Assignment 2 : 2.3

s.2.1 : 3, 4, 26, 27
s.2.2 : 23, 25, 26
s.2.3 : 14, 22, 23.

section 2.1.

#3

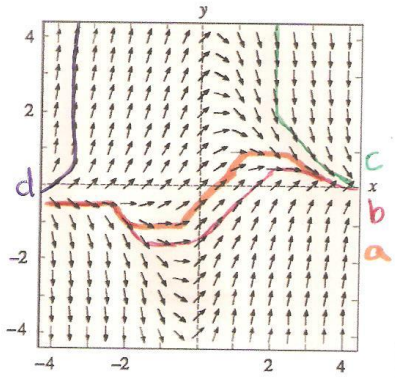

FIGURE 2.1.13 Direction field for Problem 3

#4

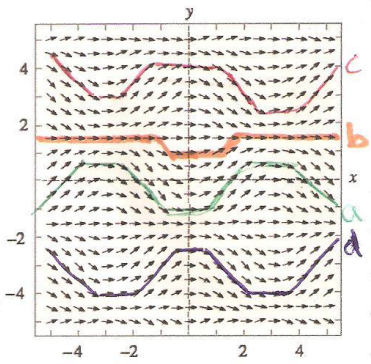

FIGURE 2.1.14 Direction field for Problem 4

#26

$$\frac{dy}{dx} = y(2-y)(4-y) = 0$$

$\Leftrightarrow y = 0$
or $2-y=0 \Leftrightarrow y=2$
or $4-y=0 \Leftrightarrow y=4$

phase portrait

critical points are
0 → unstable
2 → stable
4 → stable

#27

$$\frac{dy}{dx} = y \ln(y+2)$$

$y \ln(y+2) = 0$

$\Leftrightarrow y = 0$ or $y+2 = 1$

$\Leftrightarrow y = -1$

critical points are : $0 \rightarrow$ unstable

and $-1 \rightarrow$ stable

phase portrait

section 2.2

#23

$$\frac{dx}{dt} = 4(x^2+1) \quad , x(\pi/4) = 1$$

$$\Rightarrow \frac{dx}{(x^2+1)} = 4\,dt$$

$$\Rightarrow \int \frac{dx}{(x^2+1)} = \int 4\,dt$$

$$\Rightarrow \boxed{\tan^{-1}(x) = 4t + C}$$

$x(\pi/4) = 1$

$\Rightarrow \tan^{-1}(1) = 4(\pi/4) + C$

$\Rightarrow \frac{\pi}{4} = \pi + C$

$\Rightarrow C = \pi - \frac{\pi}{4}$

$\Rightarrow C = \frac{3\pi}{4}$ so: $\boxed{\tan^{-1}(x) = 4t - \frac{3\pi}{4}}$ implicit

$\tan(\tan^{-1}(x)) = \tan(4t - \frac{3\pi}{4})$

$\boxed{x = \tan(4t - \frac{3\pi}{4})}$ explicit

#25

$$x^2 \frac{dy}{dx} = y - xy \quad ; \quad y(-1) = -1$$

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\Rightarrow x^2 \frac{dy}{y} = (1-x)\,dx$$

$$\Rightarrow \frac{dy}{y} = \frac{(1-x)\,dx}{x^2}$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{(1-x)\,dx}{x^2}$$

$$\Rightarrow \ln|y| = \int \frac{1}{x^2} - \frac{x}{x^2}\,dx$$

$$\Rightarrow \ln|y| = \int \frac{1}{x^2} - \frac{1}{x}\,dx$$

$$\Rightarrow \ln|y| = \int \frac{1}{x^2} - \int \frac{1}{x}\,dx$$

$$\Rightarrow \ln|y| = -\frac{1}{x} - \ln|x| + C$$

$y(-1) = -1$

$$\Rightarrow \ln|-1| = -\frac{1}{-1} - \ln|-1| + C$$

$$\Rightarrow \ln|-1| = 1 - \ln|-1| + C$$

$$\Rightarrow 0 = 1 - 0 + C$$

$$\Rightarrow C = -1$$

$$\boxed{\ln|y| = -\frac{1}{x} - \ln|x| - 1}$$ Implicit

$$e^{\ln|y|} = e^{-1/x} + e^{-\ln|x|} + e^{-1}$$

$$y = e^{-1/x} + x^{-1} + e^{-1}$$

$$y = \frac{e^{-1/x} + e^{-1}}{x}$$

$$\boxed{y = \frac{e^{-(1/x+1)}}{x}}$$

EXPLICIT

#26

$$\frac{dy}{dt} + 2y = 1 \qquad y(0) = 5/2$$

$$\Rightarrow \frac{dy}{dt} = 1-2y$$

$$\Rightarrow \frac{dy}{1-2y} = dt$$

$$\Rightarrow \int \frac{dy}{(1-2y)} = \int dt$$

$$\Rightarrow -\frac{1}{2}\ln|1-2y| = t + C$$

$$y(0) = \frac{5}{2}$$

thus: $-\frac{1}{2}\ln|1-2\times\frac{5}{2}| = 0 + C$

$$\Rightarrow -\frac{1}{2}\ln|-4| = C$$

$$-\frac{1}{2}\ln|1-2y| = t - \frac{1}{2}\ln|-4|$$

$$(-2) -\frac{1}{2}\ln|1-2y| = -2t + \ln|-4|$$

$$\ln|1-2y| = -2t + \ln|-4|$$

$$e^{\ln|1-2y|} = e^{-2t} + e^{\ln|-4|}$$

$$1-2y = e^{-2t} - 4$$

$$-2y = -4e^{-2t} - 4$$

$$y = 2e^{-2t} + 4/2$$

$$\boxed{y = 2e^{-2t} + 4/2}$$

section 2.3

#19

$$(x+1)\frac{dy}{dx} + (x+2)y = 2xe^{-x}$$

① $$\frac{dy}{dx} + \underbrace{\frac{(x+2)}{(x+1)}}_{P(x)}y = \underbrace{2xe^{-x}}_{g(x)}$$

② integration factor:

$$e^{\int \frac{x+2}{x+1}dx}$$

$$\Rightarrow e^{\int 1 + \frac{1}{x+1}dx}$$

$$\Rightarrow e^{\int 1 + \int \frac{1}{x+1}dx}$$

$$\Rightarrow e^{x + \ln|x+1|}$$
$$\Rightarrow e^x \times e^{\ln|x+1|}$$
$$\Rightarrow e^x \times (x+1)$$

③ $$\frac{d}{dx}\left[e^x \times (x+1) \times y\right] = e^x(x+1) \quad x(2xe^{-x})$$

$$e^x(x+1)y = \int 2x^2 + 2x\,(dx)$$

$$\boxed{e^x(x+1)y = \frac{2}{3}x^3 + x^2 + C}$$

23

① $x\frac{dy}{dx} + (3x+1)y = e^{3x}$

$\Rightarrow \frac{dy}{dx} + \frac{(3x+1)}{x} y = \frac{e^{-3x}}{x}$

$\Rightarrow \frac{dy}{dx} + \underbrace{\frac{3x+1}{x}}_{P(x)} y = \underbrace{\frac{e^{-3x}}{x}}_{y(x)}$

$\Rightarrow$

② integration factor:

$e^{\int \frac{3x+1}{x} dx}$

$\Rightarrow e^{\int 3 + 1/x \, dx}$

$\Rightarrow e^{\int 3dx + \int \frac{1}{x} dx}$

$\Rightarrow e^{3x + \ln|x|}$

$\Rightarrow e^{3x} \times e^{\ln|x|}$

$\Rightarrow \boxed{x e^{3x}}$

③

$\frac{d}{dx}\left[x e^{3x} \times y \right] = x e^{3x} \times \frac{e^{-3x}}{x}$

$= 1$

$x e^{3x} \times y = \int 1 \, dx$

$\Rightarrow x e^{3x} \times y = x + c \Rightarrow$

$$x e^{3x} \times y = x + C$$

$$\Rightarrow y = \frac{x+C}{x e^{3x}} = \frac{x}{x e^{3x}} + \frac{C}{x e^{3x}}$$

$$\boxed{y = e^{-3x} + C x^{-1} e^{-3x}} \qquad (0; +\infty)$$

explicit.

#22

① $\frac{dP}{dt} + 2tP = P + 4t - 2$

$\Rightarrow \frac{dP}{dt} + 2tP - P = 4t - 2$

$\Rightarrow \frac{dP}{dt} + P(2t-1) = 4t - 2$

$\Rightarrow \frac{dP}{dt} + \underbrace{P(2t-1)}_{P(x)} = \underbrace{2(2t-1)}_{g(x)}$

② Integration factor:

$$e^{\int (2t-1)\,dt} = e^{t^2-t}$$

③ $\frac{d}{dt}\left[e^{t^2-t} \times y\right] = e^{t^2-t} \times (4t-2)$

$$e^{t^2-t} \times y = \int e^{t^2-t} \times (4t-2)$$