1

Assignment 3 :

2.4 : 1 - 8 - 16 - 17 - 19 - 22 - 23
2.5 : 1 - 8 - 16 - 17 - 19 - 22 - 23.

section 2.4

#1

$$\underbrace{(2x-1)}_{M(x;y)}dx + \underbrace{(3y+7)}_{N(x;y)}dy = 0$$

① $M = 2x - 1$
$N = 3y + 7$

$$\frac{\partial M}{\partial y} = 0 \qquad \boxed{\frac{dM}{dy} = \frac{dN}{dx}}$$

$$\frac{\partial N}{\partial x} = 0$$

② integral of M based on x :

$$f(x;y) = \int 2x - 1 \, dx + g(y)$$

$$\boxed{f(x;y) = x^2 - x + g(y)}$$

③ $\frac{\partial f}{\partial y} = 0 - 0 + g'(y) = 3y + 7$

$$g'(y) = 3y + 7$$

$$g(y) = \int 3y + 7$$

$$= \frac{3}{2}y^2 + 7y$$

$$\boxed{f(x;y) = x^2 - x + \frac{3}{2}y^2 + 7y = C}$$

#8

$$(1+\ln(x)+y/x)\,dx = (1-\ln(x))\,dy$$

$$\underbrace{(1+\ln(x)+y/x)}_{M(x;y)}\,dx + \underbrace{(\ln(x)-1)}_{N(x;y)}\,dy = 0$$

$M = 1+\ln(x) + \frac{y}{x}$

$N = \ln(x) - 1$

① based on y $\quad \frac{\partial M}{\partial y} = \frac{1}{x}$ $\quad \boxed{\frac{dM}{dy} = \frac{dN}{dx}}$

$\frac{\partial N}{\partial x} = \frac{1}{x}$

② integral of M based on x

$$f(x;y) = \int 1+\ln(x) + \frac{y}{x}\,dx + g(y)$$

$$= x + x\ln(x) - x + y\ln(x)$$

$$= x\ln(x) + y\ln(x) + g(y)$$

③ $\frac{\partial f}{\partial y} = 0 + \ln(x) + g'(y) = \ln(x) - 1$

based on y.

$$g'(y) = -1$$

$$g(y) = -y$$

$f(x;y) = x\ln(x) + y\ln(x) - y = C \iff x\ln(x) + y\ln(x) - y = C$

$x\ln(x) + y(\ln(x)-1) = C$

$$\boxed{y = \frac{C - x\ln(x)}{\ln(x)-1}}$$

②

#47

$$\underbrace{(\tan x - \sin x \sin y)}_{M(x;y)} dx + \underbrace{\cos x \cos y}_{N(x;y)} dy = 0$$

$M = \tan x - \sin x \sin y$
$N = \cos x \cos y$

① $\frac{\partial M}{\partial y} = -\sin x \cos y$

$\frac{N}{x} = \cos y \times (-\sin(x))$

$$\boxed{\frac{dM}{dy} = \frac{dN}{dx}}$$

② $f(x;y) = \int \tan x - \sin x \overbrace{\sin y}^{\cos y} \, dx + g(y)$

$= \ln(\sec(x)) + \cos(x)\sin(y) + g(y)$

③ $\frac{\partial f}{\partial y} = \cos x \times \cos y + g'(y) = \cos x \cos y$

$g'(y) = 0$
$g(y) = C$

$$\boxed{f(x;y) = \ln(\sec(x)) + \cos(x)\sin(y) + C}$$

#16

$$(5y-2x)y' - 2y = 0$$

$$\Leftrightarrow (5y-2x)\frac{dy}{dx} - 2y = 0$$

$$\Leftrightarrow (5y-2x)\,dy - 2y\,dx = 0$$

$$\Leftrightarrow \underbrace{(5y-2x)}_{N(x;y)}dy + \underbrace{(-2y)}_{M(x;y)}dx = 0$$

① $N = 5y-2x$ $\qquad \frac{\partial N}{\partial x} = -2$

$M = -2y$

$\frac{\partial M}{\partial y} = -2$

$$\boxed{\frac{dM}{dx} = \frac{dN}{dy}}$$

② $f(x;y) = \int -2y \; dx + g(y)$

$f(x;y) = \quad -2xy \quad + g(y)$

$$\frac{\partial f}{\partial x} = -2x + g'(y) = 5y - 2x$$

$g'(y) = 5y$

$g(y) = \frac{5}{2}y^2$

$$\boxed{f(x;y) = -2xy + \frac{5}{2}y^2 = C}$$

3

#22

$(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0$

$M = e^x + y$
$N = 2 + x + ye^y$

(1) $\frac{\partial M}{\partial y} = 1$ $\boxed{\frac{dM}{dy} = \frac{dN}{dx}}$

$\frac{\partial N}{\partial x} = 1$

(2) $f(x;y) = \int e^x + y + dx + g(y)$

$\boxed{f(x;y) = e^x + yx + g(y)}$

(3) $\frac{\partial f}{\partial y} = x + g'(y) = 2 + x + ye^y$

$g'(y) = 2 + ye^y$

$g(y) = 2y + ye^y - e^y$

ye^y ; $u = y$
$dv = e^y$
$du = 1\,dy$
$v = e^y$
$uv - \int v\,du$
$= ye^y - e^y$

$\boxed{f(x;y) = e^x + yx + 2y + ye^y - e^y = c}$

$y(0) = 1$

$e^0 + 0 + 2 + e - e = c$

$\boxed{c = 3}$

#23

$y(-1) = 2$

$$\underbrace{(4y+2t-5)}_{M(t;y)}dt + \underbrace{(6y+4t-1)}_{N(t,y)}dy = 0$$

① $M = 4y + 2t - 5$

$N = 6y + 4t - 1$

$\frac{\partial M}{\partial y} = 4$

$\frac{\partial N}{\partial t} = 4$

$$\boxed{\frac{dM}{dy} = \frac{dN}{dt}}$$

② $f(x;y) = \int 4y + 2t - 5 \, dt + g(y)$

$$\boxed{= 4yt + t^2 - 5t + g(y)}$$

$\frac{\partial f}{\partial y} = 4t + g'(y) = 6y + 4t - 1$

$g'(y) = 6y - 1$

$$\boxed{g(y) = 3y^2 - y}$$

$$\boxed{f(x;y) = 4yt + t^2 - 5t + 3y^2 - y = C}$$

$y(-1) = 2$

$4 \times 2 \times (-1) + (-1)^2 - 5 \times (-1) + 3 \times 2^2 - 2 = C$

$-8 + 1 + 5 + 12 - 2 = C$

$-10 + 18 = C$

$\boxed{C = 8}$

so : $f(x;y) = 4yt + t^2 - 5t + 3y^2 - y = 8$

4

section 2.5

#1

$(x-y)\,dx + x\,dy = 0$ $\qquad y = ux$

$$(x-ux)\,dx + x(u\,dx + x\,du) = 0$$

$$\Leftrightarrow x\,dx \cancel{-ux\,dx} + \cancel{ux\,dx} + x^2\,du = 0$$

$$\Leftrightarrow x(dx + x\,du) = 0$$

$$dx + x\,du = 0$$

$$\Leftrightarrow \frac{dx}{x} + du = 0$$

$$\Leftrightarrow \ln|x| + u = C$$

$$\Leftrightarrow \ln|x| + \frac{y}{x} = C$$

$$\Leftrightarrow \boxed{x\ln|x| + y = Cx}$$

#8

$\frac{dy}{dx} = \frac{x+3y}{3x+y}$ $\qquad y = ux$

$dy = u\,dx + x\,du$

4

#8

$$\frac{dy}{dx} = \frac{x+3y}{3x+y} \qquad \begin{aligned} y &= ux \\ dy &= u\,dx + x\,du \end{aligned}$$

$$dy(3x+y) = (x+3y)\,dx$$

$$\Rightarrow (3x+y)\,dy - (x+3y)\,dx = 0$$

$$\Rightarrow (3x+ux)(u\,dx + x\,du) - (x+3ux)\,dx = 0$$

$$\Rightarrow 3xu\,dx + 3x^2\,du + u^2x\,dx + ux^2\,du - x\,dx - 3ux\,dx = 0$$

$$\Rightarrow dx\left(\cancel{3xu} + u^2x - x - \cancel{3ux}\right) + du\left(3x^2 + ux^2\right) = 0$$

$$\Rightarrow dx\left(u^2x - x\right) + du\left(3x^2 + ux^2\right) = 0$$

$$\Rightarrow (u^2x - x)\,dx + (3x^2 + ux^2)\,du = 0$$

$$\Rightarrow x(u^2-1)\,dx + x^2(3+u)\,du = 0$$

$$\Rightarrow (u^2-1)\,dx + x(3+u)\,du = 0$$

$$\frac{dx}{x} + \frac{(3+u)}{(u^2-1)}\,du = 0$$

$$\int \frac{dx}{x} + \int \frac{(3+u)}{(u^2-1)}\,du = 0$$

$$\Rightarrow \ln|x| + \int \frac{u+3}{(u-1)(u+1)}\,du = 0$$

$$\Rightarrow \ln|x| + \int \frac{u+1-2}{(u-1)(u+1)} = \int \frac{1}{u-1} + \frac{(-2)}{(u-1)(u+1)}$$

$$= \ln|x| - \ln|u-1| + 2\ln|u+1| = C$$

$$= e^{\ln|x| - \ln|u-1| + 2\ln|u+1|} = e^C$$

$$= x - (u-1)^{-1} + (u+1)^2$$

5

$$\Rightarrow \frac{x(u+1)^2}{(u-1)} = c_1$$

$$\Rightarrow \frac{x(y/x+1)^2}{(y/x-1)} = c_1$$

$$\Rightarrow x\left(\frac{y}{x}+1\right)^2 = c_1\left(\frac{y}{x}-1\right)$$

$$\Rightarrow \boxed{(y-x)^2 = c_1(y+x)}$$

16

① $\frac{dy}{dx} - y = e^x y^2$

with $u = y^{1-n} = y^{1-2} = \boxed{y^{-1}}$

$y = u^{-1}$ $\frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

$-u^{-2}\frac{du}{dx} - u^{-1} = e^x \cdot u^{-2}$

$\Rightarrow \frac{du}{dx} + u = -e^x$

the equation is linear.

② integrating factor.

$e^{\int 1dx} = e^x$

③ $e^x \cdot u = \int e^x \cdot -e^x \, dx = \int -e^{2x} dx$

$\Leftrightarrow ue^x = -\frac{1}{2}e^{2x} + C$

$\Leftrightarrow y^{-1} = -\frac{1}{2}e^x + Ce^{-x}$

#17

$$\frac{dy}{dx} = y(xy^3-1)$$

$$\frac{dy}{dx} = xy^4 - y$$

$$y^{-3} = u$$

$$\frac{du}{dx} = -3y^{-4} \cdot \frac{dy}{dx}$$

$$-\frac{1}{3}\frac{du}{dx}y^4 = \frac{dy}{dx}$$

$$\Rightarrow -\frac{1}{3}\frac{du}{dx} - 3u = -3x$$

Integration factor.

$$e^{\int -3\,dx} = e^{-3x}$$

$$[e^{-3x}u] = \int -3x e^{-3x}\,dx$$

Integration by part :

$$u' = e^{-3x} \qquad u = -\frac{1}{3}e^{-3x}$$
$$v = -3x \qquad v' = -3.$$

$$\int -3x e^{-3x}\,dx = xe^{-3x} + \frac{1}{3}e^{-3x} + c$$

$$\Leftrightarrow \boxed{u = x + \frac{1}{3} + ce^{3x}}$$

#22

$y^{1/2} \frac{dy}{dx} + y^{3/2} = 1$ $\qquad y(0) = 4$

$\frac{dy}{dx} + y^{(3/2)-(1/2)} = y^{-1/2}$

$\Rightarrow \frac{dy}{dx} y^{1/2} + y^{3/2} = 1$

using Bernoulli substitutions :

$u = y^{3/2}$ and $\frac{du}{dx} = \frac{3}{2} y^{1/2} \frac{dy}{dx}$

$\longrightarrow \frac{dy}{dx} = \frac{2}{3} \frac{du}{dx} y^{-1/2}$

$\Rightarrow 2/3 \frac{du}{dx} + \frac{3}{2} u = \frac{3}{2}$

integration factor : $e^{\int 3/2 \, dx} = e^{3/2 x}$.

$e^{3/2 x} u = \int e^{3/2 x} = e^{3/2 x} + c$

$\Rightarrow u = 1 + c$

$\rightarrow$ get $y^{3/2} = u$

$y^{3/2} = 1 + c$

initial value : $y(0) = 4 \Leftrightarrow 4^{3/2} = 1 + c \Leftrightarrow \boxed{c = 7}$

$$\boxed{y^{3/2} = 1 + 7e^{-3/2 x}}$$

#23

$$\frac{dy}{dx} = (x+y+1)^2$$

$v = x+y+1$ $\qquad \frac{dv}{dx} = 1 + \frac{dy}{dx}$

$\iff \frac{dy}{dx} = \frac{dv}{dx} - 1$

$\frac{dv}{dx} - 1 = v^2$

$\iff \frac{dv}{dx} = v^2 + 1$

$\iff \frac{dv}{v^2+1} = dx$

$\iff \int \frac{1}{v^2+1}\,dv = \int 1\,dx$

$\iff \tan^{-1} u = x + c$

$\implies u = \tan(x+c)$

$\iff x + y + 1 = \tan(x+c)$

$\iff x + y + 1 = \tan(x+c$

$$\boxed{y = \tan(x+c) - x - 1}$$