Concordia University

ENGR 233 - Sec U - 2nd Midterm Exam - March 25, 2009 Instructor: Galia Dafni
Total time: 75 minutes Total marks: 50
Allowable materials: Pencils, pens. You may **NOT** use notes, books, calculators or any other materials. A formula sheet can be found on the back of the exam.

Write your answers in the examination booklet. Write clearly and neatly and show all your work in order to receive full marks. You do not need to simplify or approximate numerical answers but you should evaluate any trigonometric functions if the question requires it.

Problem 1 (14 marks). Consider a force given by the vector field

$$\vec{F}(x,y,z) = (2x+y^2)\hat{\imath} + xe^z\hat{\jmath} + x^2\hat{k}.$$

(a) Compute the divergence and the curl of $\vec{F}$. Is $\vec{F}$ a conservative force field?
(b) Find the work done by the force in moving a particle along the path formed by the line segments from $(0,0,0)$ to $(1,0,0)$ and from $(1,0,0)$ to $(1,1,0)$ in the xy plane ($z=0$).

Problem 2. (12 marks) This problem refers to the following vector field in the xy plane:

$$\vec{F}(x,y) = y\cos(xy)\hat{\imath} + x\cos(xy)\hat{\jmath}.$$

(a) Show that the line integrals $\int_C \vec{F}\cdot d\vec{r}$ are independent of path, and find a potential function $\phi(x,y)$ such that $\vec{F} = \vec{\nabla}\phi$.
(b) Use your answer from part (a) to find $\int_C y\cos(xy)dx + x\cos(xy)dy$, where C is any smooth path starting at the point $(0,0)$ and ending at the point $(1,\pi/2)$ in the xy plane.
(c) What is the circulation $\int_C \vec{F}\cdot\vec{T}ds$ around the closed circle $x^2+y^2=a^2$ in the xy plane, traversed counter-clockwise?

Problem 3 (12 marks). Evaluate the integral

$$\int_0^1\int_x^1 e^{y^2}\,dydx.$$

by following these steps:
(a) Sketch the region of integration.
(b) Reverse the order of integration and compute the resulting integral.

Problem 4 (12 marks).
(a) Find the mass of the lamina in the shape of the region R in xy-plane that consists of the part of the disk of radius 1 centered at the origin which lies in the first quadrant (i.e. the quarter disk bounded by the positive x axis, the positive y axis and the circle $x^2+y^2=1$) and which has density given by $\rho(x,y)=3(x^2+y^2)$.
(b) Explain why your answer to part (a) is the same as the line integral

$$\int_C -y^3dx + x^3dy,$$

where C is the curve bounding the region R, i.e. the curve consisting of the line segment from $(0,0)$ to $(1,0)$, the quarter circle from $(1,0)$ to $(0,1)$, and the line segment from $(0,1)$ to $(0,0)$, traversed counter-clockwise.