Review of Test 2 for ENGR 233

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- Test 2 covers §9.7 ~ 9.14. Test time: tutorial class, Friday, March 16^{th}, 2012.

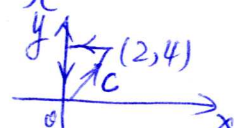
- §9.7: ① $F(x,y,z) = xye^x i - x^3yze^z j + xy^2e^y k$. Find curl F and div F. (#15 on P473)

② Verify that $f(x,y) = \arctan\left(\frac{2}{x^2+y^2-1}\right)$, $x^2+y^2 \neq 1$

is a solution of Laplace Eqn $\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. (#38 on P474)

- §9.8 — Line Integrals $\int_C G(x,y)dx$, $\int_C G(x,y)dy$, $\int_C G(x,y)ds = \int_a^b G(f(t),g(t))\sqrt{f'^2+g'^2}\,dt$.

③ Evaluate $\oint_C x^2y^3dx - xy^2dy$, where C is as



④ C is defined by: $x=t$, $y=t^2$, $0 \le t \le 2$. Evaluate $\int_C (xy)ds$, $\int_C xy\,dx + x\,dy$.

⑤ Work done W by $F = x^2 i + y j$ along curve C: $r(t) = \cos t\, i + \sin t\, j$, $0 \le t \le \frac{\pi}{2}$.

(Soln: $W = \int_C F \cdot dr$, see my lecture in §9.8).

Note: $F = Pi + Qj + Rk \Rightarrow \int_C F\cdot dr = \int_C P\,dx + Q\,dy + R\,dz$.

- §9.9: ⑥ Evaluate (a) $\int_{(1,2)}^{(3,6)} (2xy^2-3)dx + (2x^2y+4)dy$. (#7, 8 on P492)

(b) $\int_{(-1,1)}^{(0,0)} (5x+4y)dx + (4x-8y^3)dy$.

- §9.10 (Double integrals in x-y):

⑦ Evaluate $\iint_R xe^{y^2}dA$, where R is bounded by $x=0$, $y=4$, $y=x^2$. (Ex 2 on P496)

⑧ Evaluate $\iint_R (2x+4y+1)dA$: $y=x^2$, $y=x^3$ (#15 on P499)

- §9.11 (Double integrals in (r,θ)):

⑨ Region R is bounded by $r=1$, $r=3$, $x=0$, $y=0$. Find center $(\bar{x},\bar{y})$ of mass of a lamina in R with density $\rho(x,y) = x^2+y^2$.

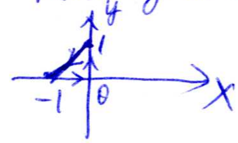
⑩ Evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2}\,dy\,dx$. (Ex 2 on P503).

I will follow this package to give you the review on Tuesday, 03/13/2012.

• §9.12: Green's Thm in R^2

(11.) Evaluate $\oint_C -y^2dx+x^2dy$, $\iint_R (2x+2y)dA$, where C — circle: $x=3\cos t$, $y=3\sin t$, $0\le t\le\pi$. R is region bounded by curve C.

(12.) Evaluate $\oint_C e^{x^2}dx + 2\tan^{-1}x\,dy$. C:



(#12 on P510).

• §9.13: (13.) Find surface area of portion of plane $2x+3y+4z=12$ that is ~~above the region~~ bounded by coordinate planes in 1st octant (#1 on P516).

(14.) Find flux of $F=\langle y^2, x^2, 5z\rangle$, S that portion of cylinder $y^2+z^2=4$ in 1st octant bounded by $x=0, y=0, z=0, x=3$. (#29 on P517).

(15.) If $G(x,y,z)=xy(9-4z)$, find $I=\iint_S G(x,y,z)\,ds$, where S — Cylinder $z=2-x^2$ in 1st octant bounded by $x=0$, $y=0$, $y=4$, $z=0$.

• §9.14: (16.) Evaluate $\oint_C F\cdot dr$, where

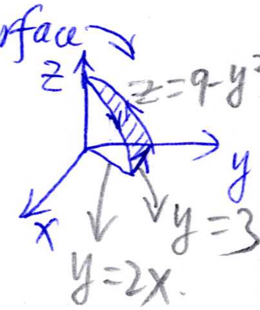
~~(a) $F=\langle x,y,z\rangle$~~, C is the boundary of triangle with vertices (1,0,0), (0,1,0), (0,0,1).

~~$F=\langle 2z+x,$~~

(#5 on P522)

(a) $F=\langle 2z+x, y-z, x+y\rangle$,

(b) $F=x^2y\,i+(x+y^2)\,j+xy^2z\,k$, C is the boundary of surface



Good Luck!