Question 1

$$m\frac{dv}{dt} = mg - F_{air\ resistance}$$

$$\frac{dv}{dt} = g - \frac{c \times v_i^2}{m}$$

Using forward scheme for differential term $\frac{dv}{dt}$ will result in the following discrete form:

$$v_{i+1} = v_i + \left(g - \frac{c \times v_i^2}{m}\right) \times \Delta t$$

We use the excel to solve the equation:

	A	B
1	gravity	9.81
2	student #	2703444
3	coeff	0.125
4	time	1
5	mass	44.07114

$$m = 35 \cdot \left(1 + \frac{Student\ ID - 19 \times 10^5}{31 \times 10^5}\right)$$

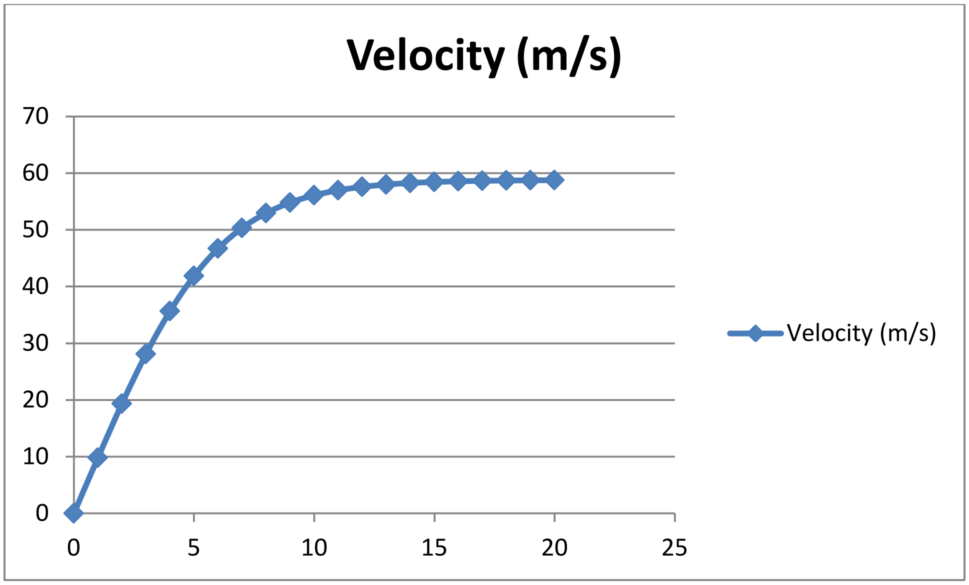
Using visual basic one can write the following function:

```
Function task8(gravity, coeff, time, mass, v)
task8 = v + (gravity - coeff / mass * v * v) * time
End Function
```

SUM | =task8(gravity,coeff,time,mass,F3)

	A	B	C	D	E	F
1	gravity	9.81				
2	student #	2703444			t	v
3	coeff	0.125			0	0.000000
4	time	1			1	=task8(gravity,coeff,time,mass,F3)
5	mass	44.07114			2	19.347043

Time (s)	Velocity (m/s)
0	0.000
1	9.810
2	19.347
3	28.095
4	35.667
5	41.868
6	46.706
7	50.329
8	52.955
9	54.811
10	56.100
11	56.983
12	57.584
13	57.989
14	58.261
15	58.444
16	58.566
17	58.647
18	58.702
19	58.738
20	58.762



Question 2

a- Because we have $f(0) \times f(0.5) < 0$ so the function changes the sign in this range. Therefore, the initial bracket includes roots of the function.

b- Using bisection method, in the second iteration, the value of $x3 = \frac{0+0.5}{2} = 0.25$ is obtained. Based on the provided figure, $f(0.25) < 0$. Therefore:

$$f(0)f(0.25) > 0$$

$$f(0.25)f(0.5) < 0$$

Thus, the root is located in $[0.25\ 0.5]$ bracket and using bisection method will lead to the third rood in the given figure.

c- We can consider the length of the bracket as an approximate error for the method. In bisection method, we divide the length of the initial bracket by half in each iteration. Generally, for an initial bracket of $[a \quad b]$ the initial length is $|(b-a)|$ and the length of the bracket in n^{th} iteration can be written as :

$$\frac{|(b-a)|}{2^n}$$

For this problem, we have:

$$E = 10^{-6} = \frac{0.5}{2^n}$$

By taking $\log_2$ from both sides of the equation and solving for n, one can find $n \approx 19$ which means 19 iterations are required.

Question 3

a- Newton method reads:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

We have $f(x) = x + \ln(x)$. Therefore,

$$f'(x) = 1 + \frac{1}{x}$$

The initial value is given as $x_0 = 0.7$. For the first iteration we have:

$$x_1 = 0.7 - \frac{(0.7 + \ln(0.7))}{(\frac{1}{0.7} + 1)} = 0.558630859269$$

The results of the other iterations along with the relative errors are given in the following table.

	Value of x	Relative error (en %)
x0	0.700000000000	
x1	0.558630859269	25.30
x2	0.567102097266	-1.49
x3	0.567143289455	-0.00726

The approximate relative error reads:

$$Approximate\ relative\ error = \frac{x_{new} - x_{old}}{x_{new}} \cdot 100\% = \frac{x_{i+1} - x_i}{x_{i+1}} \cdot 100\%$$

b- The second order Newton's method reads:

$$x_{n+1} = x_n - \frac{2! \cdot f(x_n) \cdot f'(x_n)}{\left[2! \cdot f'(x_n)^2 - f(x_n) \cdot f''(x_n)\right]}$$

we need to calculate the second-derivative of the function $f(x)$:

$$f'(x) = -\frac{1}{x^2}$$

With initial value of $x_0 = 0.7$:

$$x_1 = 0.7 - \frac{2*1*((0.7+\ln(0.7)*\left(\frac{1}{0.7}+1\right))}{2*1*\left(\frac{1}{0.7}+1\right)^2 - (0.7+\ln(0.7))*(-1/0.7^2)} = 0.566557201203$$

The results of the other iterations along with the relative errors are given in the following table.

	Value of x	Error (%)
x0	0.700000000000	
x1	0.566557201203	23.5
x2	0.567143290479	-0.10
x3	0.567143290410	1.2E-08

c- The results show that the second-order method, at each iteration, has lower approximate relative error compared to the first order method. This implies faster convergence of the second-order method. However, in this specific example, because of the selection of the initial point, the performance of the methods are very close to each other.

d- The first step is to investigate if the method with proposed forms of $x = g(x)$ converges or not. We have to check if $\left|g'(x_0)\right| < 1$.

- The first form is $x = e^{-x}$ so:

$$g(x) = e^{-x}$$
$$g'(x) = -1 \times e^{-x}$$
$$|g'(0.7)| = 0.497 < 1$$

therefore, this form will converge.

- The second form is $x = -\ln(x)$ so:

$$g(x) = -\ln(x)$$
$$g^{'}(x) = -\frac{1}{x}$$
$$\left|g^{'}(0.7)\right| = 1.430 > 1$$

therefore, this form will **not** converge.

The firs form can be used and the iterative form is

$$x_{i+1} = g(x_i)$$

$$x_{i+1} = e^{-x_i}$$

The first iteration is:

$$x_1 = e^{-0.7} = 0.497$$

$$Approximate\ relative\ error = \frac{True\ Value - x_0}{True\ Value} \cdot 100\%$$

We take the true value from part **3a:**

$$True\ Value = 0.567$$

$$Approximate\ relative\ error = \frac{0.567 - 0.497}{0.567} * 100 = 12.3\ \%$$

The x values and the corresponding approximate relative errors are presented for five iterations in the table below:

Iteration	Value of x	True relative error %
x_0	0.7	
x_1	0.499	12.3
x_2	0.609	-7.2
x_3	0.544	4
x_4	0.580	-2
x_5	0.560	1.3