



VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

**ADM2302: BUSINESS ANALYTICS
Midterm Exam Solutions - Fall 2015**

Date: November 1, 2015

Duration: 120 minutes

Professors : Montazeri (A), Chung (B & C), Manseau (D), Astaraky (E)

Last Name : _____

First Name : _____

Student Number : _____

Instructions to Students:

- Complete your name and signature in the spaces above.
- Verify that your exam copy has 7 pages (including this title page). If not, please inform the professor immediately.
- One page (8.5 by 11 inches) review sheet, both sides, is permitted.
- Answer all questions on this examination copy. Only answers in this exam booklet will be marked. Show all work.
- NO COMMUNICATION DEVICES MAY BE WITHIN SIGHT DURING THE EXAM PERIOD.
- Graphing calculators, iPads/Pods, notebooks, computers are NOT allowed.

Question	1	2	3	4	Total
Points					
Total Value	10	10	10	10	40

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Statement to be signed by the student: I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed : _____

Note: an examination copy or booklet without that signed statement will not be graded and will receive a midterm exam grade of zero.

Problem 1- Graphical Method (10 points)

Below is a linear programming formulation. Alomite and caltrine are chemicals which can be sold in any amount (fractions of tons are acceptable amount). Let :

x = number of tons of alomite to produce
 y = number of tons of caltrine to produce

Maximize $Z = 5x + y$

subject to:

$5x + 3y \leq 120$ (Labour hours constraint) (1)

$2x + 5y \leq 90$ (Plastic available) (2)

$5x + 8y \geq 80$ (Threshold requirement) (3)

$x, y \geq 0$

a) Graph the constraint lines and mark them clearly with the numbers (1), (2) and (3) to indicate which line corresponds to which constraint. Darken the feasible region. (5 points)

b) State the optimal solution by filling in the blanks below. (2 points)

Optimal value of x = _____ 24

Optimal value of y = _____ 0

Optimal value of objective function Z = _____ 120

c) Will there be a slack in any of the constraint? If yes, state the amount. (2 points)

Plastic constraint:

$2(24) + 5(0) = 48 \rightarrow \text{Slack} = 90 - 48 = 42$ units of plastic

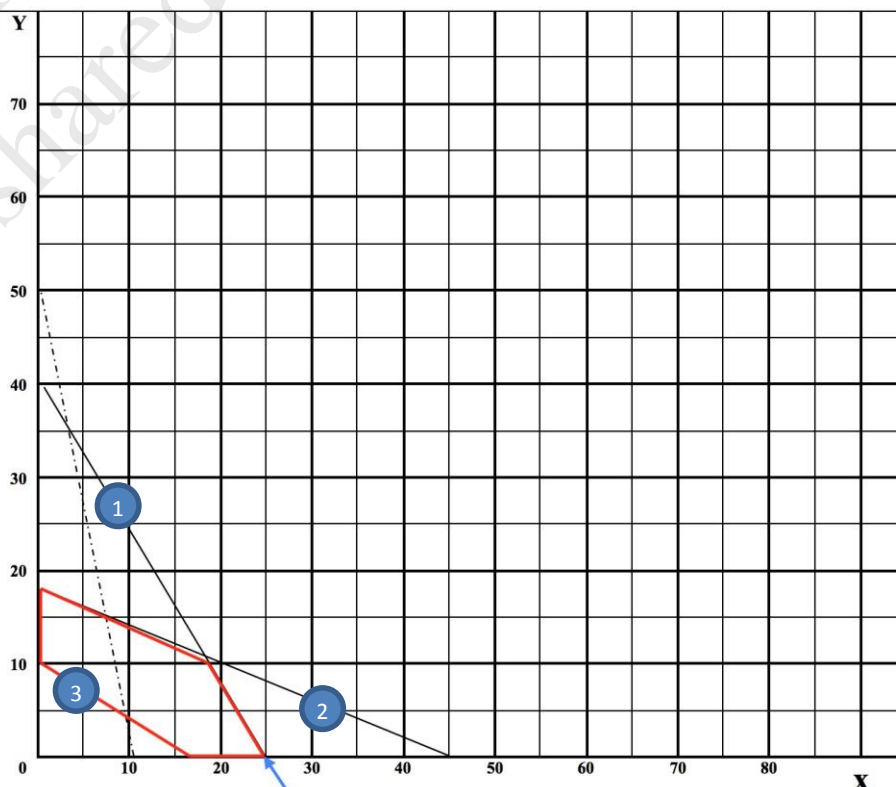
OR

Threshold constraint:

$5(24) + 8(0) = 120 \rightarrow \text{Slack} = 120 - 80 = 40$ units of plastic

d) Which constraints are binding? Please justify. (1 point)

Labor constraint because the optimal solution lies on this constraint (i.e. slack = 0)



Problem 2 - Nurse Scheduling Problem at CHEO (10 points)

David, the Head Administrator at the Children's Hospital of Eastern Ontario, wants to determine an optimal nursing schedule to make sure there are enough nurses on duty during the day. David has broken the day into twelve 2-hour periods. Throughout the day, the demand for nurses varies.

The slowest time of the day includes the three periods from 12:00 A.M. to 6:00 A.M. Beginning at midnight, the hospital requires a minimum of 30, 20, and 40 nurses, respectively.

The demand for nurses increases during the next four periods. Beginning with the 6:00 A.M. to 2:00 P.M. period, a minimum of 50, 60, 80, and 80 nurses are required for these four periods, respectively.

After 2:00 P.M., during the afternoon and evening hours, the demand for nurses decreases. For the five 2-hour periods beginning at 2:00 P.M. and ending at midnight, 70, 70, 60, 50, and 50 nurses are required, respectively.

A nurse reports for duty at the beginning of one of the 2-hour periods and works 8 consecutive hours. David's objective is to determine an optimal nursing schedule that uses the minimum number of nurses while meeting the hospital's minimum nurse requirements throughout the day.

Formulate algebraically a Linear Programming (LP) model for this problem.

(Only formulate the problem, you are not expected to solve!)

Variables: 2 points

Let X_i be the number of nurses that start the 8-hour shift in period i , where $i = 1, 2, \dots, 12$ and period 1 = 12AM - 2AM, period 2 = 2AM - 4AM, etc.

Objective function: 2 points

$$\text{Minimize } Z = X_1 + X_2 + X_3 + X_4 + \dots + X_{11} + X_{12}$$

Constraints: 5 points

0.5 point per constraint up to 5 points

Deduct 2.5 marks if a student has indicated the wrong sign throughout but everything else is good

subject to

$$X_{10} + X_{11} + X_{12} + X_1 \geq 30 \quad (\text{Period 1})$$

$$X_{11} + X_{12} + X_1 + X_2 \geq 20 \quad (\text{Period 2})$$

$$X_{12} + X_1 + X_2 + X_3 \geq 40 \quad (\text{Period 3})$$

$$X_1 + X_2 + X_3 + X_4 \geq 50 \quad (\text{Period 4})$$

$$X_2 + X_3 + X_4 + X_5 \geq 60 \quad (\text{Period 5})$$

$$X_3 + X_4 + X_5 + X_6 \geq 80 \quad (\text{Period 6})$$

$$X_4 + X_5 + X_6 + X_7 \geq 80 \quad (\text{Period 7})$$

$$X_5 + X_6 + X_7 + X_8 \geq 70 \quad (\text{Period 8})$$

$$X_6 + X_7 + X_8 + X_9 \geq 70 \quad (\text{Period 9})$$

$$X_7 + X_8 + X_9 + X_{10} \geq 60 \quad (\text{Period 10})$$

$$X_8 + X_9 + X_{10} + X_{11} \geq 50 \quad (\text{Period 11})$$

$$X_9 + X_{10} + X_{11} + X_{12} \geq 50 \quad (\text{Period 12})$$

Non negativity: 1 point

$$X_i \geq 0$$

Problem 3- Glendaruel Bottling Co. LP formulation (10 points)

Glendaruel Bottling Co. (Gaelic: Gleann Da Ruadhail) is a renowned scotch importer that bottles three blended malt scotch mixtures, the Glendaruel French Oak Reserve, the Glendaruel Green Label and the Glendaruel Special Edition, sold at respective prices of \$34, \$28.50 and \$22.50 per bottle. The blended scotches are mixed from three single malt whiskies that are imported from Scotland: type A single malt whisky, type B single malt whisky and type C single malt whisky. Although the demand for these three blended scotches is large and increasing, Glendaruel cannot import more than the amounts permitted by Canadian regulations on imports every month.

These maximum amounts as well as the purchase prices of imported single malt scotch whiskies are given in the table below:

Imported single malt whisky	Maximum number of bottles for importation	Price per bottle
A	2000	35 \$
B	2500	25 \$
C	1200	20 \$

The three blended scotches are composed as follows:

- The Glendaruel French Oak Reserve must not contain less than 60% of imported single malt whisky of type A and no more than 20% of imported single malt whisky of type C (no restrictions on imported single malt whisky of type B in the mixture);
- The Glendaruel Green Label must not contain less than 15% of imported single malt whisky of type A and no more than 60% of imported single malt whisky of type C (no restrictions on imported single malt whisky of type B in the mixture);
- The Glendaruel Special Edition must not contain more than 50% of imported single malt whisky of type C (no restrictions on imported single malt whiskies of type A and B in the mixture).

Formulate the linear programming model to help Glendaruel Bottling Co. to determine the number of bottles for each blended scotch to produce with their appropriate composition to maximize the contribution to earnings. Define the decision variables, objective function, and constraints.

(Only formulate the problem, you are not expected to solve!)

Alternative 1

Variables: 2 points

Let x_{ij} be the quantity (in bottles) of imported single malt whisky of type i in the blended scotch j
 Where $i =$ type A, type B, type C single malt whiskies and $j = 1, 2, 3$ (Glendaruel French Oak Reserve, Glendaruel Green Label and Glendaruel Special Edition)

Objective function: 1 point

$$\begin{aligned} \text{Maximize } Z = & 34(x_{A1} + x_{B1} + x_{C1}) + 28,50(x_{A2} + x_{B2} + x_{C2}) \\ & + 22,50(x_{A3} + x_{B3} + x_{C3}) - 35(x_{A1} + x_{A2} + x_{A3}) \\ & - 25(x_{B1} + x_{B2} + x_{B3}) - 20(x_{C1} + x_{C2} + x_{C3}) \end{aligned}$$

subject to:

Constraints:

Resource available (2 points)

$$\begin{aligned} x_{A1} + x_{A2} + x_{A3} & \leq 2000 \text{ single malt A} \\ x_{B1} + x_{B2} + x_{B3} & \leq 2500 \text{ single malt B} \\ x_{C1} + x_{C2} + x_{C3} & \leq 1200 \text{ single malt C} \end{aligned}$$

Mixture requirements (4 points)

$$\begin{aligned} 0,4x_{A1} - 0,6x_{B1} - 0,6x_{C1} & \geq 0 \\ -0,2x_{A1} - 0,2x_{B1} + 0,8x_{C1} & \leq 0 \\ 0,85x_{A2} - 0,15x_{B2} - 0,15x_{C2} & \geq 0 \\ -0,6x_{A2} - 0,6x_{B2} + 0,4x_{C2} & \leq 0 \\ -0,5x_{A3} - 0,5x_{B3} + 0,5x_{C3} & \leq 0 \end{aligned}$$

Non negativity (1 point)

$$x_{ij} \geq 0 \text{ for } i = A, B, C \text{ et } j = 1, 2, 3$$

Alternative 2**Variables: 2 points**

Let x_{ij} be the quantity (in bottles) of imported single malt whisky of type i in the blended scotch j
 Where $i =$ type A, type B, type C single malt whiskies and $j = 1, 2, 3$ (Glendaruel French Oak Reserve, Glendaruel Green Label and Glendaruel Special Edition)

x_i = quantity of single malt whisky ingredient i to blend, $i = A, B, C$
 y_j = quantities of blended scotch whiskies sold, $j = 1, 2, 3$

Objective function: 1 point

Maximize $Z = 34Y_1 + 28,50Y_2 + 22,50Y_3 - 35X_A - 25X_B - 20X_C$
 subject to:

Constraints:

Resource available (2 points)

$X_A \leq 2000$ single malt A

$X_B \leq 2500$ single malt B

$X_C \leq 1200$ single malt C

Mixture requirements (4 points)

$$X_{A1} \geq 0.6Y_1 \rightarrow X_{A1} - 0.6Y_1 \geq 0$$

$$X_{C1} \leq 0.25Y_1 \rightarrow X_{C1} - 0.25Y_1 \leq 0$$

$$X_{A2} \geq 0.15Y_2 \rightarrow X_{A2} - 0.15Y_2 \geq 0$$

$$X_{C2} \leq 0.6Y_2 \rightarrow X_{C2} - 0.6Y_2 \leq 0$$

$$X_{C3} \leq 0.5Y_3 \rightarrow X_{C3} - 0.5Y_3 \leq 0$$

Linkage variable for ingredient of single malt whiskies

$$x_{A1} + x_{A2} + x_{A3} = X_A \rightarrow x_{A1} + x_{A2} + x_{A3} - X_A = 0$$

$$x_{B1} + x_{B2} + x_{B3} = X_B \rightarrow x_{B1} + x_{B2} + x_{B3} - X_B = 0$$

$$x_{C1} + x_{C2} + x_{C3} = X_C \rightarrow x_{C1} + x_{C2} + x_{C3} - X_C = 0$$

Linkage variable for blended scotch

$$x_{A1} + x_{B1} + x_{C1} = Y_1 \rightarrow x_{A1} + x_{B1} + x_{C1} - Y_1 = 0$$

$$x_{A2} + x_{B2} + x_{C2} = Y_2 \rightarrow x_{A2} + x_{B2} + x_{C2} - Y_2 = 0$$

$$x_{A3} + x_{B3} + x_{C3} = Y_3 \rightarrow x_{A3} + x_{B3} + x_{C3} - Y_3 = 0$$

Deduct 2.5 if the linkage variables are missing, this would indicate wrong logic for this approach

Non negativity (1 point)

Deduct 0.5 if student indicated integer

$$X_{ij}, X_i, Y_j \geq 0 \text{ for } i = A, B, C \text{ et } j = 1, 2, 3$$

Problem 4 - Sensitivity Analysis (10 points)

Jai Hind Cycles (JHC) is a new bicycle manufacturing company in Hyderabad India. JHC is interested in optimizing and learning more about its production process. It produces 3 types of steel bikes: road (R), mountain (M), and hybrid (H) bikes. Each bike requires material (steel), machine time, and labour resources. The manager has formulated a linear programming model to maximize profits based on his knowledge of the manufacturing process. He needs your help to understand the results and answer his what-if questions.

$$\text{Maximize } Z = 120R + 160M + 140H \text{ (profit)}$$

subject to:

$$6R + 5M + 3H \leq 300 \text{ pounds (Material)}$$

$$9R + 4M + 5H \leq 280 \text{ hours (Machine Time)}$$

$$2R + 8M + 4H \leq 320 \text{ hours (Labour)}$$

$$M \geq 18 \text{ (Mountain Bike Requirement)}$$

$$R, M, H \geq 0$$

The following correct but incomplete output for this problem is provided below as an Excel Solver output. You will need this output to answer the following questions (a) through (f) below where each part is to be considered independently of all others.

Excel formulation:

	Road	Mountain	Hybrid			
Number of Bikes to Produce	0	20	40		Objective (total profit)	
Unit Profit	120	160	140			
Constraints				LHS	Sign	RHS
Material	6	5	3	220	<=	300
Machine Time	9	4	5	280	<=	280
Labour	2	8	4	320	<=	320
Mountain Bike Requirement		1		20	>=	18

Sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$2	Number of Bikes to Produce Road	0	-80	120	A	B
\$C\$2	Number of Bikes to Produce Mountain	20	0	160	73.84615385	48
\$D\$2	Number of Bikes to Produce Hybrid	40	0	140	60	30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$6	Material LHS	220	0	C	D	E
\$E\$7	Machine Time LHS	280	20	280	12	120
\$E\$8	Labour LHS	320	10	320	147.6923077	9.6
\$E\$9	Mountain Bike Requirement LHS	F	G	18	2	1E+30

- a) What is the optimal solution to JHC's production problem? How much profit can JHC expect if it follows the production schedule suggested by the optimal solution? (1 point)

Values of R, M & H (0.5 point)

Value of Z (0.5 point)

The optimal solution to JHCs production problem is to produce 0 road bikes, 20 mountain bikes, and 40 hybrid bikes for a profit of \$8800 dollars. $(0 \times 120 + 20 \times 160 + 40 \times 140)$.

- b) The manager has indicated that machine time can be increased by 10 hours if JHC rents additional equipment at a cost of 15\$ per hour. Is this worth it and why? (1 point)

0.5 point for right answer: yes

0.5 point for explanation around shadow price being more than 15\$, leaving 5\$ per hour additional, knowing we are in the allowable increase

Yes it is worth it because the shadow price is \$20 while it only costs \$15 to acquire an additional hour. For every unit increase of the machine time, the profit will increase by \$5. The shadow price is valid as long as the increase is less than the allowable increase of 12 hours, in this case it is.

- c) The manager is concerned about the recommendation of not producing any road bikes. What would the unit profit of road bikes have to be before it becomes worthwhile to produce? (1 point)

Full mark for the number 200, otherwise no points, no need to demonstrate the how

The reduced cost of road bikes is -80 meaning that the unit profit for road bikes must be \$200 $(120+80)$ before they become worthwhile to produce.

- d) What is the impact on the total profit, if JHC wants to increase the labour hours to 440 hours by hiring additional workers at a premium of \$8 per hour? Justify your answer. (2.5 points)

1 point for answer which is 9040\$ after the increase of 240\$ $(120*2)$

1.5 point for the explanation that it's the shadow price minus the hourly rate that gives 2\$ per additional hour

We're increasing the RHS of the labour constraint by 120 units (within the allowable increase). The shadow price is \$10 per hour but we must also pay the workers \$8 per hour. Therefore we can expect an increase in profit of \$240 (120×2) . The new profit will be \$9040 after the additional workers have been paid.

- e) A company in the area has offered to sell the manager 50 pounds of steel for a discounted price of \$1 per pound. Is this something the manager should be interested in? Justify. (1 point)

0.5 for answer which is no

0.5 for explanation around slack or non binding or having too much material already

No, material is a non-binding constraint and there is slack. JHC has too much material right now.

- f) Complete the table below to fill in the missing value of the sensitivity report.(3.5 points)

0.5 per answer

Accept 1E30 or infinity or very large number for infinity for B and D

Item	Value
A	80
B	∞
C	300
D	∞
E	80
F	20
G	0