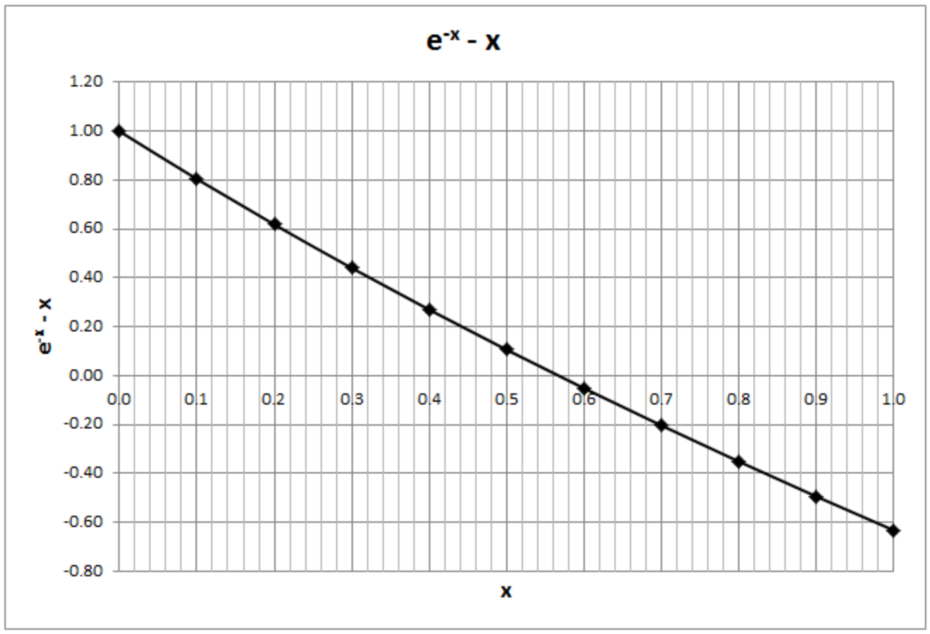
Question 1

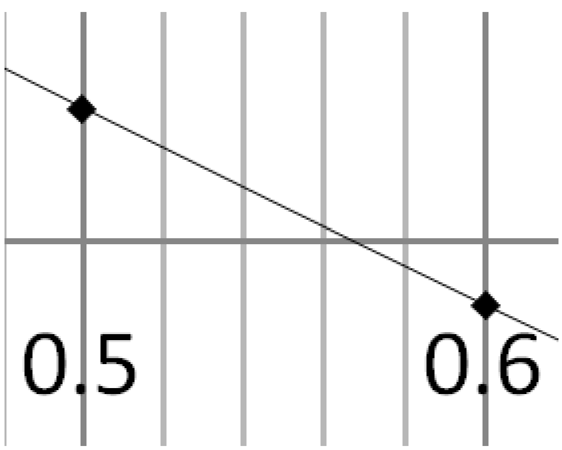
we consider e^x with **exp()** function in Excel.
For instance for $e^x - x$ we will have :

=exp(B4) – B4

then we introduce a range and check where the plot crosses the x-axis:

x	e^{-x} - x
0.0	1.00
0.1	0.80
0.2	0.62
0.3	0.44
0.4	0.27
0.5	0.11
0.6	-0.05
0.7	-0.20
0.8	-0.35
0.9	-0.49
1.0	-0.63





Roughly one can say the root is $x \approx 0.567$.

Question 2

$$a = \frac{m\ g\ sin\theta - F}{m}$$

The value of the acceleration for the measured values is as follows:

$$a = \frac{50 * 10 * \sin(0.5) - 100}{50} = 2.794$$

There is no uncertainty associated with $\boldsymbol{g}$, so we consider the error of other inputs:

$$\Delta a = \left|\frac{da}{dm}\right| \Delta m + \left|\frac{da}{dF}\right| \Delta F + \left|\frac{da}{d\theta}\right| \Delta \theta$$

$$\frac{da}{dm} = \frac{F}{m^2}$$

$$\frac{da}{dF} = -\frac{1}{m}$$

$$\frac{da}{d\theta} = gcos\theta$$

$$\Delta a = \frac{100}{50^2} * 1 + \frac{1}{50} * 1 + 10 * \cos(0.5) * 0.005 = 0.04 + 0.02 + 0.0439 = 0.1039$$

We note the plausible range of acceleration will be $a \pm \Delta a$,that is :

$$a = [2.690, 2.898]$$

Question 3

a & b)

$$f(t) = \frac{1}{t} + \sin(t)$$

Forward method:

$$f'(t_i) = \frac{f(t_{i+1}) - f(t_i)}{\Delta t}$$

$$t_i = 2$$

$$t_{i+1} = t_i + h = 2 + 0.1 = 2.1$$

$$f(2) = 1.409$$

$$f(2.1) = 1.339$$

$$f'(2) = \frac{f(2.1) - f(2)}{0.1} = \frac{1.339 - 1.409}{0.1} = -0.7$$

Backward method:

$$f'(t_i) = \frac{f(t_i) - f(t_{i-1})}{\Delta t}$$

$$t_i = 2$$

$$t_{i-1} = t_i - h = 2 - 0.1 = 1.9$$

$$f(2) = 1.409$$

$$f(1.9) = 1.473$$

$$f'(2) = \frac{f(2) - f(1.9)}{0.1} = \frac{1.409 - 1.473}{0.1} = -0.64$$

Central method:

$$f'(t_i) = \frac{f(t_{i+1}) - f(t_{i-1})}{2\Delta t}$$

$$t_i = 2$$

$$t_{i-1} = t_i - h = 2 - 0.1 = 1.9$$

$$t_{i+1} = t_i + h = 2 + 0.1 = 2.1$$

$$f'(2) = \frac{f(2.1) - f(1.9)}{2 * 0.1} = \frac{1.339 - 1.473}{0.2} = -0.67$$

$$Forward\ error = \frac{central - forward}{central} * 100 = \frac{-0.67 - (-0.7)}{-.67} = -4.47\ \%$$

$$Backward\ error = \frac{central - Backward}{central} * 100 = \frac{-0.67 - (-.064)}{-.67} = -4.47\ \%$$

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c) The exact value can be obtained from the function derivative:

$$f'(t) = -\frac{1}{t^2} + \cos(t)$$

$$f'(2) = -0.666$$

$$True\ error = \frac{exact - central}{exact} * 100 = \frac{-0.666 - (-0.67)}{-0.67} * 100 = 0.6\ \%$$

d) In function derivative approximation the error of a n^{th}-order method is proportional to:

$$E \sim h^n$$

The central method is second-order so n=2.

$$E_1 \sim h_1^2 = 0.1^2$$

$$E_2 \sim h_2^2$$

$$\frac{E_1}{E_2} = \left(\frac{h_1}{h_2}\right)^2$$

We want the error to be 50 times smaller, it means:

$$E_2 = \frac{E_1}{50}$$

$$\frac{E_1}{E_2} = 50$$

$$50 = \left(\frac{0.1}{h_2}\right)^2$$

$$h_2 = 0.014$$

e) Taylor series read:

$$f(t_{i+1}) = f(t_i) + \frac{f'(t_i)}{1!}h^1 + \frac{f''(t_i)}{2!}h^2 + \cdots$$
$$t_{i+1} - t_i = h$$

for the first order we just need two terms so:

$$f(t_{i+1}) = f(t_i) + \frac{f'(t_i)}{1!}h^1$$

We need the value of $f(2)$, so:

$$t_{i+1} = 2$$
$$t_{i+1} - t_i = h$$
$$2 - t_i = 0.1$$
$$t_i = 1.9$$

At the right-hand-side we need:

$$f(1.9) = 1.473$$
$$f'(1.9) = -0.600$$

Then,

$$f(2) = 1.473 - 0.600 * 0.1 = 1.413$$

f) Condition_number

$$CN(\tilde{t}) = \left|\frac{\tilde{t}f'(\tilde{t})}{f(\tilde{t})}\right|$$

for $\tilde{t} = 2$:

$$CN(1) = \left|\frac{2 * f'(2)}{f(2)}\right| = \left|\frac{2 * -0.666}{1.409}\right| = 0.945 < 1$$

The function is called ill-conditioned if its condition number is greater than 1. In this case, the function is well-conditioned, that is to say, the error terms will not be magnified.

Question 4

the answers are presented in the following table:

$$f(x) = e^x - \frac{1}{x^2}$$

$$Initial\ bracket: [x_1 \quad x_2] = [0.68, 0.72]$$

Iteration	x_1	$f(x_1)$	x_2	$f(x_2)$	x_3	$f(x_3)$	New Interval (bracket)			Approximate relative error
1	0.68	-0.188	0.72	0.125	0.70	-0.027	$f(x_3)f(x_2) < 0$	0.70	0.72	-
2	0.70	-0.027	0.72	0.125	0.71	0.050	$f(x_1)f(x_2) < 0$	0.70	0.71	1.41%
3	0.70	-0.027	0.71	0.050	0.705	0.012	$f(x_1)f(x_2) < 0$	0.70	0.705	-0.71%

<u>First iteration calculation sample:</u>

$$f(x_1) = f(0.68) = -0.188$$

$$f(x_2) = f(0.72) = 0.125$$

$$x_3 = \frac{x_1 + x_2}{2} = \frac{0.68 + 0.72}{2} = 0.70$$

$$f(x_3) = f(0.70) = 0.027$$

The new interval is chosen by replacing x1, x2, x3 in the conditions $f(x_1)\,f(x_3) < 0$ and $f(x_2)\,f(x_3) < 0$ by considering the sign of the value of the function. In this case, $f\ (x_1)$ & $f\ (x_3)$ are negative and the condition is not satisfied but the condition $f(x_2)\,f(x_3) < 0$ is satisfied and therefore the interval for the next iteration will be:

$$[x_1, x_2]^1 = [0.70, 0.72]$$

$$approximate\ relative\ error = \frac{x_{3New} - x_{3Previous}}{x_{3New}} \cdot 100\%$$