Midterm Exam Emat 233

October 18, 2005

Allowable materials: Pencils, pens. You may **NOT** use notes, books, calculators or any other materials.

Self-serve formula sheet (not marked). For a vector-valued functions $\vec{r}(t)$ in three dimension defining a smooth curve for $t \in [a, b]$

$\vec{B}(t) =$	$a_T(t) =$	$\vec{T}(t) =$	$\kappa(t) =$	$s(t) =$	$a_N(t) =$	$\vec{N}(t) =$

$\frac{\|\vec{r}'(t)\times\vec{r}''(t)\|}{\|\vec{r}'(t)\|}$	$\frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$	$\int_a^t \|\vec{r}'(\tau)\|\mathrm{d}\tau$	$\frac{\|\vec{r}'(t)\times\vec{r}''(t)\|}{\|\vec{r}'(t)\|^3}$	$\vec{T}(t) \times \vec{N}(t)$	$\frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$	$\frac{\vec{r}'(t)\cdot\vec{r}''(t)}{\|\vec{r}'(t)\|}$
(a)	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**	**(g)**

[10 points] Problem 1.

Find the length of the curve traced by the given vector function on the indicated interval.

$$\vec{r}(t) = \mathrm{e}^t \cos(2t)\,\mathbf{i} + \mathrm{e}^t \sin(2t)\,\mathbf{j} + \mathrm{e}^t\,\mathbf{k}\ , \qquad 0 \le t \le 3\pi.$$

[10 points] Problem 2.

The osculating plane to a curve, at a given point, is the plane containing the unit tangent vector $\vec{T}$ and the principal normal vector $\vec{N}$ to the curve at that point. Find the (scalar) equation of the osculating plane to the curve below, at the indicated point.

$$\vec{r}(t) = t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j} + \frac{1}{3}t^3\,\mathbf{k}\ , \qquad t = 1.$$

[10 points] Problem 3.

A particle moves such that its position $\vec{r}$ at time t is given by the function

$$\vec{r}(t) = t\ln t\,\mathbf{i} + t\,\mathbf{j} + e^{-t}\,\mathbf{k}, \qquad t > 0$$

(i) Find the velocity $\vec{v}(t)$ and the acceleration $\vec{a}(t)$ of the particle, as functions of time.

(ii) At time $t = 1$, what are the tangential and normal components of the acceleration a_T and a_N (such that $\vec{a} = a_T\vec{T} + a_N\vec{N}$), and what is the curvature κ of the curve described by $\vec{r}(t)$?

[10 points] Problem 4.

(i) Compute the gradient of the function

$$F(x, y, z) = x^2 + 4xz + 2yz^2.$$

(ii) Find the direction along which the function F increases as fast as possible at the point $(2, 1, -1)$, and find the maximum rate of change.

[10 points] Problem 5.

Find all points on the surface $x^2 + 3y^2 + z^2 + 2z - 2xy = 16$ at which the tangent plane is horizontal.