

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Pages
Final	April 2014	3
Instructors:	D. Dryanov, L. Dube, H. Greenspan, M. Hadid, Z. Li	Course Examiner
		A. Atoyan & H. Proppe
Special Instructions:	Only calculators approved by the Department are allowed	
	Show your work for full marks	

MARKS

- [9] 1. (a) Let $f(x) = \ln(1 + x^2)$ and $g(x) = \sqrt{4 + x}$. Find $f \circ g$ and $g \circ f$ and determine the domain of each of these composite functions.
- (b) Find the range of the function $f = e^{2x} + 3$, the inverse function f^{-1} , and the range of f^{-1} .
- [12] 2. Evaluate the limits **Do not use l'Hôpital rule:**
- (a) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 + x - 6}$ (b) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 5x - 5} - x}{x - 1}$ (c) $\lim_{x \rightarrow \infty} \ln \left(\frac{1 + x + 2x^3}{3 + 2x + x^3} \right)$
- [6] 3. Calculate both one-sided limits of $f(x) = \frac{|x^2 - 9|}{x + 3}$ at the point(s) where the function f is discontinuous.
- [15] 4. Find the derivatives of the following functions:
- (a) $f(x) = \frac{\sqrt{x^7} + x^{5/2}}{x^3}$
- (b) $f(x) = \ln(x^4 \cdot \sqrt{x + 3}) + \ln e$
- (c) $f(x) = \frac{\arctan(2x)}{\tan(x)}$
- (d) $f(x) = \sin[x^2 \cos(e^x)]$
- (e) $f(x) = (1 + 2x)^{x^2}$ (use logarithmic differentiation)

- [15] 5. (a) Verify that the point (2,1) belongs to the curve defined by the equation $xy + 2\sqrt{3 + y^2} = x^3 - 2$, and find the equation of the tangent line to the curve at this point.
- (b) Two cars start simultaneously moving away from the intersection of two orthogonal streets at the speeds $v_1 = 12$ m/s going west, and $v_2 = 16$ m/s going north. How fast is the distance between the cars increasing at the instant $t = 5$ seconds after they start moving from the intersection?
- (c) Use the l'Hôpital's rule to evaluate the $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{\cos(2x) - 1}$.
- [6] 6. Let $f(x) = 3 + x + 3x^2 - x^3$.
- (a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(3, f(3))$.
- (b) Find all points $x = c$ (if any) on the interval $[0,3]$ such that $f'(c) = m$.
- [9] 7. Consider the function $f(x) = \sqrt{2x + 1}$.
- (a) Use the **definition of the derivative** to find the formula for $f'(x)$.
- (b) Write the linearization formula for f at $a = 4$
- (c) Use this linearization to approximate the value of $f(3) = \sqrt{7}$
- [12] 8. (a) Find the absolute extrema of $f(x) = \frac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.
- (b) A box with a square base is to be constructed with a volume of 54 m^3 . The material for the box costs $\$2/\text{m}^2$, and the material for the top costs $\$6/\text{m}^2$. Find the dimensions that minimize the cost of the box.

[16] **9.** Given the function $f(x) = 2x^2 - x^4$.

- (a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).
- (b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).
- (c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).
- (d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question:** Let f be a function which is monotonically decreasing (strictly) and differentiable everywhere on the real axis. Let also $g = x^2 + 1$. Prove that the composite function $h = f \circ g$ has one and only one local extremum and determine whether it corresponds to a maximum or minimum of $h(x)$.