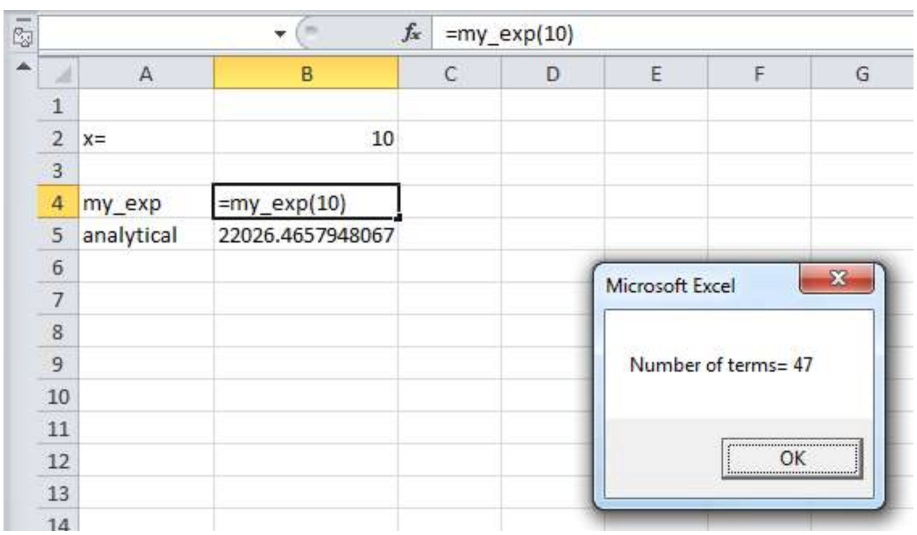
Question 1

```
Option Explicit
Function my_exp(x)
            Dim sum As Double, sum_old As Double, term As Double
   Dim i As Integer, max_terms As Integer
term = 1
max_terms = 1000
sum = term
For i = 1 Tomax_terms
sum_old = sum
term = term * x / i
sum = sum + term
   If Abs((sum - sum_old) / sum) * 100 < 0.00000000000001 Then Exit For
Next i
MsgBox " Number of terms= " &i + 1
my_exp = sum
End Function
```



Question 2

a- There should not be any zero in the main diagonal of the coefficient matrix. For the given system, the coefficient matrix is:

$$\begin{cases} 7y + 2z = 3 \\ 18z = 2 \\ x + 2y = 1 \end{cases}$$

$$A = \begin{bmatrix} 0 & 7 & 2 \\ 0 & 0 & 18 \\ 1 & 2 & 0 \end{bmatrix}$$

There are zeros in the main diagonal, so it cannot be solved by LU method in the current form. We have to change the order of the equations, so we swap the equations as follows,

$$\begin{cases} x + 2y = 1 \\ 7y + 2z = 3 \\ 18z = 2 \end{cases}$$

The matrix form of the new system is:

$$Ax = b$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 7 & 2 \\ 0 & 0 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

As can be seen in the new matrix, there is no zero in the main diagonal.

b-

$$\begin{cases} 9a + 3b + 5c = 17 \\ 7a + 5b + c = 13 \\ a + 2b + 4c = 7 \end{cases}$$

The matrix form of the system is:

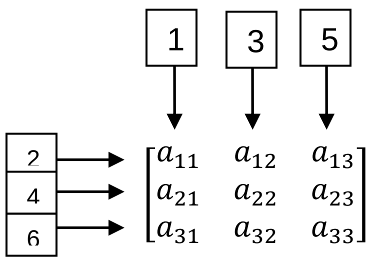
$$\begin{bmatrix} 9 & 3 & 5 \\ 7 & 5 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 17 \\ 13 \\ 7 \end{bmatrix}$$

As can be seen in the coefficient matrix, there is no zero in the main diagonal.

$$A = LU$$

$$\begin{bmatrix} 9 & 3 & 5 \\ 7 & 5 & 1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Now, we complete the matrix multiplication for each element of matrix A, based on the following steps:



Step1:

$$a_{11} = 9 = [l_{11} \quad 0 \quad 0] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad => l_{11} = 9$$

$$a_{21} = 7 = [l_{21} \quad l_{22} \quad 0] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad => l_{21} = 7$$

$$a_{31} = 1 = [l_{31} \quad l_{32} \quad l_{33}] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad => l_{31} = 1$$

Step2:

$$a_{12} = 3 = [l_{11} \quad 0 \quad 0] \begin{bmatrix} u_{12} \\ 1 \\ 0 \end{bmatrix} \quad => u_{12} = \frac{1}{3}$$

$$a_{13} = 5 = [l_{11} \quad 0 \quad 0] \begin{bmatrix} u_{13} \\ u_{23} \\ 1 \end{bmatrix} \quad => u_{13} = \frac{5}{9}$$

Step3:

$$a_{22} = 5 = [l_{21} \quad l_{22} \quad 0] \begin{bmatrix} u_{12} \\ 1 \\ 0 \end{bmatrix} => l_{22} = \frac{8}{3}$$

$$a_{32} = 2 = [l_{31} \quad l_{32} \quad l_{33}] \begin{bmatrix} u_{12} \\ 1 \\ 0 \end{bmatrix} \quad => l_{32} = \frac{5}{3}$$

Step4:

$$a_{23} = 1 = [l_{21} \quad l_{22} \quad 0]\begin{bmatrix} u_{13} \\ u_{23} \\ 1 \end{bmatrix} => u_{23} = -\frac{13}{12}$$

step5:

$$a_{33} = 4 = [l_{31} \quad l_{32} \quad l_{33}]\begin{bmatrix} u_{13} \\ u_{23} \\ 1 \end{bmatrix} \qquad => l_{33} = \frac{21}{4}$$

so:

$$l = \begin{bmatrix} 9 & 0 & 0 \\ 7 & \frac{8}{3} & 0 \\ 1 & \frac{5}{3} & \frac{21}{4} \end{bmatrix}$$

$$u = \begin{bmatrix} 1 & \frac{1}{3} & \frac{5}{9} \\ 0 & 1 & -\frac{13}{12} \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = LU$$

$$Ax = LUx = b$$

$$\begin{cases} Ux = z \\ Lz = b \end{cases}$$

$$Lz = b$$

$$\begin{bmatrix} 9 & 0 & 0 \\ 7 & \frac{8}{3} & 0 \\ 1 & \frac{5}{3} & \frac{21}{4} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 13 \\ 7 \end{bmatrix}$$

$9z_1 = 17 \quad => z_1 = \frac{17}{9}$

$7z_1 + \frac{8}{3}z_2 = 13 \quad => z_2 = -\frac{1}{12}$

$z_1 + \frac{5}{3}z_2 + \frac{21}{4}z_3 = 7 \quad => z_3 = 1$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \frac{17}{9} \\ -\frac{1}{12} \\ 1 \end{bmatrix}$$

$$Ux = z$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{5}{9} \\ 0 & 1 & -\frac{13}{12} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{17}{9} \\ -\frac{1}{12} \\ 1 \end{bmatrix}$$

$c = 1$

$b - \frac{13}{12}c = -\frac{1}{12} \quad => b = 1$

$a + \frac{1}{3}b + \frac{5}{9}c = \frac{17}{9} \quad => a = 1$

$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ we can check the solutions by plugging them into one of the equations:

$$9a + 3b + 5c = 17$$
$$9(1) + 3(1) + 5(1) = 17$$
$$17 = 17 \quad OK$$

Question 3

a- First, we write the matrix form of the system:

$$\begin{cases} 6x + y + 2z = 7 \\ x + 7y + 5z = -1 \\ 2\alpha x + (\frac{\beta}{4})y + 17z = 14 \end{cases}$$

$$\begin{bmatrix} 6 & 1 & 2 \\ 1 & 7 & 5 \\ 2\alpha & \frac{\beta}{4} & 17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 14 \end{bmatrix}$$

In order to use Cholesky's method, the coefficient matrix must be symmetric, so:

$2\alpha = 2 \qquad => \alpha = 1$

$\frac{\beta}{4} = 5 \qquad => \beta = 20$

b-

$$Ax = b$$

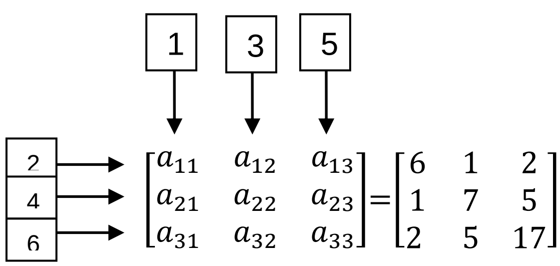
$$\begin{bmatrix} 6 & 1 & 2 \\ 1 & 7 & 5 \\ 2 & 5 & 17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 14 \end{bmatrix}$$

$$A = SS^T$$

We write the S matrix by their elements and transpose it inorder to form the S^T:

$$A = \begin{bmatrix} 6 & 1 & 2 \\ 1 & 7 & 5 \\ 2 & 5 & 17 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} s_{11} & 0 & 0 \\ s_{21} & s_{22} & 0 \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{bmatrix} s_{11} & s_{21} & s_{31} \\ 0 & s_{22} & s_{32} \\ 0 & 0 & s_{33} \end{bmatrix}$$

Now, we complete the matrix multiplication for each element of coefficient matrix A, based on the following steps. Because matrix A is symmetric, we can skip the symmetric values (i.e. we just use them once):



$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 6 & 1 & 2 \\ 1 & 7 & 5 \\ 2 & 5 & 17 \end{bmatrix}$$

Step1:

$$a_{11} = 6 = [s_{11} \quad 0 \quad 0] \begin{bmatrix} s_{11} \\ 0 \\ 0 \end{bmatrix} \quad => s_{11} = \sqrt{6}$$

$$a_{21} = 1 = [s_{21} \quad s_{22} \quad 0] \begin{bmatrix} s_{11} \\ 0 \\ 0 \end{bmatrix} => s_{21} = \frac{\sqrt{6}}{6}$$

$$a_{31} = 2 = [s_{31} \quad s_{32} \quad s_{33}] \begin{bmatrix} S_{11} \\ 0 \\ 0 \end{bmatrix} \qquad => s_{31} = 2\frac{\sqrt{6}}{6}$$

Step2:

a_{12} *is a symmetric value (we already used* a_{21}*)*

a_{13} *is a symmetric value (we already used* a_{31}*)*

step3:

$$a_{22} = 7 = [s_{21} \quad s_{22} \quad 0]\begin{bmatrix} s_{21} \\ s_{22} \\ 0 \end{bmatrix} \qquad => s_{22} = \sqrt{\frac{41}{6}}$$

$$a_{32} = 5 = [s_{31} \quad s_{32} \quad s_{33}]\begin{bmatrix} s_{21} \\ s_{22} \\ 0 \end{bmatrix} \qquad => s_{32} = \sqrt{\frac{392}{123}}$$

step4:

a_{23} *is a symmetric value (we already used* a_{32}*)*

step5:

$$a_{33} = 17 = [s_{31} \quad s_{32} \quad s_{33}]\begin{bmatrix} s_{31} \\ s_{32} \\ s_{33} \end{bmatrix} \qquad => s_{33} = \sqrt{\frac{539}{41}}$$

so:

$$S = \begin{bmatrix} s_{11} & 0 & 0 \\ s_{21} & s_{22} & 0 \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} \sqrt{6} & 0 & 0 \\ \frac{\sqrt{6}}{6} & \sqrt{\frac{41}{6}} & 0 \\ \frac{2\sqrt{6}}{6} & \sqrt{\frac{392}{123}} & \sqrt{\frac{539}{41}} \end{bmatrix} = \begin{bmatrix} 2.4495 & 0 & 0 \\ 0.4082 & 2.6141 & 0 \\ 0.8165 & 1.7852 & 3.6258 \end{bmatrix}$$

$$s^T = \begin{bmatrix} 2.4495 & 0.4082 & 0.8165 \\ 0 & 2.6141 & 1.7852 \\ 0 & 0 & 3.6258 \end{bmatrix}$$

$$A = SS^T$$
$$Ax = b$$
$$SS^T x = b$$

$$\begin{cases} S^T x = z \\ Sz = b \end{cases}$$

$$Sz = b$$

$$\begin{bmatrix} 2.4495 & 0 & 0 \\ 0.4082 & 2.6141 & 0 \\ 0.8165 & 1.7852 & 3.6258 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 14 \end{bmatrix}$$

$2.4495z_1 = 7 \quad => z_1 = 2.8577$
$0.4082z_1 + 2.6141z_2 = -1 \quad => z_2 = -0.8288$
$0.8165z_1 + 1.7852z_2 + 3.6258z_3 = 14 \quad => z_3 = 3.6258$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2.8577 \\ -0.8288 \\ 3.6258 \end{bmatrix}$$

$$S^T x = z$$

$$\begin{bmatrix} 2.4495 & 0.4082 & 0.8165 \\ 0 & 2.6141 & 1.7852 \\ 0 & 0 & 3.6258 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.8577 \\ -0.8288 \\ 3.6258 \end{bmatrix}$$

$3.6258z = 3.6258 \quad => z = 1$
$2.6141y + 1.7852z = -0.8288 \quad =>y= -1$
$2.4495x + 0.4082y + 0.8165z = 2.8577 \quad => x = 1$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ we can check the solutions by plugging them into one of the equations:

$$6x + y + 2z = 7$$
$$6(1) + (-1) + 2(1) = 7$$
$$7 = 7 \quad OK$$

Question 4

a- We know that:

$$AA^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From problem 3-a we have : $A = SS^T$

$$SS^T A^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can write the inverse matrix by its columns:

$$A^{-1} = [C1 \quad C2 \quad C3]$$

$$C1 = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} \; C2 = \begin{bmatrix} c_{12} \\ c_{22} \\ c_{32} \end{bmatrix} \; C3 = \begin{bmatrix} c_{13} \\ c_{23} \\ c_{33} \end{bmatrix}$$

$$SS^T [C1 \quad C2 \quad C3] = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

part1- $SS^T C1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

part2- $SS^T C2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

part3- $SS^T C3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Solving part1:

$$SS^T C1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} S^T C1 = z1 \\ Sz1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

$$\begin{bmatrix} 2.4495 & 0 & 0 \\ 0.4082 & 2.6141 & 0 \\ 0.8165 & 1.7852 & 3.6258 \end{bmatrix} \begin{bmatrix} z1_1 \\ z1_2 \\ z1_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$2.4495z1_1 = 1 \quad => \quad z1_1 = 0.4082$

$0.4082z1_1 + 2.6141z1_2 = 0 \quad => \quad z1_2 = -0.0638$

$0.8165z1_1 + 1.7852Z1_2 + 3.6258z1_3 = 0 \quad => \quad z1_3 = -0.0605$

$$z1 = \begin{bmatrix} 0.4082 \\ -0.0638 \\ -0.0605 \end{bmatrix}$$

$$S^T C1 = z1 = \begin{bmatrix} 0.4082 \\ -0.0638 \\ -0.0605 \end{bmatrix}$$

$$\begin{bmatrix} 2.4495 & 0.4082 & 0.8165 \\ 0 & 2.6141 & 1.7852 \\ 0 & 0 & 3.6258 \end{bmatrix} C1 = \begin{bmatrix} 0.4082 \\ -0.0638 \\ -0.0605 \end{bmatrix}$$

$$C1 = \begin{bmatrix} 0.1744 \\ -0.0130 \\ -0.0167 \end{bmatrix}$$

Solving part2:

similar to part1, $C2 = \begin{bmatrix} -0.0130 \\ 0.1818 \\ -0.0519 \end{bmatrix}$

Solving part3:

similar to part1, $C3 = \begin{bmatrix} -0.0167 \\ -0.0519 \\ 0.0761 \end{bmatrix}$

So: $A^{-1} = \begin{bmatrix} 0.1744 & -0.0130 & -0.0167 \\ -0.0130 & 0.1818 & -0.0519 \\ -0.0167 & -0.0519 & 0.0761 \end{bmatrix}$

b- To solve the system we have

$$Ax = b$$
$$A^{-1}Ax = A^{-1}b$$
$$Ix = A^{-1}b$$
$$x = A^{-1}b$$

$$x = \begin{bmatrix} 0.1744 & -0.0130 & -0.0167 \\ -0.0130 & 0.1818 & -0.0519 \\ -0.0167 & -0.0519 & 0.0761 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \\ 14 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$