

Concordia University
Department of Electrical and
Computer Engineering

ELEC-331
Lecture 4
Magnetic circuits
Chapter 1

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Introduction and justification

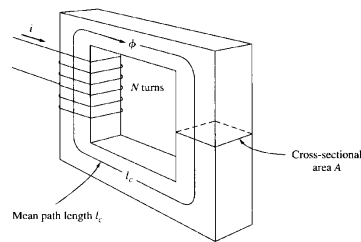
- Electro(**magneto**)mechanical devices: A magnetic field is often the means by which conversion from electrical to mechanical energy takes place.
- A current-carrying wire produces a magnetic field.
- A time varying magnetic field induces a voltage in a coil (*transformer action*).
- A current-carrying wire in the presence of a magnetic field has a force induced on it (*motor action*)
- A moving wire in the presence of a magnetic field has a voltage induced on it (*generator action*)

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Production of a magnetic field

- Ampere's Law: "The tangential component of the *magnetic field intensity* vector (H) integrated along a closed path equals the net current enclosed by this path." $\oint H \cdot dl = I_{net}$

- Example: Rectangular core with a winding with N turns.
Assumption: The magnetic field remains within the core ($H = \text{cte}$). Path of integration: the mean path length of the core (l_c).



$$H l_c = N i$$

$$H = \frac{N i}{l_c}$$

(Ampere-turn/m)

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The *magnetic flux density* (B)

- The strength of the magnetic field flux depends on the material of the core (μ : magnetic permeability)

$$B = \mu H, \quad \mu = \mu_r \mu_0 \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$B = \frac{\mu N i}{l_c} \text{ (Tesla)}$$

- The total flux (ϕ) in a given area

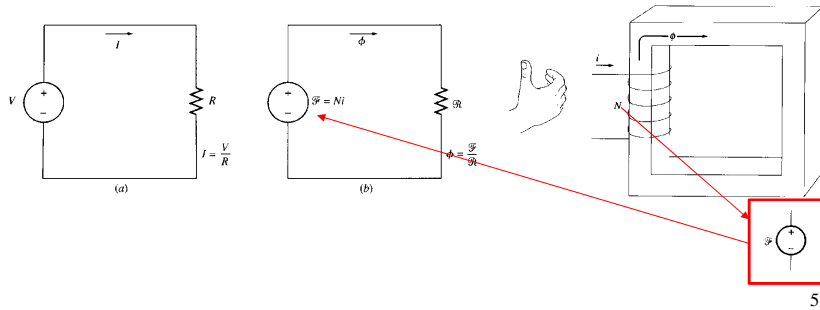
$$\phi = \int_A B \cdot dA \quad \phi = BA \quad \phi = \frac{\mu N i A}{l_c} \text{ (Weber - Wb)}$$

B is assumed constant and perpendicular to the plane of A

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Magnetic circuits

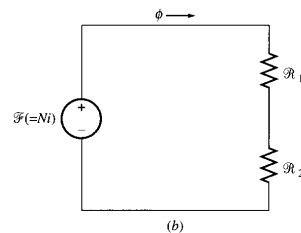
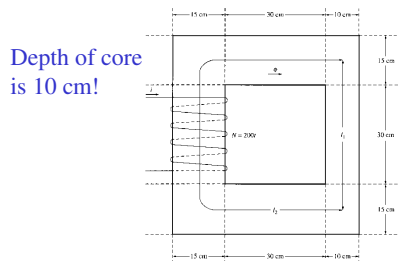
- Analogy: Magnetic and electric circuits.
- $V = IR$ (Ohm's law) $\mathcal{F} = \phi \mathcal{R}$
- \mathcal{F} is the magnetomotive force (mmf), ϕ is the flux and \mathcal{R} is the reluctance of the magnetic circuit.
- Direction of the flux: Right-hand rule.



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Magnetic circuits

- Magnetomotive force: $\mathcal{F} = Ni$
- Reluctance: $\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{l_c}{\mu A}$ from $\phi = \frac{\mu Ni A}{l_c}$
- **Examples:**
 - Core with different areas (Example 1.1: $\phi = ?$)

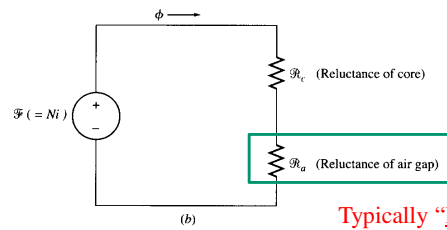
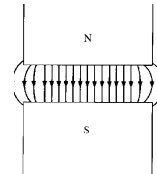
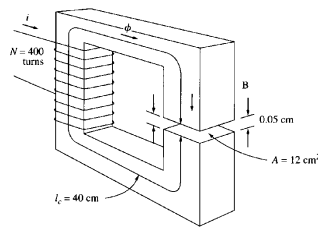


How to choose the number of reluctances (\mathcal{R} s)?

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Magnetic circuits: Examples

- Core with air gap (Example 1.2 (with an error) and 1.3)
- Fringing effect. Increased gap area (+ 5%)

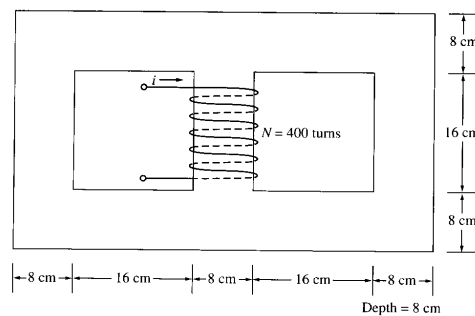


Typically “large” or “small?”

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Magnetic circuits

- Core with multiple legs (used in transformers.)
- Example (problem 1.13): $N = 400$, “depth” = 8 cm, $i = ?$ for $B = 0.5$ T in the central leg?

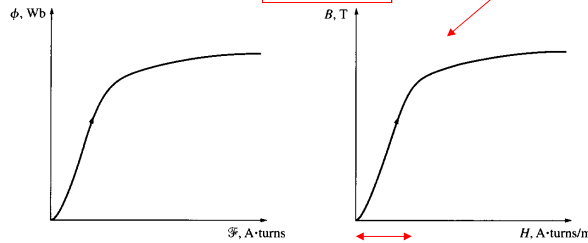


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Magnetic behavior of ferromagnetic materials

- The magnetic permeability (μ) is not constant: Saturation effect.

$$B = \mu H$$

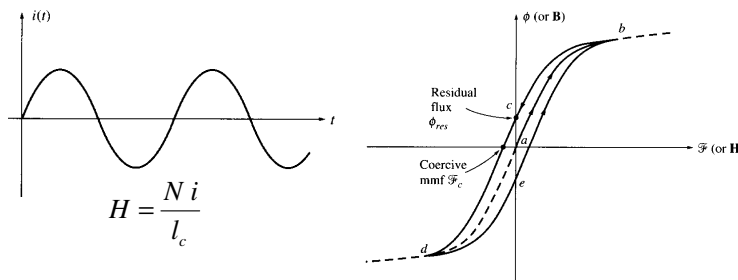


- Devices such as transformers have to operate in the linear (nonsaturated) region.

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Hysteresis effect

- What happens to the flux when an **ac current** is applied to the winding (transformer case)?

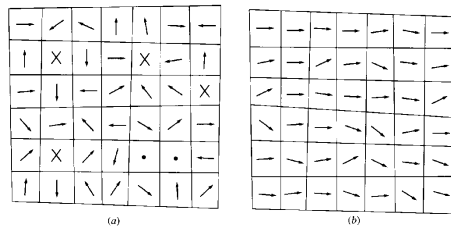


- Phenomenon that explains the production of *permanent magnets* (PMs)

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Hysteresis effect explained

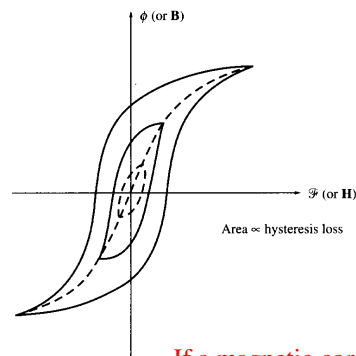
- The atoms in small regions (domains) of the core have their magnetic fields aligned arbitrarily. An external magnetic field can realign the magnetic domains, **increasing the magnetic field in a certain direction as per μ_r** , at the expense of some “losses.”



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Hysteresis losses

- The magnitude of the hysteresis losses can be estimated by the area of the hysteresis loop



If a magnetic core is excited with sinusoidal currents of 1 A and 2 A, when would the hysteresis losses be higher?

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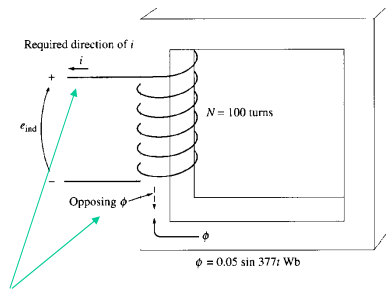
Induced voltage from a time changing magnetic field: Faraday's law

- If a flux passes through N turns of a coil of wire, a voltage will be induced in the winding in direct proportion to the rate of change in the flux with respect to time.

$$e_{ind} = (-)N \frac{d\phi}{dt}$$

$$e_{ind} = 100 \frac{d}{dt} 0.05 \sin 377t$$

$$e_{ind} = 1885 \cos 377t$$

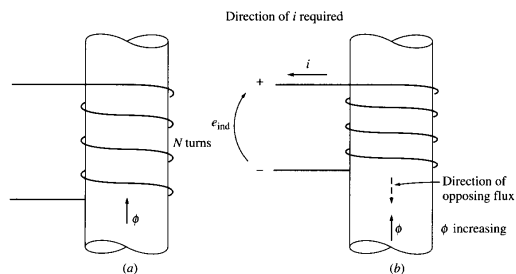


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Induced voltage from a time changing magnetic field: Faraday's law

- The polarity of the induced voltage is such that if the coil ends were short-circuited, the resulting current would oppose the original flux change:

$$e_{ind} = (-)N \frac{d\phi}{dt}$$



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Eddy current losses: Can be reduced by laminating the ferromagnetic core

- The time varying flux also induces voltages in the ferromagnetic core, causing the flow of (eddy) currents and ohmic losses. These can be reduced by reducing the length of the “core turns” by laminating the core.

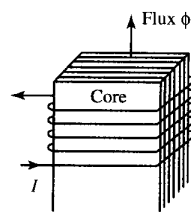


Figure 11.1.4 Part of a laminated core.

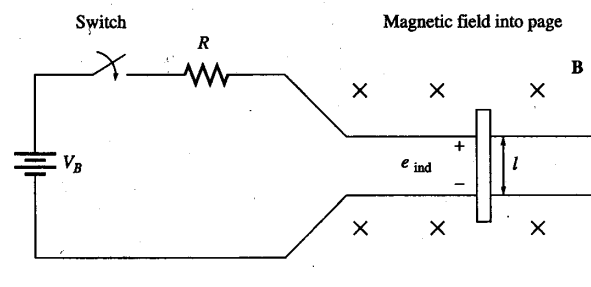
Eddy current losses are proportional to the square of the lamination thickness.

Would laminations “perpendicular to the direction of the magnetic flux” be useful?

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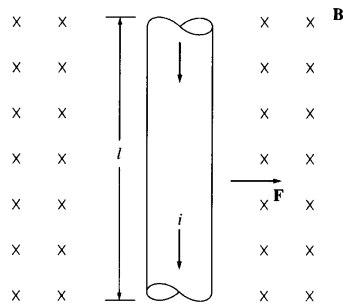
Linear dc machine

Simplest machine to understand the concepts of *motor* (conversion of electrical into mechanical power) and *generator*.



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Production of induced force on a wire



“A magnetic field induces a force on a current-carrying wire within the field”

$$F = i(l \times B)$$

$$|F| = i l B \sin \theta,$$

θ = angle between the wire and field vector

Direction of the force?

Right hand rule: l (index finger), B (middle finger) and F (Thumb).

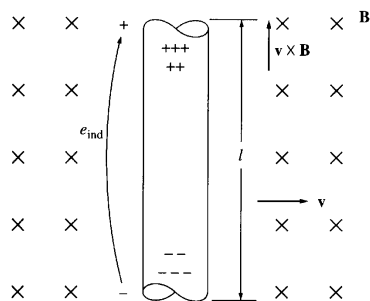
Numerical example: (1.7) textbook!

Java applet (animation):

<http://www.walter-fendt.de/ph11e/lorenzforce.htm>

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Induced voltage on a conductor moving in a magnetic field



“A magnetic field induces a voltage on a conductor (wire) moving within the field”

$$e_{ind} = (v \times B) \cdot l$$

Numerical example: (1.8) textbook!

The result of a “x product “ is a vector.
Direction? (Right hand rule...)

The result of the “x product” is aligned with “ l .” Thus: $e_{ind} = v B l$, pointing up.

Java applet (animation):

<http://micro.magnet.fsu.edu/electromag/java/faraday2/index.html>

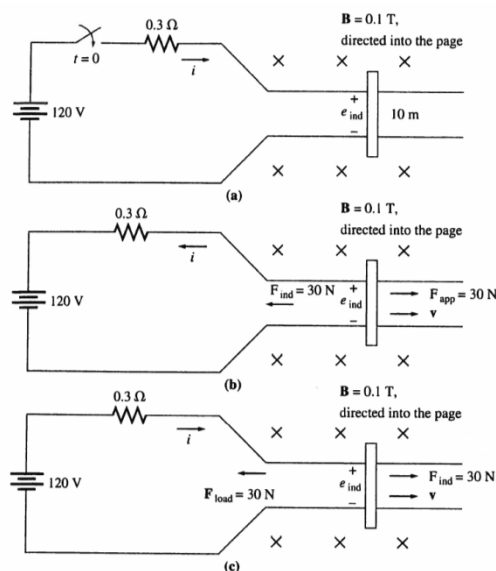
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Example 1.10: Linear DC machine.

- A linear dc machine has a battery voltage of 120 V, an internal resistance of 0.3Ω and a magnetic flux density (B) of 0.1 T :
 - What is the machine's maximum starting current? What is the steady-state velocity at **no load**?
 - Suppose that a 30 N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming?
 - Repeat the above for the force at the opposite direction.
 - If the bar is unloaded, and runs into a region where $B = 0.08 \text{ T}$. How fast will it go?

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Example 1.10: Linear DC machine.



- Kirchhoff' voltage law:
 $V_B = e_{ind} + iR$
- Conditions for steady-state? Current (i) and bar velocity (v) are constant.
- i is constant only if e_{ind} is constant. This requires that v is constant. That is a (acceleration) is zero!
- Newton's law (eq. 1.7):
 $F_{net} = ma$
- Neglecting friction on the rail and air, for $F_{net} = 0$:
 $F_{ind} = F_{load} \text{ or } F_{app}$
- Same magnitudes but opposite directions...

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