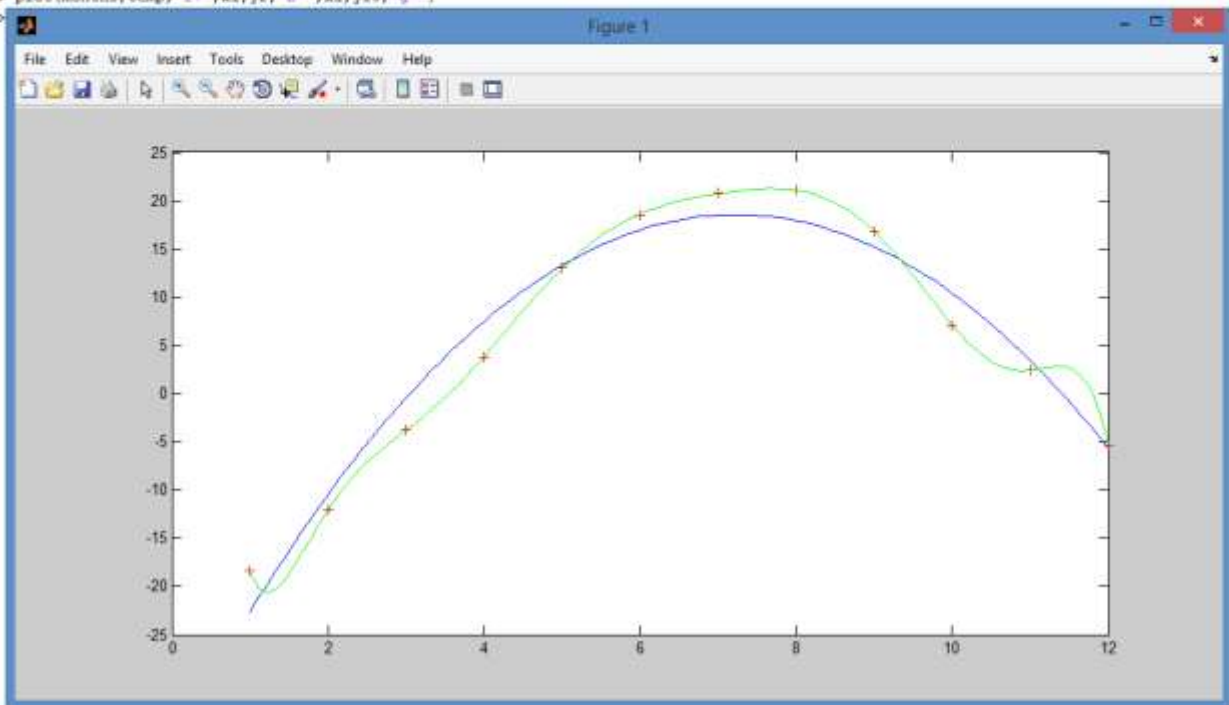


Assignment #5 - Solutions

Problem 1

```
>> months = [1 2 3 4 5 6 7 8 9 10 11 12];
>> temp = [-18.4 -12.1 -3.8 3.8 13 18.5 20.8 21 16.8 7.1 2.4 -5.3];
>> x1 = linspace(1,12,100);
>> p2 = polyfit(months,temp,2);
>> p10 = polyfit(months,temp,10);
Warning: Polynomial is badly conditioned. Add points with distinct X values, reduce the degree of the polynomial, or try centering
and scaling as described in HELP POLYFIT.
> In polyfit at 18
>> y2 = polyval(p2,x1);
>> y10 = polyval(p10,x1);
>> plot(months,temp,'c+',x1,y2,'b-',x1,y10,'g-')
```



Note that the polynomial of degree 10 passes through all of the points. However, we note that some anomalies to the extremities are due to the nature of the polynomial extrapolation. For a practical matter, a spline approximation gives a much more "smooth" curve.

Problem 2

a) We start by writing the formula second-order forward method:

$$f'_i = \frac{-f_{i+2} + 4f_{i+1} - 3f_i}{2h}$$

we are looking for $f'(0)$ so we set $x_i = 0$. In the RHS of the equation we need the values of function at x_{i+2} , x_{i+1} , and x_i . The step size is $h = 2$.

$$x_i = 0$$

$$x_{i+2} = x_i + 2h = 0 + 2 * 2 = 4$$

$$x_{i+1} = x_i + h = 0 + 1 * 2 = 2$$

we can now get the corresponding f values from the table so we can write:

$$f'(0) = \frac{-4.01 + 4 * 2.05 - 3 * 0.37}{2 * 2} = 1.01975$$

b) We start by writing the formula second-order forward method:

$$f'_i = \frac{-f_{i+2} + 4f_{i+1} - 3f_i}{2h}$$

we are looking for $f'(0)$ so we set $x_i = 0$. In the RHS of the equation we need the values of function at x_{i+2} , x_{i+1} , and x_i . The step size is $h = 1$.

$$x_i = 0$$

$$x_{i+2} = x_i + 2h = 0 + 2 * 1 = 2$$

$$x_{i+1} = x_i + h = 0 + 1 * 1 = 1$$

we can now get the corresponding f values from the table so we can write:

$$f'(0) = \frac{-2.05 + 4 * 1.14 - 3 * 0.37}{2 * 1} = 1.1995$$

Assignment #5 - Solutions

c) We start by writing the formula second-order centred method:

$$f'_i = \frac{f_{i+1} + f_{i-1}}{2h}$$

we are looking for $f'(0)$ so we set $x_i = 0$. In the RHS of the equation we need the values of function at x_{i-1}, x_{i+1} . The step size is $h = 2$.

$$x_i = 0$$

$$x_{i+1} = x_i + h = 0 + 1 * 2 = 2$$

$$x_{i-1} = x_i - h = 0 - 1 * 2 = -2$$

we can now get the corresponding f values from the table so we can write:

$$f'(0) = \frac{2.05 - .072}{2 * 2} = 0.04945$$

d) We start by writing the formula fourth-order centred method:

$$f'_i = \frac{-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}}{12h}$$

we are looking for $f'(0)$ so we set $x_i = 0$. In the RHS of the equation we need the values of function at $x_{i-1}, x_{i+1}, x_{i-2}, x_{i+2}$. The step size is $h = 1$.

$$x_i = 0$$

$$x_{i+1} = x_i + h = 0 + 1 * 1 = 1$$

$$x_{i-1} = x_i - h = 0 - 1 * 1 = -1$$

$$x_{i+2} = x_i + 2h = 0 + 2 * 1 = 2$$

$$x_{i-2} = x_i - 2h = 0 - 2 * 1 = -2$$

we can now get the corresponding f values from the table so we can write:

$$f'(0) = \frac{-2.05 + 8 * 1.14 - 8 * 0 + 0.72}{12 * 1} = 0.6491$$

e) We know that for using Richardson method (Romberg method) we need the results of two approximation resulted from the same method with the same order of accuracy with various step size h values. This is only valid for parts **a** and **b**. Please note that the result of the improved estimation using the results of parts **a** and **b** will be of 4th order (why?).

f) the results of part **e** and **d** are comparable since both are of order 4. The result of part **e** is of order 4 because the results we used for this Richardson's approximation are of 2nd order ($r = 2$) and the result will be of order of $r + 2 = 2 + 2 = 4$.

Problem 3

a) First of all, we need to find out how many times we need to apply Trapezoidal method. The step size is given $h = 2$:

$$m = \frac{b - a}{h}$$

where a and b are the lower and upper bounds of the integral: $a = -1$ and $b = 3$:

$$m = \frac{3 - (-1)}{2} = 2$$

So we need to apply this method twice. Each application of trapezoidal method needs two points (what about 1/3 and 3/8 Simpson's methods?) and therefore, we need to have 2 groups of 2-points. In order to find the required points we start from the lower bound ($a = -1$) and proceed by step size $h = 2$ to get the upper bound. It will result in the following table:

x	f
-1	f(-1)
1	f(1)
3	f(3)

first group one with two points : -1 and 1

second group with two points : 1 and 3

The equation for each single application of Trapezoidal is: $I_{Trapezoidal} = \frac{h}{2}(f_1 + f_2)$. So by applying the method to each group :

$$I = \frac{2}{2}(f(-1) + f(1)) + \frac{2}{2}(f(1) + f(3))$$

$$I = \frac{2}{2}(0 + 2) + \frac{2}{2}(2 + 84) = 88$$

b) First of all, we need to find out how many times we need to apply 1/3 Simpson's method. The step size is given $h = 2$:

$$m = \frac{b - a}{2h}$$

where a and b are the lower and upper bounds of the integral: $a = -1$ and $b = 3$:

$$m = \frac{3 - (-1)}{2 * 2} = 1$$

Assignment #5 - Solutions

So we need to apply this method once. Each application of 1/3 Simpson's method needs three points (what 3/8 Simpson's methods?) and therefore, we need to have 1 group of 3-points. In order to find the required points we start from the lower bound ($a = -1$) and proceed by step size $h = 2$ to get the upper bound. It will result in the following table:

x	f
-1	f(-1)
1	f(1)
3	f(3)

group with two points : $-1, 1, 3$

The equation for each single application of 1/3 Simpson's is: $I_{1/3\text{Simpson's}} = \frac{h}{3}(f_1 + 3f_2 + f_3)$. So by applying the method to the only group :

$$I = \frac{2}{3}(f(-1) + 3f(1) + f(3))$$

$$I = \frac{2}{3}(0 + 3 * 2 + 84) = 60$$

c) First of all, we need to find out how many times we need to apply Trapezoidal method. The step size is given $h = 1$:

$$m = \frac{b - a}{h}$$

where a and b are the lower and upper bounds of the integral: $a = -1$ and $b = 3$:

$$m = \frac{3 - (-1)}{1} = 4$$

So we need to apply this method for four times. Each application of trapezoidal method needs two points (what about 1/3 and 3/8 Simpson's methods?) and therefore, we need to have 4 group of 2-points. In order to find the required points we start from the lower bound ($a = -1$) and proceed by step size $h = 1$ to get the upper bound. It will result in the following table:

X	f
-1	f(-1)
0	f(0)
1	f(1)
2	f(2)
3	f(3)

first group one with two points : -1 and 0

second group with two points : 0 and 1

third group one with two points : 1 and 2

fourth group with two points : 2 and 3

Assignment #5 - Solutions

The equation for each single application of Trapezoidal is: $I_{Trapezoidal} = \frac{h}{2}(f_1 + f_2)$. So by applying the method to each group :

$$I = \frac{1}{2}(f(-1) + f(0)) + \frac{1}{2}(f(0) + f(1)) + \frac{1}{2}(f(1) + f(2)) + \frac{1}{2}(f(2) + f(3))$$

$$I = \frac{1}{2}(0 + 0) + \frac{1}{2}(0 + 2) + \frac{1}{2}(2 + 18) + \frac{1}{2}(18 + 84) = 62$$

d) We start by writing the formula:

$$I_{rombrg} = I_2 + \check{E}_2$$

$$\check{E}_2 = \frac{I_2 - I_1}{\left(\frac{h_1}{h_2}\right)^r - 1}$$

For the Romberg method, we need results of the same method with different step sizes. This is valid for results of part a and c. Based on the RHS of this equation, we need to decide about the indices. We know that $h_2 < h_1$. Here this would be valid if select:

$$h_1 = h_a = 2 \qquad h_2 = h_c = 1$$

$$I_1 = I_a = 88 \qquad I_2 = I_c = 62$$

We also need to determine the value of r which is the order of the methods we used. We used composite Trapezoidal method so $r = 2$.

$$I_{romberg} = 62 + \frac{62 - 88}{\left(\frac{2}{1}\right)^2 - 1} = 53.333$$

e)

$$I_{exact} = \int_{-1}^3 (x^4 + x) dx = \left[\frac{1}{5}x^5 + \frac{1}{2}x^2 \right] \Big|_{-1}^3 = 52.8$$

f) The true error can be estimated by

$$Error = I_{exact} - I_{approximation}$$

$$E_a = 35.2,$$

$$E_b = 7.2,$$

$$E_c = 9.2,$$

$$E_d = 0.5333$$

Assignment #5 - Solutions

Result of part **d** is more accurate (why?).

g) The function is a polynomial of degree 4 so we need a method whose error is proportional to $f^{(5)}$ or higher. Among the methods we did learn in this course, the only choice is 3-point Gaussian method:
 $E \sim f^{(2*3)} = f^{(6)}$

Problem 4

We know that each two consecutive polynomials (g functions) in cubic spline method must have equal: 1) value 2) slope (first derivative) and 3) curvature (second derivative) at the connecting points. Here, the connection point is $x_1 = 1$ therefore,

$$\begin{cases} P_l(x_1) = P_r(x_1) \\ P'_l(x_1) = P'_r(x_1) \\ P''_l(x_1) = P''_r(x_1) \end{cases}$$

so:

$$\begin{cases} a + b + c = 6 \\ 3a + 2b + c = 10 \\ 6a + 2b = 12 \end{cases}$$
$$a = 1 \quad b = 2 \quad c = 3$$

Problem 5

a)

$$\frac{dy}{dx} = -2xy$$

$$f(x, y) = -2xy$$

the step size is $h = 0.1$ and initial condition is given at $x = 0$:

$$\begin{cases} i = 0 \rightarrow x_0 = 0; y_0 = 1 \\ i = 1 \rightarrow x_1 = 0.2; y_1 = y(0.1) = ? \\ i = 2 \rightarrow x_2 = 0.2; y_2 = y(0.2) = ? \end{cases}$$

Assignment #5 - Solutions

So we need to take two iterations in order to find the value of y at 0.2. We write the formula of the Euler method:

$$y(x_{i+1}) = y(x_i) + f(x_i, y_i)h$$

1st iteration: $i=0$;

$$y(x_1) = y(x_0) + f(x_0, y_0)h$$

$$y(0.1) = y_0 + f(0,1)h = 1 - 2 * 0 * 1 * 0.1 = 1$$

2nd iteration: $i=1$;

$$y(x_2) = y(x_1) + f(x_1, y_1)h$$

$$y(0.2) = y_1 + f(0.1,1)h = 1 - 2 * 0.1 * 1 * 0.1 = 0.98$$

b)

$$\frac{dy}{dx} = -2xy$$

$$f(x, y) = -2xy$$

the step size is $h=0.1$ and initial condition is given at $x = 0$:

$$i = 0 \rightarrow x_0 = 0; y_0 = 1$$

$$i = 1 \rightarrow x_1 = 0.1; y_1 = ?$$

$$i = 2 \rightarrow x_2 = 0.2; y_2 = ?$$

So we need to take two iterations in order to find the value of y at 0.2. We write the formula of the Heun:

$$\left\{ \begin{array}{l} y(x_{i+1}) = y(x_i) + \left[\frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^*)}{2} \right] h \\ y_{i+1}^* = y(x_i) + f(x_i, y_i)h \end{array} \right.$$

1st iteration: $i=0$;

Assignment #5 - Solutions

$$\left\{ \begin{array}{l} y(x_1) = y(x_0) + \left[\frac{f(x_0, y_0) + f(x_1, y_1^*)}{2} \right] h \\ y_1^* = y(x_0) + f(x_0, y_0)h \end{array} \right.$$

So:

$$\left\{ \begin{array}{l} y(0.1) = y(0) + \left[\frac{f(0,1) + f(0.1, y_1^*)}{2} \right] 0.1 \\ y_1^* = y(0) + f(0.1, 1)0.1 = 1 + 0 = 1 \end{array} \right.$$

$$y(0.1) = y(0) + \left[\frac{f(0,1) + f(0.1,1)}{2} \right] 0.1 = 1 + \left[\frac{(-2 * 0 * 1) + (-2 * 0.1 * 1)}{2} \right] 0.1 = 0.99$$

2nd iteration: i=1;

$$\left\{ \begin{array}{l} y(x_2) = y(x_1) + \left[\frac{f(x_1, y_1) + f(x_2, y_2^*)}{2} \right] h \\ y_2^* = y(x_1) + f(x_1, y_1)h \end{array} \right.$$

So:

$$\left\{ \begin{array}{l} y(0.2) = y(0.1) + \left[\frac{f(0.1, 0.99) + f(0.2, y_2^*)}{2} \right] 0.1 \\ y_2^* = y(0.1) + f(0.1, 0.99)0.1 = 0.99 - 2 * .1 * .99 * 0.1 = 0.9702 \end{array} \right.$$

$$\begin{aligned} y(0.2) &= y(0.1) + \left[\frac{f(0.1, 0.99) + f(0.2, 0.9702)}{2} \right] 0.1 \\ &= 0.99 + \left[\frac{(-2 * 0.1 * 0.99) + (-2 * 0.2 * 0.9702)}{2} \right] 0.1 = 0.9707 \end{aligned}$$

c)

$$\frac{dy}{dx} = -2xy$$

$$f(x, y) = -2xy$$

Assignment #5 - Solutions

the step size is $h=0.1$ and initial condition is given at $x = 0$:

$$i = 0 \rightarrow x_0 = 0; y_0 = 1$$

$$i = 1 \rightarrow x_1 = 0.2; y_1 = ?$$

$$i = 2 \rightarrow x_2 = 0.4; y_2 = ?$$

So we need to take two iterations in order to find the value of y at 0.2 . We write the formula of the Mid-point:

$$\left\{ \begin{array}{l} y(x_{i+1}) = y(x_i) + \left[f\left(x_{i+\frac{1}{2}}, y_{i+\frac{1}{2}}\right) \right] h \\ y_{i+\frac{1}{2}} = y(x_i) + f(x_i, y_i) \frac{h}{2} \end{array} \right.$$

1st iteration: $i=0$;

$$\left\{ \begin{array}{l} y(x_1) = y(x_0) + f\left(x_{0+\frac{1}{2}}, y_{0+\frac{1}{2}}\right) h \\ y_{0+\frac{1}{2}} = y(x_0) + f(x_0, y_0) \frac{h}{2} \end{array} \right.$$

So:

$$\left\{ \begin{array}{l} y(0.1) = y(0) + f\left(0.05, y_{0+\frac{1}{2}}\right) h \\ y_{0+\frac{1}{2}} = y(0) + f(0,1) \frac{0.1}{2} = 1 + 0 = 1 \end{array} \right.$$

$$y(0.1) = y(0) + f(0.05,1)0.1 = 1 - 2 * 0.05 * 1 * 0.1 = 0.99$$

2nd iteration: $i=1$;

$$\left\{ \begin{array}{l} y(x_2) = y(x_1) + f\left(x_{1+\frac{1}{2}}, y_{1+\frac{1}{2}}\right) h \\ y_{1+\frac{1}{2}} = y(x_1) + f(x_1, y_1) \frac{h}{2} \end{array} \right.$$

Assignment #5 - Solutions

So:

$$\left\{ \begin{array}{l} y(0.2) = y(0.1) + f\left(0.15, y_{1+\frac{1}{2}}\right)h \\ y_{1+\frac{1}{2}} = y(0.1) + f(0.1, 0.99) \frac{0.1}{2} = 0.99 - 2 * 0.1 * 0.99 * \frac{0.1}{2} = 0.9801 \end{array} \right.$$

$$y(0.2) = 0.99 + f(0.15, 0.9801)0.1 = 0.99 - 2 * 0.15 * 0.9801 * 0.1 = 0.9606$$

d)

$$\frac{dy}{dx} = -2xy$$

$$f(x, y) = -2xy$$

the step size is $h=0.2$ and initial condition is given at $x = 0$:

$$i = 0 \rightarrow x_0 = 0; y_0 = 1$$

$$i = 1 \rightarrow x_1 = 0.2; y_1 = ?$$

So we need to take one iterations in order to find the value of y at 0.2 . We write the formula of the Rung-Kutta:

$$k_1 = f(x_0, y_0) = f(0, 1) = 0$$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1 h}{2}\right) = f(0.1, 1 + 0) = -0.2$$

$$k_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2 h}{2}\right) = f\left(0.1, 1 + \frac{-0.2 * 0.2}{2}\right) = -0.196$$

$$k_4 = f(x_0 + h, y_0 + k_3 h) = f(0.2, 1 - 0.196 * 0.2) = -0.38432$$

$$y(0.2) = y(0) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y(0.2) = 1 + \frac{0.2}{6}(0 + 2 * -0.2 + 2 * -0.196 - 0.38432) = 0.960789$$

e) exact solution for $x = 0.2$:

$$y = e^{-x^2}$$

$$y(0.2) = e^{-(0.2)^2} = 0.960789$$

Assignment #5 - Solutions

True relative errors:

$$E_a = \frac{0.98 - 0.960789}{0.960789} = 0.02$$

$$E_b = \frac{0.9707 - 0.960789}{0.960789} = 0.01 \%$$

$$E_c = \frac{0.9606 - 0.960789}{0.960789} = 0.02 \%$$

$$E_d = \frac{0.960789 - 0.960789}{0.960789} \approx 0$$

The results show that even with larger step size, the runge-kutta method is more accurate than the other methods.

Problem 6

First we need to determine the independent variables and assign indices to them in order to represent the Temperature (T) in the domain:

$$x \sim i$$

$$y \sim j$$

$$t \sim n$$

then we have to discretize the governing equation based on a general point $T_{i,j}^n$. The governing equation will become:

$$\left[\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \alpha T \right]_{i,j}^n$$
$$\left(\frac{\partial T}{\partial t} \right)_{i,j}^n = \alpha \left[\left(\frac{\partial^2 T}{\partial x^2} \right)_{i,j}^n + \left(\frac{\partial^2 T}{\partial y^2} \right)_{i,j}^n \right] + \alpha (T)_{i,j}^n$$

Now we have to replace the derivative terms with the corresponding discretized form:

First we do so for differential term respect to the time. Using forward Euler scheme:

$$\left(\frac{\partial T}{\partial t} \right)_{i,j}^n = \frac{T_{i,j}^{(n+1)} - T_{i,j}^{(n)}}{\Delta t}$$

Then we discretize the spatial differential terms. Using 2nd order centred scheme:

Assignment #5 - Solutions

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_{i,j}^n = \frac{T_{i+1,j}^{(n)} - 2T_{i,j}^{(n)} + T_{i-1,j}^{(n)}}{\Delta x^2}$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j}^n = \frac{T_{i,j+1}^{(n)} - 2T_{i,j}^{(n)} + T_{i,j-1}^{(n)}}{\Delta y^2}$$

Then we replace the differential terms with the discrete forms in the governing equation. (Please note that we also replace the $5T$ term in the governing equation with $5T_{i,j}^{(n)}$ since we are making the governing equation discrete for a general point with i, j and n indices)

$$\frac{T_{i,j}^{(n+1)} - T_{i,j}^{(n)}}{\Delta t} = 0.5 \frac{T_{i+1,j}^{(n)} - 2T_{i,j}^{(n)} + T_{i-1,j}^{(n)}}{\Delta x^2} + 0.5 \frac{T_{i,j+1}^{(n)} - 2T_{i,j}^{(n)} + T_{i,j-1}^{(n)}}{\Delta y^2} + 0.5T_{i,j}^{(n)}$$

Then we need to discretize the domain, using $\Delta x = 1$ and $\Delta y = 1$:

we have :

$$x_i = i\Delta x$$

$$y_j = j\Delta y$$

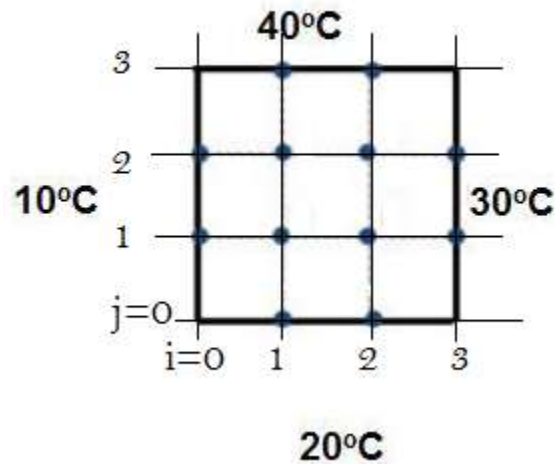
so we can write (we continue on i and j indices until we get to the end of the domain in x and y direction, here both are 3):

i	X_i
0	0
1	1
2	2
3	3

j	y_j
0	0
1	1
2	2
3	3

Assignment #5 - Solutions

we can discrete the domain based on the i and j values:



with the following i, j indices:

	$T_{1,3}$	$T_{2,3}$	
$T_{0,2}$	$T_{1,2}$	$T_{2,2}$	$T_{3,2}$
$T_{0,1}$	$T_{1,1}$	$T_{2,1}$	$T_{3,1}$
	$T_{1,0}$	$T_{2,0}$	

where only 4 highlighted points of the interior of the domain are computational nodes (or unknowns). So we need to apply the discrete governing equation to these 4 points.

Initial condition gives the $T_{i,j}^0$ values for the interior parts of the domain (only at $n = 0$)

Boundary Condition Gives the $T_{i,j}$ for the boundary points of the domain (for all n)

at $n = 0$ we have $t = n\Delta t = 0$ and we have : $T_{1,1}^0 = 5$; $T_{1,2}^0 = 5$; $T_{2,1}^0 = 5$; $T_{2,2}^0 = 5$

point(1,1)

$i=1; j=1; n=0;$

$$\frac{T_{1,1}^1 - T_{1,1}^0}{\Delta t} = 0.5 \frac{T_{2,1}^0 - 2T_{1,1}^0 + T_{0,1}^0}{\Delta x^2} + 0.5 \frac{T_{1,2}^0 - 2T_{1,1}^0 + T_{1,0}^0}{\Delta y^2} + 0.5T_{1,2}^0$$

$$T_{1,1}^{(1)} = T_{1,1}^{(0)} + 0.1 \left(0.5 [T_{2,1}^{(0)} - 2T_{1,1}^{(0)} + 10] + 0.5 [T_{1,2}^{(0)} - 2T_{1,1}^{(0)} + 20] + 0.5T_{1,2}^{(0)} \right)$$

$$T_{1,1}^{(1)} = 5 + 0.1(0.5 * [5 - 2 * 5 + 20] + 0.5 * [5 - 2 * 5 + 10] + 0.5 * 5)$$

$$T_{1,1}^{(1)} = 6.25^\circ C$$

Assignment #5 - Solutions

point(1,2)

i=1;j=2;n=0;

$$\frac{T_{1,2}^1 - T_{1,2}^0}{\Delta t} = 0.5 \frac{T_{2,2}^0 - 2T_{1,2}^0 + T_{0,2}^0}{\Delta x^2} + 0.5 \frac{T_{1,3}^0 - 2T_{1,2}^0 + T_{1,1}^0}{\Delta y^2} + 0.5T_{1,2}^0$$

$$T_{1,2}^{(1)} = T_{1,2}^{(0)} + 0.1 \left(0.5 [T_{2,2}^{(0)} - 2T_{1,2}^{(0)} + 20] + 0.5 [30 - 2T_{1,2}^{(0)} + T_{1,1}^{(0)}] + 0.5T_{1,2}^{(0)} \right)$$

$$T_{1,2}^{(1)} = 5 + 0.1(0.5 * [5 - 2 * 5 + 10] + 0.5 * [40 - 2 * 5 + 5] + 0.5 * 5)$$

$$T_{1,2}^{(1)} = 7.25^\circ C$$

point(2,1)

i=2;j=1;n=0;

$$\frac{T_{2,1}^1 - T_{2,1}^0}{\Delta t} = 0.5 \frac{T_{3,1}^0 - 2T_{2,1}^0 + T_{1,1}^0}{\Delta x^2} + 0.5 \frac{T_{2,2}^0 - 2T_{2,1}^0 + T_{2,0}^0}{\Delta y^2} + 0.5T_{2,1}^0$$

$$T_{2,1}^{(1)} = T_{2,1}^{(0)} + 0.1 \left(0.5 [30 - 2T_{2,1}^{(0)} + T_{1,1}^{(0)}] + 0.5 [T_{2,2}^{(0)} - 2T_{2,1}^{(0)} + 20] + 0.5T_{2,1}^{(0)} \right)$$

$$T_{2,1}^{(1)} = 5 + 0.1(0.5 * [30 - 2 * 5 + 5] + 0.5 * [5 - 2 * 5 + 20] + 0.5 * 5)$$

$$T_{2,1}^{(1)} = 7.25^\circ C$$

point(2,2)

i=2;j=2;n=0;

$$\frac{T_{2,2}^1 - T_{2,2}^0}{\Delta t} = 0.5 \frac{T_{3,2}^0 - 2T_{2,2}^0 + T_{1,2}^0}{\Delta x^2} + 0.5 \frac{T_{2,3}^0 - 2T_{2,2}^0 + T_{2,1}^0}{\Delta y^2} + 0.5T_{2,2}^0$$

$$T_{2,2}^{(1)} = T_{2,2}^{(0)} + 0.1 \left(0.5 [30 - 2T_{2,2}^{(0)} + T_{1,2}^{(0)}] + 0.5 [40 - 2T_{2,2}^{(0)} + T_{2,1}^{(0)}] + 0.5T_{2,2}^{(0)} \right)$$

$$T_{2,2}^{(1)} = 10 + 0.1(0.5 * [30 - 2 * 5 + 5] + 0.5 * [40 - 2 * 5 + 5] + 0.5 * 5)$$

$$T_{2,2}^{(1)} = 8.25^\circ C$$