Concordia University
Department of Electrical and Computer Engineering

ELEC-331

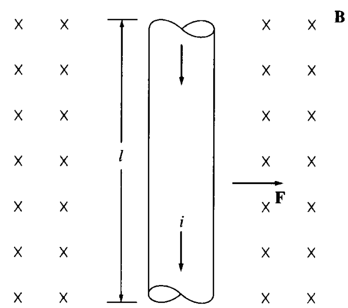
Lecture 7

Ac machinery fundamentals (4.1, 4.2)

Outline of the lecture

- Fundamental concepts (Chapter 1)
- Introduction on ac machines
- Voltage induced in a rotating loop
- Torque induced in a current-carrying loop
- The rotating magnetic field
- The effect of the number of poles
- Reversing the direction of the magnetic field

Production of induced force on a wire



"A magnetic field induces a force on a current-carrying wire within the field"

$\mathrm{F} = i(\mathrm{l} \times \mathrm{B})$

$|\mathrm{F}| = ilB \sin\theta,$

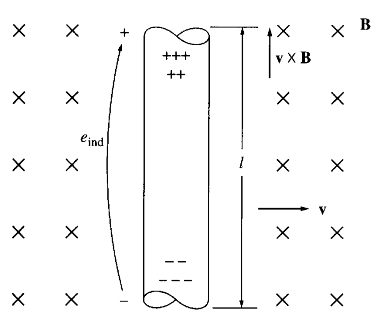
θ = angle bewteen the wire and field vector

Direction of the force?
Right hand rule: l (index finger), B (middle finger) and F (Thumb).

Java applet (animation):

http://www.walter-fendt.de/ph14e/lorentzforce.htm

Induced voltage on a conductor moving in a magnetic field



"A magnetic field induces a voltage on a conductor (wire) moving within the field"

$$e_{ind} = (\mathrm{v} \times \mathrm{B}) \bullet \mathrm{l}$$

Java applet (animation):

http://micro.magnet.fsu.edu/electromag/java/faraday2/index.html

Introduction on ac machines

- **Motors** convert electrical to mechanical energy while **generators** convert mechanical to electrical energy.

Voltage induced in a rotating loop (operation as a generator)

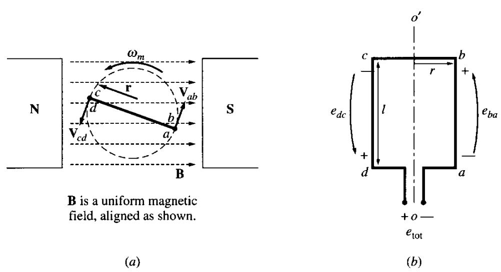


FIGURE 4–1
A simple rotating loop in a uniform magnetic field. (*a*) Front view; (*b*) view of coil.

- Java applet showing the basic system:
- http://www.walter-fendt.de/ph14e/generator_e.htm

Voltage induced in a rotating loop (operation as a **generator**)

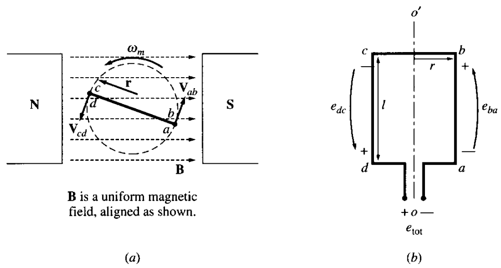


FIGURE 4–1
A simple rotating loop in a uniform magnetic field. (*a*) Front view; (*b*) view of coil.

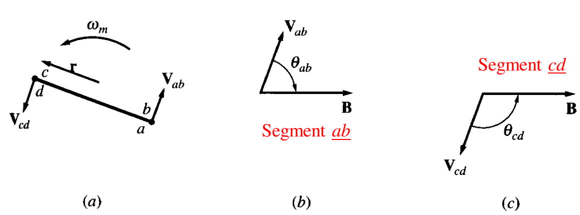
- Basic equation (section 1.7): $e_{ind} = (v \times B) \bullet l$
- Polarity of the induced voltage: Right hand rule. $v \rightarrow$ index finger, B $\rightarrow$ middle finger and thumb $\rightarrow e_{ind}$)

Voltage induced in a rotating loop

- The total voltage is the sum of the voltages in each segment: $e_{ind} = e_{ba} + e_{cb} + e_{dc} + e_{ab}$

$e_{ab} = (v \times B) \bullet l = vBl \sin\theta_{ab}$ $\qquad e_{cd} = (v \times B) \bullet l = vBl \sin\theta_{cd}$

$e_{bc} = e_{da} = 0$ because the direction of $v \times B$ is perpendicular to l.



"The velocity of the wire is tangential to the path of rotation"

Voltage induced in a rotating loop

- Total voltage in the loop: $e_{ind} = e_{ab} + e_{cd} = vBl\sin\theta_{ab} + vBl\sin\theta_{cd}$

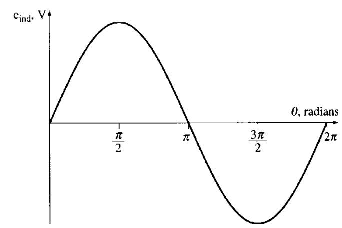
  For: $\theta = \theta_{ab}$, $\theta_{ab} = 180^\circ - \theta_{cd}$, $\sin\theta = \sin(180^\circ - \theta)$, $e_{ind} = 2vBl\sin\theta$
- Considering that:

  $\theta = \omega t$, ω is the angular velocity of the loop

  $v = r\omega$, v is the tangential speed and r the radius

  $e_{ind} = 2r\omega Bl\sin\omega t$, Area of the loop: $A = 2rl$ and $\phi_{Max} = AB$

  $$e_{ind} = \phi_{Max}\omega\sin\omega t$$



- Factors that affect e_{ind}?
- *Comments for lab #3!*

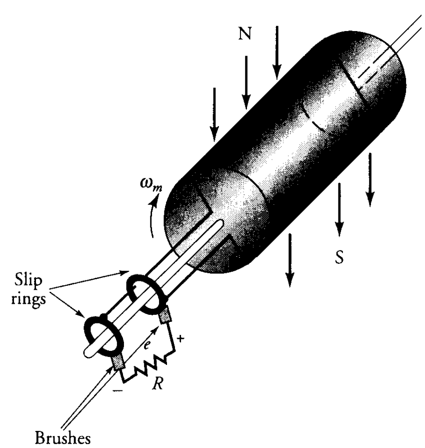
Note: $\theta = 0°$ means that the loop is in the vertical position.

Rotor circuit of an elementary machine

How to supply the voltage induced in the loop to a load? Via *slip* (metal) *rings* and *brushes*.

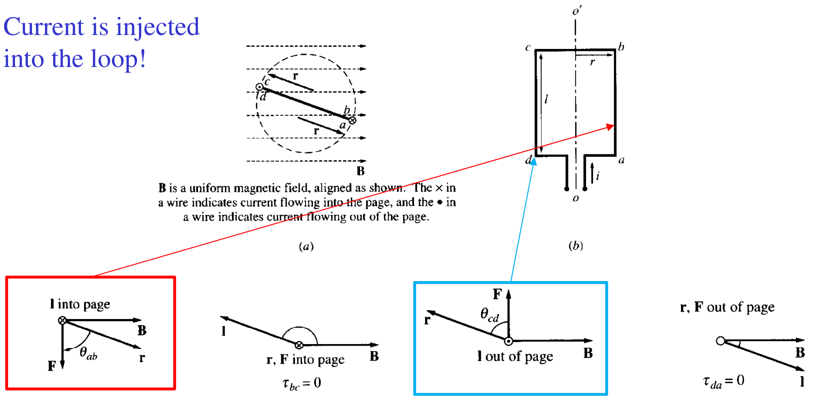
Segments *ab* and *cd* are always connected to the same slip ring.

Brushes are made of graphite like material, highly conductive and with low friction not to wear down the slip rings.



Torque induced in a current-carrying loop (operation as a **motor**)

Current is injected into the loop!



B is a uniform magnetic field, aligned as shown. The × in a wire indicates current flowing into the page, and the • in a wire indicates current flowing out of the page.

(a) (b)

$\tau_{bc} = 0$ $\tau_{da} = 0$

- Basic equation (Force & torque): $F = i(l \times B)$ $\tau = F(r \sin\theta)$

How are θ_{ab} and θ_{cd} defined? From the center towards the sides of the loop!

Torque induced in a current-carrying loop (operation as a **motor**)

The total torque induced on the loop is: $\tau_{ind} = \tau_{ba} + \tau_{cb} + \tau_{dc} + \tau_{ab}$

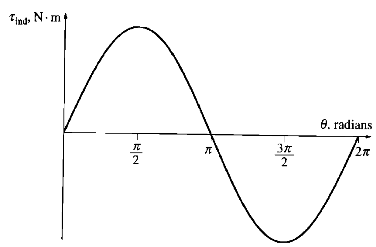
$F_{ab} = i(l \times B) = ilB \qquad \tau_{ab} = (F_{ab})(r \sin\theta_{ab}) = rilB \sin\theta_{ab}$ (clockwise)

$\tau_{cd} = (F_{cd})(r \sin\theta_{cd}) = rilB \sin\theta_{cd}$ (clockwise)

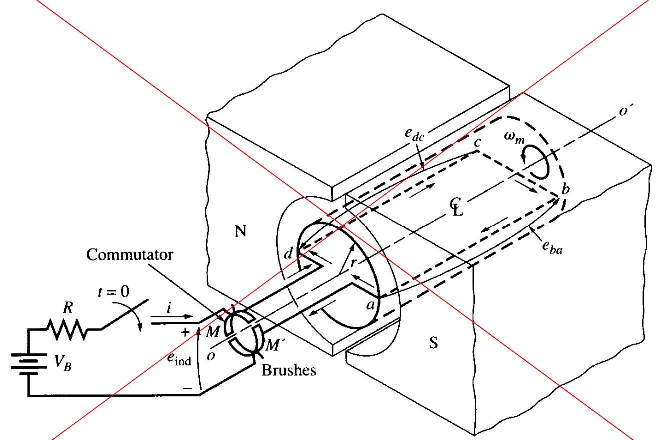
$\tau_{bc} = \tau_{da} = 0$

$\tau_{ind} = 2rilB \sin\theta$

(clockwise), since $\theta_{ab} = \theta_{cd}$

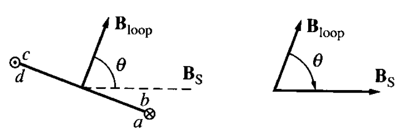


Rotor circuit of an elementary DC machine



Torque induced in a current-carrying loop (operation as a motor)

- One can also say that the torque induced on a loop tend to align the magnetic field produced by this loop with the external one.



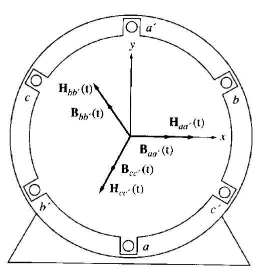
- The torque induced in the loop is proportional to the strength of the loop's magnetic field, the strength of the external magnetic field and the sine of the angle between them.

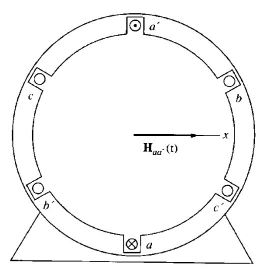
$$\tau_{ind} = kB_{loop}B_S \sin\theta = k\mathrm{B}_{loop} \times \mathrm{B}_S$$

The *rotating* magnetic field

- It is required so that the rotor actually rotates due to a torque that tends to line up the magnetic field of the *rotor* with the magnetic field of the *stator*.
- In conventional synchronous AC machines, a DC current is applied to the *rotor* circuit as described in the simplified model of previous slides.
- A three-phase set of currents with equal magnitude and phase shifted by 120° flowing in a three-phase winding produces a *rotating magnetic field* of constant magnitude.

The *rotating* magnetic field





Stator of a three-phase rotating machine.

A set of three-phase balanced currents is applied to it!

The windings are phase-shifted in space by 120°.

$$i_{aa'}(t) = I_M \sin \omega t$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120^\circ)$$
$$i_{cc'}(t) = I_M \sin(\omega t + 120^\circ)$$

$$B_{aa'}(t) = B_M \sin \omega t \angle 0^\circ$$
$$B_{bb'}(t) = B_M (\omega t - 120^\circ) \angle 120^\circ$$
$$B_{cc'}(t) = B_M (\omega t + 120^\circ) \angle 240^\circ$$

$$B_{net}(t) = B_{aa'}(t) + B_{bb'}(t) + B_{cc'}(t)$$

Applet of the rotating magnetic field:

http://www.uno.it/utenti/tetractys/askapplets/rotatingField1Applet.htm

The *rotating* magnetic field

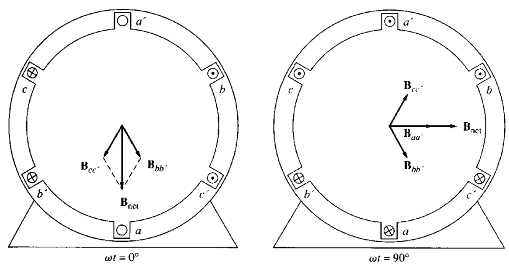
$B_{aa'}(t) = B_M \sin \omega t \angle 0^\circ \quad B_{bb'}(t) = B_M \sin(\omega t - 120^\circ) \angle 120^\circ \quad B_{cc'}(t) = B_M \sin(\omega t + 120^\circ) \angle 240^\circ$

$$B_{net}(0^\circ) = B_{aa'}(0^\circ) + B_{bb'}(0^\circ) + B_{cc'}(0^\circ) = 0 + \left(\frac{-\sqrt{3}}{2} B_M\right)\angle 120^0 + \left(\frac{\sqrt{3}}{2} B_M\right)\angle 240^0$$

$$B_{net}(0^\circ) = 1.5 B_M \angle -90^0$$

$$B_{net}(90^\circ) = B_{aa'}(90^\circ) + B_{bb'}(90^\circ) + B_{cc'}(90^\circ) = B_M \angle 0^0 + (-0.5 B_M)\angle 120^0 + (-0.5 B_M)\angle 240^0$$

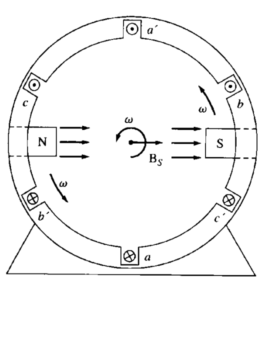
$$B_{net}(90^\circ) = 1.5 B_M \angle 0^0$$



The magnitude of the rotating magnetic field is constant!

The electrical frequency and the speed of the magnetic field rotation

- For the case we have seen so far (2 poles) the mechanical speed (revolutions/s) is equal to the electric frequency in Hz.

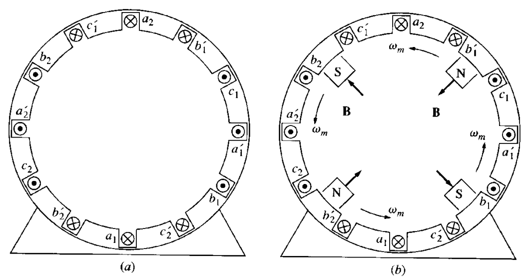


$f_e = f_m$ 2poles

$\omega_e = \omega_m$ 2poles

The electrical frequency and the speed of the magnetic field rotation

- For a P=4 poles machine, each pole moves only halfway the stator in one electrical cycle.



$$f_e = \frac{P}{2} f_m \quad \text{P poles} \qquad \omega_e = \frac{P}{2} \omega_m \quad \text{P poles}$$

Reversing the direction of the magnetic field rotation

- If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed (4-35).
- The result is that the shaft would rotate in the reverse direction since β_{loop} follows β_S.