Equations of Statics

Vectors

Cartesian Vector $\bar{\boldsymbol{A}} = A_x\,\bar{\imath} + A_y\,\bar{\jmath} + A_z\bar{\boldsymbol{k}}$

Magnitude $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Unit vector $\bar{\boldsymbol{u}}_A = \frac{\bar{A}}{A} = \frac{A_x}{A}\bar{\imath} + \frac{A_y}{A}\bar{\jmath} + \frac{A_z}{A}\bar{\boldsymbol{k}}$

Unit vector using direction cosines

$$\bar{\boldsymbol{u}}_A = \cos(\alpha)\,\bar{\imath} + \cos(\beta)\,\bar{\jmath} + \cos(\gamma)\,\bar{\boldsymbol{k}}$$

Dot Product $\bar{\boldsymbol{A}} \cdot \bar{\boldsymbol{B}} = A\,B\,Cos(\theta)$

$\bar{\boldsymbol{A}} \cdot \bar{\boldsymbol{B}} = A_xB_x + A_yB_y + A_zB_z$

Cross Product

$$\bar{\boldsymbol{C}} = \bar{\boldsymbol{A}} \times \bar{\boldsymbol{B}} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\bar{\boldsymbol{r}}_{AB} = (x_B - x_A)\,\bar{\imath} + (y_B - y_A)\,\bar{\jmath} + (z_B - z_A)\,\bar{\boldsymbol{k}}$$

Cartesian Force Vector

$$\bar{\boldsymbol{F}}_{AB} = F\,\bar{\boldsymbol{u}}_{AB} = F\,\frac{\bar{\boldsymbol{r}}_{AB}}{|\bar{\boldsymbol{r}}_{AB}|}$$

Moments

Moment of a force

Scalar Formulation $M_o = F\,d$

Vector Formulation $\bar{\boldsymbol{M}}_o = \bar{\boldsymbol{r}} \times \bar{\boldsymbol{F}}$

$$\bar{\boldsymbol{M}}_o = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a force about a specified axis aa (unit vector $\bar{u}_{aa}$)

$$M_{aa} = \bar{u}_{aa} \cdot (\bar{\boldsymbol{r}} \times \bar{\boldsymbol{F}}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a force and couple system

$$\bar{\boldsymbol{F}}_R = \sum \bar{\boldsymbol{F}}\,;\ (\bar{\boldsymbol{M}}_R)_O = \sum \bar{\boldsymbol{M}} + \sum (\bar{\boldsymbol{M}})_O$$

Equilibrium

Particle equilibrium

$$\sum F_x = 0;\ \sum F_y = 0;\ \sum F_z = 0;$$

Rigid body equilibrium in 2D

$$\sum F_x = 0;\ \sum F_y = 0;\ \sum M_O = 0;$$

Rigid body equilibrium in 3D

$$\sum F_x = 0;\ \sum F_y = 0;\ \sum F_z = 0;$$
$$\sum M_x = 0;\ \sum M_y = 0;\ \sum M_z = 0;$$

Equilibrium equations: Vector formulation

$$\sum \bar{\boldsymbol{F}} = 0;\ \sum \bar{\boldsymbol{M}}_o = 0;$$

Centre of Gravity

$$\bar{x} = \frac{\sum x_i W_i}{\sum W_i}\ \ or\ \bar{x} = \frac{\int x\,dW}{\int dW}$$

Unit Conversion: $1m = 3.28\,ft.$

Equations of Dynamics

Particle Rectilinear Motion

Basic Relationships	For Constant acceleration a_c
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$ads = vdv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates

$v_x = \dot{x}$ $\quad a_x = \dot{v}_x = \ddot{x}$

$v_y = \dot{y}$ $\quad a_y = \dot{v}_y = \ddot{y}$

$v_z = \dot{z}$ $\quad a_z = \dot{v}_z = \ddot{z}$

n, t, b Coordinates

$v = \frac{ds}{dt} = \dot{s}$ $\quad a_t = \dot{v} = v\frac{dv}{ds}$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \qquad a_n = \frac{v^2}{\rho}$$

r, θ, z Coordinates

$v_r = \dot{r}$ $\quad a_r = \ddot{r} - r\dot{\theta}^2$

$v_\theta = r\dot{\theta}$ $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$v_z = \dot{z}$ $\quad a_z = \dot{v}_z = \ddot{z}$

Relative Motion

$\bar{\boldsymbol{v}}_B = \bar{\boldsymbol{v}}_A + \bar{\boldsymbol{v}}_{B/A}$ $\quad \bar{\boldsymbol{a}}_B = \bar{\boldsymbol{a}}_A + \bar{\boldsymbol{a}}_{B/A}$

Equations of Motion

Particle	Rigid Body as particle
$\sum \boldsymbol{F} = m\bar{\boldsymbol{a}}$	$\sum \boldsymbol{F} = m\bar{\boldsymbol{a}}_G$
$\sum F_x = ma_x$	$\sum F_x = m(a_G)_x$
$\sum F_y = ma_y$	$\sum F_y = m(a_G)_y$
$\sum F_z = ma_z$	$\sum F_z = m(a_G)_z$

Work- Energy

Work $\quad U_{1\to 2} = \int \bar{F}.d\bar{r} = \int F\cos(\theta)\,ds$

Principle of Work-energy

$$\frac{1}{2}mv_1^2 + U_{1\to 2} = \frac{1}{2}mv_2^2$$

Conservation of Mechanical energy

$$\frac{1}{2}mv^2 + mgh + \frac{1}{2}ks^2 = constant$$

Where $\frac{1}{2}mv^2 \Rightarrow Kinetic\ energy$

$mgh \Rightarrow Gravitational\ Potential\ energy$

$\frac{1}{2}ks^2 \Rightarrow Elastic\ Potential\ energy$