Assignment 5 **Due: Monday, 11 April 2016, 4:00 pm**

Problem 1) Solve the specified problem of lab 6 (Only problem 3 of task 5).

Problem 2) The following data is provided. Calculate the value of the function derivative f' at point $x = 0$ using:

x	f(x)
-4	16.09
-3	4.39
-2	0.72
-1	0.00
0	0.37
1	1.14
2	2.05
3	3.02
4	4.01
5	5.00

(a) 2^{nd}-order forward method with $h = 2$.
(b) 2^{nd}-order forward method with $h = 1$.
(c) 2^{nd}-order central method with $h = 2$.
(d) 4^{th}-order central method with $h = 1$.
(e) If you want to improve the approximation of function derivative $f^{'}(0)$ using Richardson method, the results of which parts you would suggest to be used. (The solution of Richardson method **is not required**).
(f) comment on the order of accuracy of methods in part **d** and **e**.

Problem 3) Consider the following integral:

$$I = \int_{-1}^{3} (x^4 + x)dx$$

(a) Approximate the integral I using the trapezoidal method by step size of $h = 2$.

(b) Approximate the integral I using the 1/3 Simpson's method by step size of $h = 2$.

(c) Approximate the integral I using the trapezoidal method by step size of $h = 1$.

(d) Using Romberg method, improve the approximation of integral I using your results of parts **a** and **c**.

(e) Calculate the exact solution of integral I.

(f) Calculate the true error for parts **a, b , c,** and **d**. Discuss your results.

(g) Propose a method that gives the exact solution for integral I (The solution of the method **is not required**).

Problem 4) The following data are considered:

x	x_0	x_1	x_2	x_3
	0	1	2	3
f(x)	y_0	y_1	y_2	y_3

Applying the cubic spline method, the two polynomials at left, $P_l(x)$, and right, $P_r(x)$, of the point x_1 are obtained as follows,

For the interval $[x_0, x_1]$: $P_l(x) = ax^3 + bx^2 + cx + 2$

For the interval $[x_1, x_2]$: $P_d(x) = 2x^3 + 4x + 2$

Calculate the values of a, b, and c to satisfy all the required conditions of the polynomials in the cubic spline method.

Problem 5) The following ODE is given,

$$\frac{dy}{dx} = -2xy$$

where the initial value is $y(0) = 1$. Find the value of $y(0.2)$:

(a) Using Euler method with $h = 0.1$.

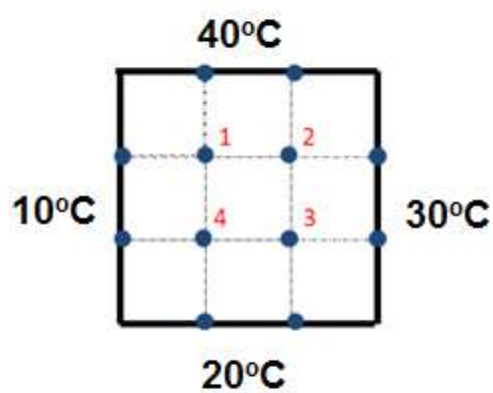
(b) Using Heun method with $h = 0.1$.

(c) Using mid-point method with $h = 0.1$.

(d) Using the 4th-order Rung-Kutta method with $h = 0.2$.

(e) The analytical solution for this ODE is $y = e^{-x^2}$. Find the exact solution for $y(0.2)$ and calculate the true relative error for previous parts. Discuss the result.

Problem 6) Consider a $3m \times 3m$ rectangular metal plate with initial temperature of 5^{oC} while the temperature around the plate is as given in the figure.



The following equation of plate temperature T is given:

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \alpha T$$

where $a = 0.5$. Using the time step $\Delta t = 0.1$:

(a) Find the temperature of the plate at points $1, 2, 3$ and 4 at time $t = 0.1$.

(b) Having your results of part **(a)**, determine the temperature at point 1 at time $t = 0.2$.

Use the **first-order forward method** for time derivative $\frac{\partial T}{\partial t}$, and use the **second order centred scheme** for spatial derivatives $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$.