancel{\frac{1}{r}}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$$

$$\cancel{r}\frac{\partial u}{\cancel{\partial r}} = \frac{k}{r}\,\partial r$$

$$u = \cancel{k}\ln r^k$$

$$u = \ln r^k + C$$

AK/ss

Table of Laplace Transforms

	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$	Sec.
1	$1/s$	1	
2	$1/s^2$	t	
3	$1/s^n, \quad (n = 1, 2, \cdots)$	$t^{n-1}/(n-1)!$	6.1
4	$1/\sqrt{s}$	$1/\sqrt{\pi t}$	
5	$1/s^{3/2}$	$2\sqrt{t/\pi}$	
6	$1/s^a \quad (a > 0)$	$t^{a-1}/\Gamma(a)$	
7	$\dfrac{1}{s-a}$	e^{at}	6.1
8	$\dfrac{1}{(s-a)^2}$	te^{at}	
9	$\dfrac{1}{(s-a)^n} \quad (n = 1, 2, \cdots)$	$\dfrac{1}{(n-1)!}t^{n-1}e^{at}$	6.3
10	$\dfrac{1}{(s-a)^k} \quad (k > 0)$	$\dfrac{1}{\Gamma(k)}t^{k-1}e^{at}$	
11	$\dfrac{1}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(e^{at} - e^{bt})$	6.1
12	$\dfrac{s}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(ae^{at} - be^{bt})$	
13	$\dfrac{1}{s^2+\omega^2}$	$\dfrac{1}{\omega}\sin \omega t$	
14	$\dfrac{s}{s^2+\omega^2}$	$\cos \omega t$	
15	$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\sinh at$	6.1
16	$\dfrac{s}{s^2-a^2}$	$\cosh at$	
17	$\dfrac{1}{(s-a)^2+\omega^2}$	$\dfrac{1}{\omega}e^{at}\sin \omega t$	6.3
18	$\dfrac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$	
19	$\dfrac{1}{s(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}(1 - \cos \omega t)$	
20	$\dfrac{1}{s^2(s^2+\omega^2)}$	$\dfrac{1}{\omega^3}(\omega t - \sin \omega t)$	6.2
21	$\dfrac{1}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega^3}(\sin \omega t - \omega t \cos \omega t)$	
22	$\dfrac{s}{(s^2+\omega^2)^2}$	$\dfrac{t}{2\omega}\sin \omega t$	
23	$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	6.5
24	$\dfrac{s}{(s^2+a^2)(s^2+b^2)} \quad (a^2 \neq b^2)$	$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$	
25	$\dfrac{1}{s^4+4k^4}$	$\dfrac{1}{4k^3}(\sin kt \cosh kt - \cos kt \sinh kt)$	
26	$\dfrac{s}{s^4+4k^4}$	$\dfrac{1}{2k^2}\sin kt \sinh kt$	6.7
27	$\dfrac{1}{s^4-k^4}$	$\dfrac{1}{2k^3}(\sinh kt - \sin kt)$	
28	$\dfrac{s}{s^4-k^4}$	$\dfrac{1}{2k^2}(\cosh kt - \cos kt)$	
29	$\sqrt{s-a} - \sqrt{s-b}$	$\dfrac{1}{2\sqrt{\pi t^3}}(e^{bt} - e^{at})$	
30	$\dfrac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-(a+b)t/2}I_0\left(\dfrac{a-b}{2}t\right)$	5.7
31	$\dfrac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$	5.5
32	$\dfrac{s}{(s-a)^{3/2}}$	$\dfrac{1}{\sqrt{\pi t}}e^{at}(1+2at)$	
33	$\dfrac{1}{(s^2-a^2)^k} \quad (k > 0)$	$\dfrac{\sqrt{\pi}}{\Gamma(k)}\left(\dfrac{t}{2a}\right)^{k-1/2} I_{k-1/2}(at)$	5.7
34	e^{-as}/s	$u(t-a)$	6.3
35	e^{-as}	$\delta(t-a)$	6.4
36	$\dfrac{1}{s}e^{-k/s}$	$J_0(2\sqrt{kt})$	5.5
37	$\dfrac{1}{\sqrt{s}}e^{-k/s}$	$\dfrac{1}{\sqrt{\pi t}}\cos 2\sqrt{kt}$	
38	$\dfrac{1}{s^{3/2}}e^{k/s}$	$\dfrac{1}{\sqrt{\pi k}}\sinh 2\sqrt{kt}$	
39	$e^{-k\sqrt{s}} \quad (k > 0)$	$\dfrac{k}{2\sqrt{\pi t^3}}e^{-k^2/4t}$	
40	$\dfrac{1}{s}\ln s$	$-\ln t - \gamma \quad (\gamma \approx 0.5772)$	5.7
41	$\ln \dfrac{s-a}{s-b}$	$\dfrac{1}{t}(e^{bt} - e^{at})$	
42	$\ln \dfrac{s^2+\omega^2}{s^2}$	$\dfrac{2}{t}(1 - \cos \omega t)$	6.5
43	$\ln \dfrac{s^2-a^2}{s^2}$	$\dfrac{2}{t}(1 - \cosh at)$	
44	$\arctan \dfrac{\omega}{s}$	$\dfrac{1}{t}\sin \omega t$	
45	$\dfrac{1}{s}\operatorname{arc\,cot} s$	$\operatorname{Si}(t)$	App. 3

Laplace Transform: General Formulas

Formula	Name, Comments	Sec.
$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)\,dt$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Definition of Transform Inverse Transform	6.1
$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$	Linearity	6.1
$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$ $\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$ $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{1}{s}\mathcal{L}(f)$	Differentiation of Function Integration of Function	6.2
$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$	s-Shifting (1st Shifting Theorem)	6.3
$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	t-Shifting (2nd Shifting Theorem)	6.3
$\mathcal{L}\{tf(t)\} = -F'(s)$ $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \int_s^\infty F(\tilde{s})\,d\tilde{s}$	Differentiation of Transform Integration of Transform	6.5
$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau = \int_0^t f(t-\tau)g(\tau)\,d\tau$ $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$	Convolution	6.6
$\mathcal{L}(f) = \dfrac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt$	f Periodic with Period p	6.8

Concordia
UNIVERSITY

Department of Mathematics & Statistics

(14)

Course	Number	Section(s)
Engineering Mathematics	311/2	all

Examination	Date	Time	Pages
Final	December 17, 1997	3 hours	~~3~~ 2 only Rec'd

Instructor

J. Hayes, H. Kisilevsky

Special Instructions

- Calculators allowed.
- Laplace Transforms attached.
- All problems have equal value.

1. Suppose that $f(t)$ has the Laplace transform $\mathcal{L}(f(t)) = F(s)$.

   a). Find $f(t)$ if

$$F(s) = \frac{s}{(s^2+2s+2)(s+2)}.$$

$\frac{s}{s^2+2s+2}$ $\frac{s}{s+2}$

$\frac{s}{((s+1)^2+1)(s+2)}$

   b). Compute $F(s)$ if $f(t) = te^{-t}\cos t$.

2. Using Laplace transforms, solve the initial value problem

$$y'' + 5y' + 7y = u(t-1) + \delta(t-1), \qquad y(0) = y'(0) = 0.$$

3. Let

$$f(x) = \begin{cases} 1, & \text{for } 0 < x < 1 \\ 0, & \text{for } 1 \le x \le 2. \end{cases}$$

$\frac{s}{((s+1)^2+1)}$ $\frac{s}{(s+2)}$

$e^{-t}\sin t *$

   a). Find the Fourier cosine series for the *even* periodic extension of the function f.

   b). To what values does the Fourier series converge if $x = 3.7$, and if $x = 5$?

4. Obtain the Fourier integral representation of $f(x)$ if

$$f(x) = \begin{cases} 0, & \text{for } x < 0 \\ x, & \text{for } 0 < x < 1 \\ 0, & \text{for } 1 \le x < \infty. \end{cases}$$

$(e^{-t}\cos t - e^{-t}\sin t) * e^{-2t}$

$\int e^{-3(t-\tau)}\cos\tau - \int e^{-3(t-\tau)}\sin\tau$

$e^{-3t}\left[\int e^{3\tau}\cos\tau - \int e^{3\tau}\sin\tau\right]$

5. Use the method of separation of variables to find the temperature $u(x,t)$ in a rod of length 2 if

$$\partial u/\partial t = \partial^2 u/\partial x^2,$$

with $u(0,t) = u(2,t) = 0$, (and $c = 1$) and

$$u(x,0) = \begin{cases} x, & \text{for } 0 \le x \le 1 \\ 2-x, & \text{for } 1 \le x \le 2. \end{cases}$$

6. Find the deflection $u(x,t)$ in a vibrating string (length $= \pi$, ends fixed, and $c^2 = 1$) if the initial deflection is $u(x,0) = 2(\sin x - \sin 3x)$ and the initial velocity $u_t(x,0) = 0$. What is $u(\pi/2, 3)$?

7. Find the temperature in the square $0 \le x \le 2$, $0 \le y \le 2$ if the upper side is kept at a temperature of $\sin \pi x/2$, and the other sides are kept at 0.

8. Solve the the following equation using the indicated change of variables:

$$u_{xx} - 4u_{xy} + 3u_{yy} = 0, \quad (v = x + y, z = 3x + y).$$

$\frac{du}{dx} = \frac{du}{dv}\frac{dv}{dx} + \frac{du}{dz}\frac{dz}{dx}$

$2-x \quad + \quad \frac{\sin n\pi x}{2}$

$-1 \quad \frac{n\pi}{2} \cos \frac{n\pi x}{2}$

0

$(-2+x)\frac{n\pi}{2} \cos \frac{n\pi x}{2}) - \frac{n\pi}{2} \cos \frac{n\pi x}{2}$

$\left(-\frac{n\pi}{2} \cos \frac{\pi x}{2}\right) + \frac{n\pi x}{2} \cos \frac{\pi x}{2} - \frac{n\pi x}{2} \cos \frac{n\pi x}{2}$

CONCORDIA UNIVERSITY — **Department of Mathematics & Statistics**

Course	**Number**		**Section(s)**
Engineering Mathematics	311/2		All
Examination	**Date**	**Time**	**# of pages**
Final	December 1996	3 hours	3
Instructor			**Course Examiner**
J. Hayes, J.C. Turgeon			J. Hayes

Materials Allowed:
- Calculators Permitted
- Ruled Paper Booklets
- Laplace Transform Table attached

Marks

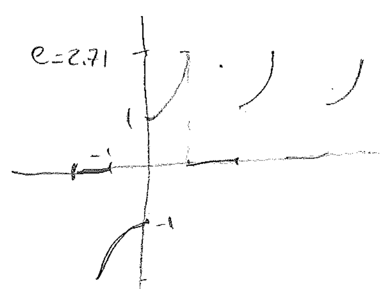
14 1. a) Find the Laplace transform of the periodic function $f(t) = e^{-t} \quad 0 < t < 3$, $f(t+3) = f(t)$

b) Use the Laplace transform to solve $y'' + 6y' + 5y = 1 - 2\,u(t-2)$, $y(0) = y'(0) = 0$.

16 2. a) Find the Laplace transform of $f(t) = \int_0^t \frac{\sin u}{u}\,du$ (the sine integral).

b) Use the Laplace transform to solve the integral equation

$$y(t) = 1 - \int_0^t (t-\tau)\, y(\tau)\, d\tau$$



10 3. We define $f(x) = \begin{cases} e^x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$

Sketch the odd periodic extension of this function for $-6 < x < 6$. To what values will the corresponding Fourier series converge at $x = 2$, $x = -1$, $x = 5$?

12 4. Obtain the fourier sine integral representation for $f(x) = \begin{cases} a - x & 0 < x < a \\ 0 & x > a \end{cases}$.

Marks

12 5. Use the method of separation of variables to find the temperature $u(x,t)$ in a bar of length $L = 4$ that is perfectly insulated, (also at the ends).

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u_x(0, t) = u_x(4, t) = 0$$

$$u(x,0) = \begin{cases} 10 & 0 < x < 2 \\ 0 & 2 < x < 4 \end{cases}$$

What does $\lim_{t \to \infty} u(x,t)$ equal ?

$\frac{\partial u}{\partial t} = 0 \to \frac{\partial^2 u}{\partial x^2} = 0$

$\frac{\partial u}{\partial x} = C_1$ $\boxed{u(x) = C_1 x + C_2}$

steady-state sol.

12 6. The faces of a thin square plate (side length 24) are perfectly insulated. The upper side is kept at a temperature of $20^\circ C$ and the other sides are kept at $0^\circ C$. Find the steady-state temperature $u(x,y)$ in the plate. (Use the method of separation of variables).

12 7. Find the deflection $u(x,t)$ of the vibrating string (length $L = \pi$, ends fixed, with $c^2 = \frac{T}{\rho} = 1$) corresponding to zero initial velocity and initial deflection $u(x,0) = .1 \sin x - .02 \sin 3x$. (Use the method of separation of variables).

$\frac{d^2 u}{dr^2}$

12 8. a) Solve the differential equation $u_{xx} - 2u_{xy} + u_{yy} = 0$ by using the transformation $v = x$, $z = x + y$.

$z = 3x + y$ $z = xy$

b) Find the Fourier cosine series for $f(x) = \sin^2 x$ $0 < x < \pi$.

$z = 3x + y$

$z = xy$

$\frac{\partial z}{\partial x}$ $\frac{\partial z}{\partial y}$

$\frac{dz}{dx} = y$

$\frac{dz}{dy} = x$

JCT/bjc

Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. t^n, n = positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}$, n = positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$

$$u_c(t) \equiv u(t-c)$$

0.13

2

K.3

Concordia
UNIVERSITY

Department of Mathematics & Statistics

Course	Number		Section(s)
Engineering Mathematics	311/2		V
Examination	**Date**	**Time**	**Pages**
Final	December 1995	3 hours	2
Instructors		**Course Examiner**	
M. Zaki		M. Zaki	

Materials allowed:
Calculators permitted.

MARKS

[10] **1** Let $f(t)$ be defined by:

(a) Express $f(t)$ in terms of unit step function.
(b) Find the Laplace transform of this representation.
(c) Evaluate $L^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\}$

[10] **2** Use the Laplace transform methods to solve

$$y'' + 6y' + 5 = t - tu(t-2)$$
$$y(0) = y'(0) = 0$$

[10] **3** Using the Laplace transform, solve the integral equation

$$\frac{dy}{dt} = \cos t + \int_0^t y(w)\ \cos(t-w)\ dw$$
$$y(0) = 1$$

[10] **4** Let $f(x) = \begin{cases} \sin\frac{\pi x}{L} & \text{if } 0 \le x \le \frac{L}{2} \\ 0 & \text{if } \frac{L}{2} < x < L \end{cases}$

(a) Graph an odd periodic extension of $f(x)$ on $[-2L, 2L]$.
(b) Expand $f(x)$ in a sine series.
(c) Find $f(-\frac{L}{2})$, $f(\frac{L}{2})$, $f(L)$

[10] 5 Obtain the Fourier cosine integral representation for

$$f(x) = \begin{cases} 1-x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

[10] 6 Use the method of separating variables to find the temperature $u(x,t)$ in a rod of length 2 if

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

$$u(o,t) = 0, \quad u(2,t) = 0$$

$$u(x,0) = \begin{cases} x & 0 < x < 1 \\ 0 & 1 \leq x < 2 \end{cases}$$

[15] 7 Find the deflection $u(x,t)$ of the vibrating string (length $= \pi$, ends fixed, and $c^2 = 1$) corresponding to zero initial velocity and initial deflection $3(\sin x - \sin 5x)$.

[12] 8 The faces of a thin square plate, $a = 10$, are perfectly insulated. The upper side is kept at a temperature 200°C and the other sides are kept at 0°C. Find the steady-state temperature $u(x,y)$ in the plate.

[16] 9 (a) Find solution $U(x,y)$ of $U_x + U_y = 2(x+y)U$ by separating variables.

(b) Solve: $U_{xy} - U_{yy} = 0$ using the transformation $v = x,\ z = x + y$.

CONCORDIA UNIVERSITY — Department of Mathematics & Statistics

Course	Number	Section(s)
Engineering Mathematics	311/2	All

Examination	Date	Time	# of pages
Final	December 1994	3 hours	3

Instructors	Course Examiner:
M. Bobetic, M. Zaki	M. Zaki

Materials Allowed: Calculators permitted.

MARKS

10 1. a) Find the convolution $u(t - 1) * t$.

b) Solve: $y(t) = \sin t + \int_0^t y(w) \sin(t - w)dw$

18 2. a) Find the Laplace transform of $|\sin \frac{t}{2}|$.

b) Find $f(t)$ if $\mathcal{L}(f) = \ln \frac{s^2 - 1}{(s - 1)^2}$.

c) Solve: $y'' + 3y' + 2y = \delta(t - 4)$, $y(0) = y'(0) = 0$

10 3. Find the Fourier series of $f(x) = x$ $(-4 < x < 4)$ with period $p = 2L = 8$.

12 4. a) Represent $f(x) = x$ $(0 < x < L)$ by a Fourier <u>cosine series</u> and sketch the corresponding periodic extension of $f(x)$.

b) Using the Fourier integral representation, show that:

$$\int_0^\infty \frac{\sin w \cos xw}{w} dw = \begin{cases} \frac{\pi}{2} & \text{if } 0 \le x < 1 \\ \frac{\pi}{4} & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

16 5. a) Find solution $U(x, y)$ of $U_x + U_y = 2(x + y)U$ by separating variables.

b) Solve: $U_{xy} - U_{yy} = 0$ using the transformation $v = x$, $z = x + y$.

MARKS

12 6. Find the deflection $u(x,t)$ of the vibrating string (length = Π, ends fixed, and $c^2 = \frac{T}{\rho} = 1$) corresponding to zero initial velocity and initial deflection $2(\sin x - \sin 3x)$.

12 7. Find the temperature $u(x,t)$ in a bar of silver (length 10 cm, constant cross section of area 1 cm^2, density 10.6 gm/cm^3 thermal conductivity 1.04 cal/cm sec ^{o}C, specific heat .056 cal/gm ^{o}C) that is perfectly insulated laterally, whose ends are kept at temperature O ^{o}C and whose initial temperature (in ^{o}C) is $f(x)$ where

$f(x) = \sin(0.4\, \Pi x)$.

10 8. The faces of a thin rectangular plate (12 x 5) are perfectly insulated. The upper edge, of length 12, is kept at temperture 30^{o} C, and other sides are kept at O^{o} C. Find the steady-state temperature $u(x,y)$ in the plate.

MZ/bjc

TABLE OF LAPLACE TRANSFORMS

	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$
1	$1/s$	1
2	$1/s^2$	t
3	$1/s^n, \quad (n = 1, 2, \cdots)$	$t^{n-1}/(n-1)!$
4	$1/\sqrt{s}$	$1/\sqrt{\pi t}$
5	$1/s^{3/2}$	$2\sqrt{t/\pi}$
6	$1/s^a \quad (a > 0)$	$t^{a-1}/\Gamma(a)$
7	$\dfrac{1}{s-a}$	e^{at}
8	$\dfrac{1}{(s-a)^2}$	te^{at}
9	$\dfrac{1}{(s-a)^n} \quad (n = 1, 2, \cdots)$	$\dfrac{1}{(n-1)!}t^{n-1}e^{at}$
10	$\dfrac{1}{(s-a)^k} \quad (k > 0)$	$\dfrac{1}{\Gamma(k)}t^{k-1}e^{at}$
11	$\dfrac{1}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(e^{at} - e^{bt})$
12	$\dfrac{s}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(ae^{at} - be^{bt})$
13	$\dfrac{1}{s^2+\omega^2}$	$\dfrac{1}{\omega}\sin \omega t$
14	$\dfrac{s}{s^2+\omega^2}$	$\cos \omega t$
15	$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\sinh at$
16	$\dfrac{s}{s^2-a^2}$	$\cosh at$
17	$\dfrac{1}{(s-a)^2+\omega^2}$	$\dfrac{1}{\omega}e^{at}\sin \omega t$
18	$\dfrac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$
19	$\dfrac{1}{s(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}(1 - \cos \omega t)$
20	$\dfrac{1}{s^2(s^2+\omega^2)}$	$\dfrac{1}{\omega^3}(\omega t - \sin \omega t)$
21	$\dfrac{1}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega^3}(\sin \omega t - \omega t \cos \omega t)$
22	$\dfrac{s}{(s^2+\omega^2)^2}$	$\dfrac{t}{2\omega}\sin \omega t$
23	$\dfrac{s^2}{(s^2+\omega^2)^2}$	$\dfrac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$
24	$\dfrac{s}{(s^2+a^2)(s^2+b^2)} \quad (a^2 \neq b^2)$	$\dfrac{1}{b^2-a^2}(\cos at - \cos bt)$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\mathcal{L}[u(t-a)] = \frac{e^{-as}}{s}, \quad \mathcal{L}[\delta(t-a)] = e^{-as}.$$

CONCORDIA UNIVERSITY

Dept. of Mathematics & Statistics

Course	Number	Section(s)
Engineering Mathematics	311/2	All

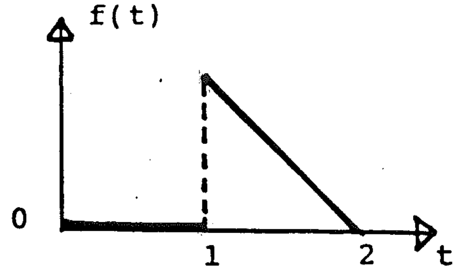
Examination	Date	Time	# of pages
Final	December 1992	3 hours	4

Instructors	Course Examiner:
Bobetic, Zaki	M. Bobetic

Materials Allowed: Calculators Permitted.

marks

10 1. Let $f(t)$ be defined by:



a) Express $f(t)$ in terms of unit step function

b) Find the Laplace transform of this representation

c) Evaluate $L^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\}$

10 2. Use the Laplace transform methods to solve

5y

$$y'' + 6y' + 5 = t - tu(t-2)$$

$$y(0) = y'(0) = 0$$

10 3. Using the Laplace transform solve the integral equation

$$y' = \cos t + \int_0^t y(\tau)\cos(t-\tau)\,d\tau$$

$$y(0) = 1$$

.../2

MARKS

15 4. Let $f(x) = \begin{cases} \sin \frac{\pi x}{L} & \text{if } 0 \le x \le \frac{L}{2} \\ 0 & \text{if } \frac{L}{2} < x < L \end{cases}$

a) Graph an odd periodic extension of $f(x)$ on $[-2L, 2L]$.

b) Expand $f(x)$ in a sine series.

c) Find $f\left(-\frac{L}{2}\right)$, $f\left(\frac{L}{2}\right)$, $f(L)$

10 5. Let $f(x)$ be periodic with the period 2π defined

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

Find the complex form of its Fourier expansion.

10 6. Obtain the Fourier cosine integtral representation for

$$f(x) = \begin{cases} 1 - x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

10 7. Use the method of separating variables to find the temperature $u(x, t)$ in a rod of length $L = 2$, if

$$\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad K = \text{constant}$$

$$u(o,t) = 0, \quad u(L, t) = 0$$

$$u(x,0) = \begin{cases} x & 0 < x < 1 \\ 0 & 1 \le x < 2 \end{cases}$$

(continued next page)

MARKS

15 8. Solve Laplace equation for steady-state temperature $U(x,y)$ in a rectangular plate with indicated boundaries if

$$u_{xx} + u_{yy} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$u(x,0) = 0, \quad u(x,b) = f(x)$$
$$u(0,y) = 0, \quad u(a,y) = 0$$

10 9. Using the method of separation of variables, find the solution to the vibrating string problem described by

$$u_{tt} = 4u_{xx}$$

$$u(0,t) = u(1,t) = 0$$
$$u(x,0) = x(1 - x), \quad u_t(x,0) = 1 - x$$

	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$	Cf. Sec.
1	$1/s$	1	5.1
2	$1/s^2$	t	
3	$1/s^n, \quad (n = 1, 2, \cdots)$	$t^{n-1}/(n-1)!$	
4	$1/\sqrt{s}$	$1/\sqrt{\pi t}$	
5	$1/s^{3/2}$	$2\sqrt{t/\pi}$	
6	$1/s^a \quad (a > 0)$	$t^{a-1}/\Gamma(a)$	
7	$\dfrac{1}{s-a}$	e^{at}	5.1
8	$\dfrac{1}{(s-a)^2}$	te^{at}	5.3
9	$\dfrac{1}{(s-a)^n} \quad (n = 1, 2, \cdots)$	$\dfrac{1}{(n-1)!} t^{n-1}e^{at}$	
10	$\dfrac{1}{(s-a)^k} \quad (k > 0)$	$\dfrac{1}{\Gamma(k)} t^{k-1}e^{at}$	
11	$\dfrac{1}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(e^{at} - e^{bt})$	5.1
12	$\dfrac{s}{(s-a)(s-b)} \quad (a \neq b)$	$\dfrac{1}{(a-b)}(ae^{at} - be^{bt})$	
13	$\dfrac{1}{s^2 - \omega^2}$	$\dfrac{1}{\omega}\sin \omega t$	5.1
14	$\dfrac{s}{s^2 - \omega^2}$	$\cos \omega t$	
15	$\dfrac{1}{s^2 - a^2}$	$\dfrac{1}{a}\sinh at$	
16	$\dfrac{s}{s^2 - a^2}$	$\cosh at$	
17	$\dfrac{1}{(s-a)^2 + \omega^2}$	$\dfrac{1}{\omega} e^{at}\sin \omega t$	5.3
18	$\dfrac{s-a}{(s-a)^2 + \omega^2}$	$e^{at}\cos \omega t$	
19	$\dfrac{1}{s(s^2 + \omega^2)}$	$\dfrac{1}{\omega^2}(1 - \cos \omega t)$	5.2
20	$\dfrac{1}{s^2(s^2 + \omega^2)}$	$\dfrac{1}{\omega^3}(\omega t - \sin \omega t)$	
21	$\dfrac{1}{(s^2 + \omega^2)^2}$	$\dfrac{1}{2\omega^3}(\sin \omega t - \omega t \cos \omega t)$	5.5
22	$\dfrac{s}{(s^2 + \omega^2)^2}$	$\dfrac{t}{2\omega}\sin \omega t$	
23	$\dfrac{s^2}{(s^2 + \omega^2)^2}$	$\dfrac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	
24	$\dfrac{s}{(s^2 + a^2)(s^2 + b^2)} \quad (a^2 \neq b^2)$	$\dfrac{1}{b^2 - a^2}(\cos at - \cos bt)$	