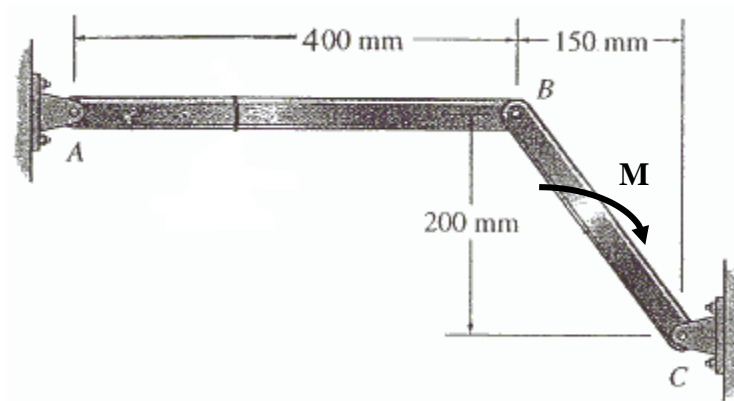


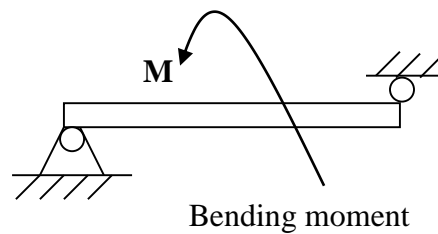
Question 1 (20%)

Multiple Choice Questions (Answer will be only **one** among A to D for each sub-question.)

- (1) The members in the structure below are pin-jointed, loaded as shown. The moment load M is applied in the paper plane. Which of the following statements for the structure is incorrect?

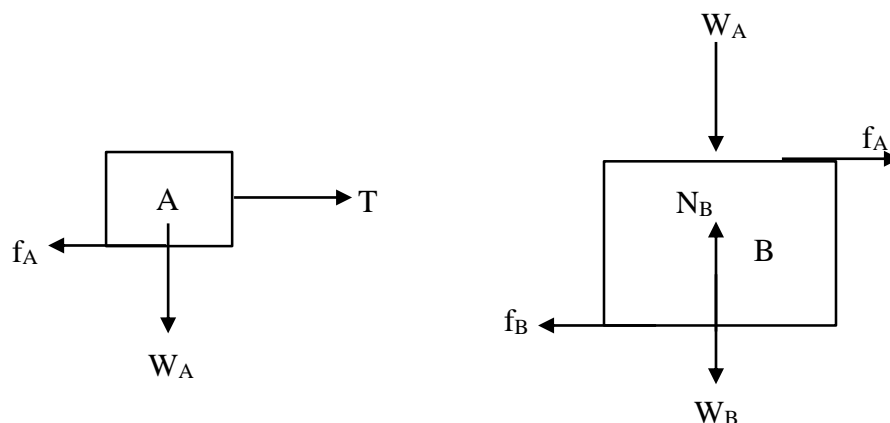
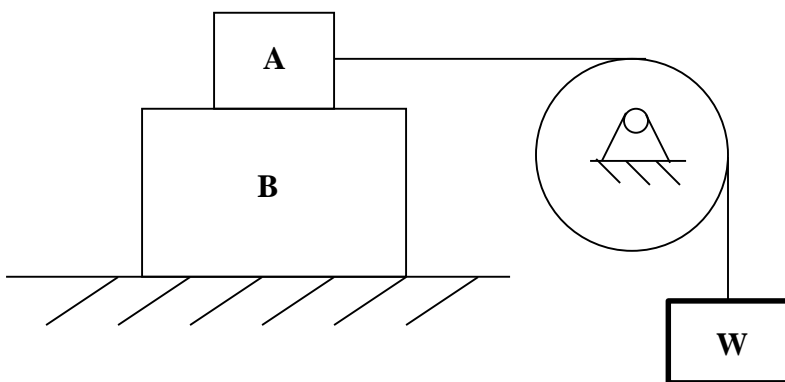


- (A) Both normal and shear stresses are induced in member AB .
(B) Normal stress is induced in member AB .
(C) Shear stress is not induced in member AB .
(D) Both normal and shear stresses are induced in member BC .
- (2) For the load mode below, which of the following statements is correct? The moment load M is applied in the paper plane.



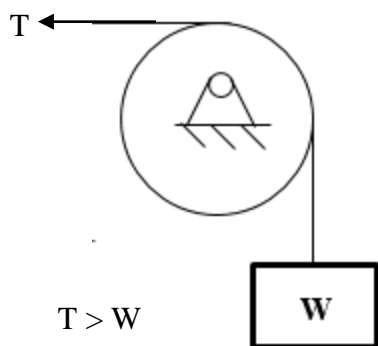
- (A) Shear stress is constant throughout the beam.
(B) Bending moment is constant along the beam.
(C) Bending stress (normal stress) is constant throughout the beam.
(D) Shear force is constant along the beam.

- (3) In the figure below, the weights of block A and block B are W_A and W_B , respectively. Friction cannot be neglected between the wheel and the rope, between block A and block B , between block B and the support. Under the weight W applied at the free end of the rope, the system is static. For force equilibrium of the system, which of the following Free Body Diagrams (FBDs) is correct?

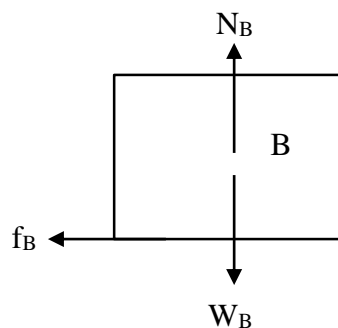


(A)

(B)

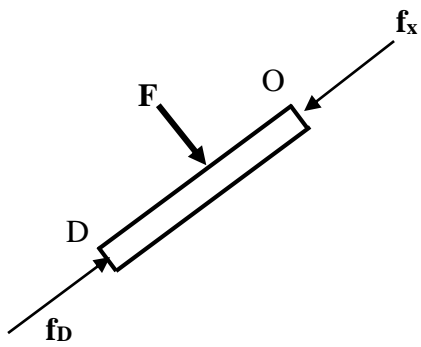
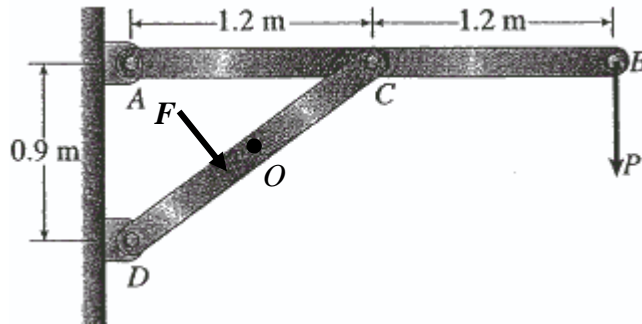


(C)

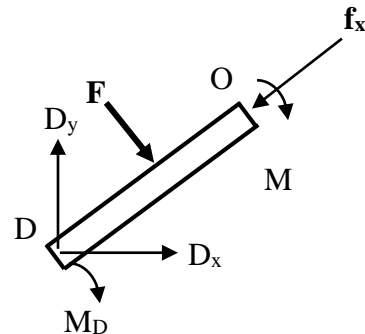


(D)

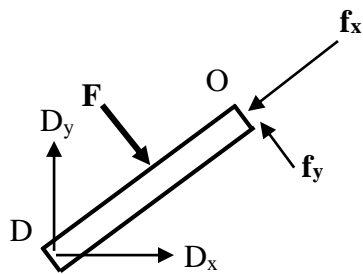
- (4) For the loaded structure below, to determine the internal components at the center O of member CD , which of the following FBDs is correct?



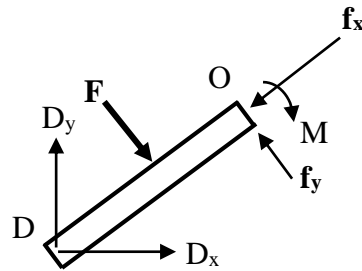
(A)



(B)



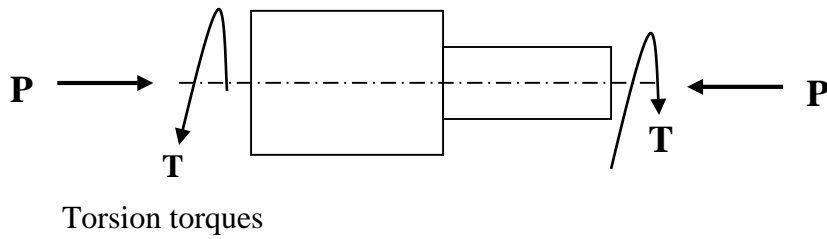
(C)



(D)

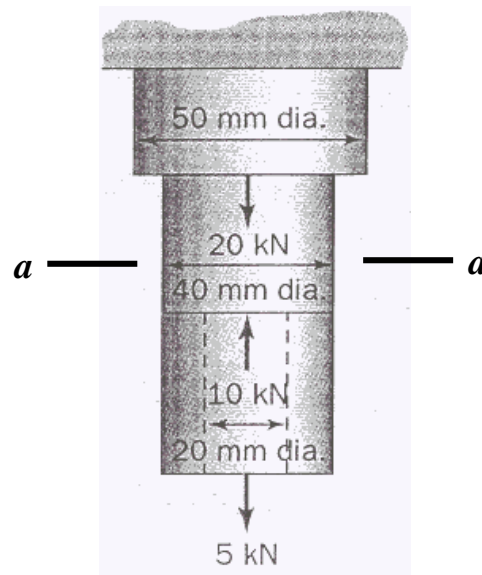
- (5) For a thin-walled cylinder subjected to internal air pressure, which of the following statements is reasonable?
- (A) Axial, circumferential and radial stresses vary linearly through the wall thickness.
 - (B) Radial stress is negligible.
 - (C) Circumferential stress is smaller than axial stress.
 - (D) Axial stress increases with the wall thickness.

(6) For the loaded solid shaft below, which of the following statements is incorrect?



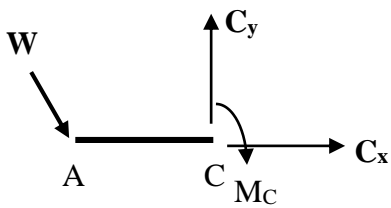
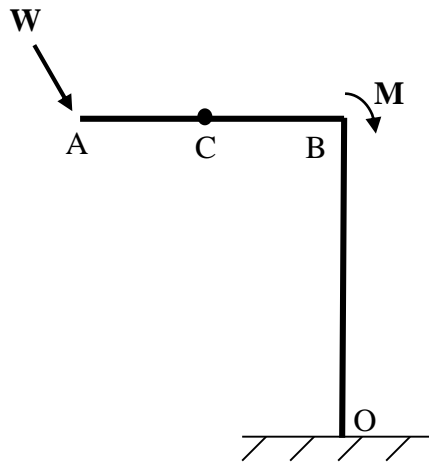
- (A) Normal stress is constant throughout the shaft.
- (B) Shear stress is constant throughout the shaft.**
- (C) Maximum shear stress occurs on the surface of the shaft.
- (D) Shear stress is zero at the axis of the shaft.

(7) In the axially loaded structure below, what is the internal force of section $a - a$?

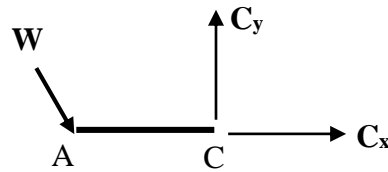


- (A) 20 kN
- (B) 5 kN**
- (C) 10 kN
- (D) 15 kN

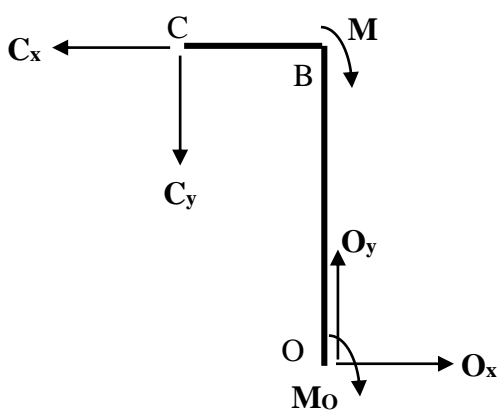
- (8) Bracket ABO is welded at B and built-in at O , as shown below. To determine the internal components at C , which of the following FBDs is correct?



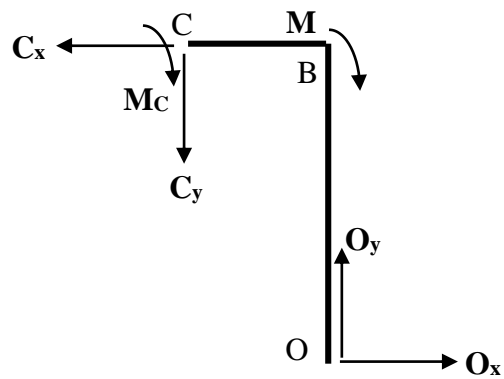
(A)



(B)

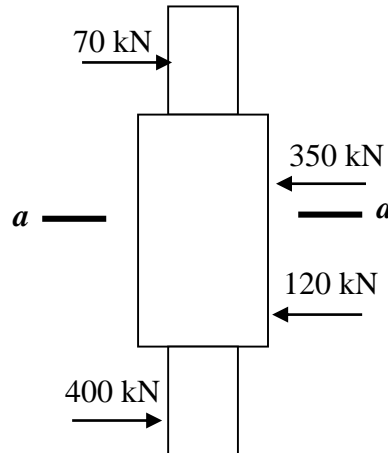


(C)

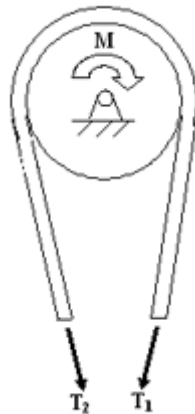


(D)

- (9) For the shaft subjected to shear forces, as shown below, what is the internal force value of section $a - a$?

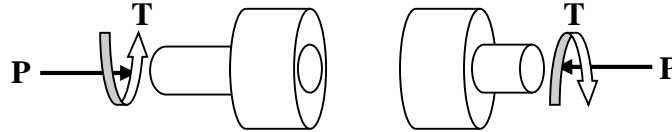


- (A) 120 kN
 (B) 150 kN
 (C) 350 kN
 (D) 280 kN
- (10) Flexible flat belt is applied on cylindrical wheel to transfer power, as shown below. Suppose that the friction between the belt and wheel reaches the maximum static friction. Which of the following statements is incorrect?



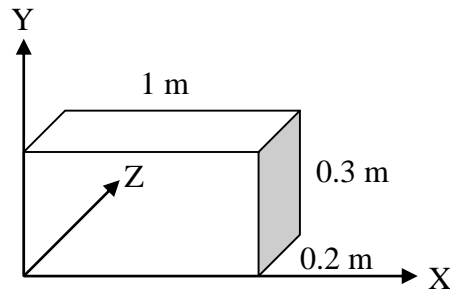
- (A) The ratio of the internal forces T_2/T_1 increases with the friction coefficient between the cylinder and belt.
 (B) The ratio of the internal forces T_2/T_1 increases with the contact angle between the cylinder and belt.
 (C) The contact angle between the cylinder and belt is always less than 180° .
 (D) The ratio of the internal forces T_2/T_1 is constant for a given belt/cylinder system.

- (11) Clutch plates and disc brakes are the main applications of friction between coaxial discs. Coaxial discs are pressed together by an axial force P , as shown below. For the maximum torque T that can be transmitted from one to the other by frictional contact, which of the following statements is incorrect?

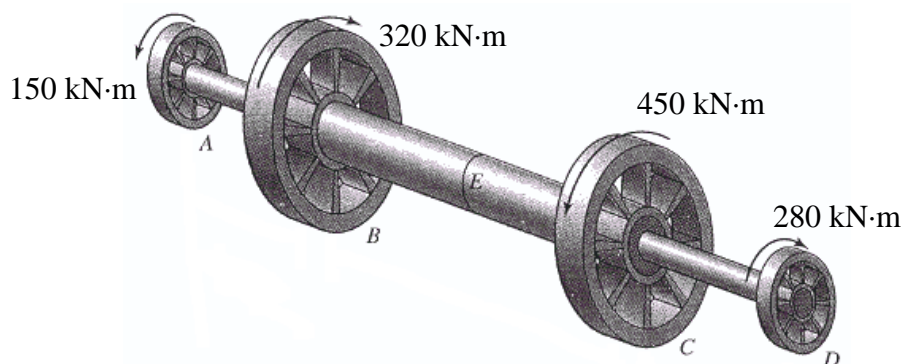


- (A) The contact pressure is always uniform over the disc surfaces.
 (B) The torque T increases as the contact pressure increases.
 (C) The torque varies with the radius of the disc.
 (D) The torque varies with the friction coefficient between the discs.
- (12) For three-dimensional stress state, which of the following statements is incorrect?
- (A) At a point of a loaded body, there are three normal stresses and six shear stresses.
 (B) On each plane there are two shear stresses that are equal.
 (C) A shear stress on one plane is always accompanied by a complementary shear stress acting on the perpendicular plane.
 (D) The three normal stresses at a point are generally not equal.
- (13) For three-dimensional stress ~ strain relations, which of the following statements is incorrect?
- (A) Volume strain is the sum of normal strains.
 (B) In plane stress condition, one of normal stresses is zero.
 (C) In plane strain condition, all normal strains are zero.
 (D) For a hydrostatic state of stress, $\sigma_x = \sigma_y = \sigma_z$.
- (14) For stress ~ strain behavior of materials, which of the following statements is incorrect?
- (A) Tensile strength is the ultimate stress that a material can take before failure.
 (B) The materials exhibiting significant yielding are classed into ductile materials.
 (C) Below proportion limit, stress increases linearly with strain.
 (D) Due to strain hardening stress does not increase with strain after yielding occurs.

- (15) For the block shown below, in high temperature environments, which of the following statements is incorrect?

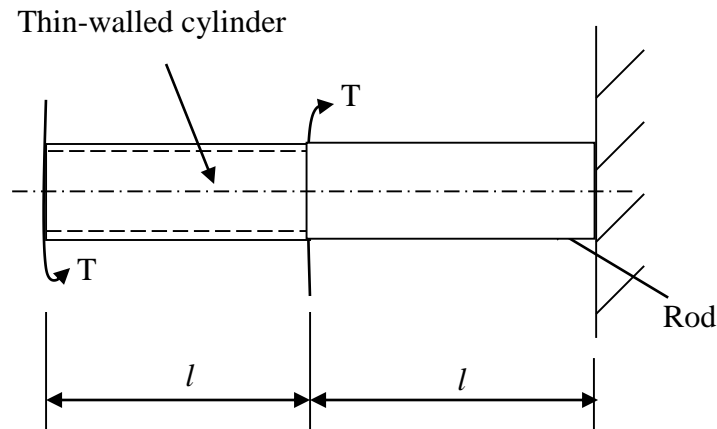


- (A) The thermal deformations of the block in X , Y and Z directions are equal.
 (B) Thermal strain is proportional to the thermal coefficient of the block material.
 (C) Thermal strain in X , Y and Z directions are equal.
 (D) Thermal strain is proportional to the temperature change.
- (16) A solid circular steel shaft is subjected to the torques shown below. What is the value of torque in the shaft at the middle point E between B and C ?

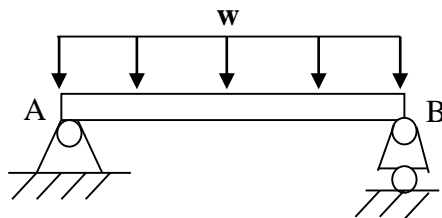


- (A) 450 N·m
 (B) 320 N·m
 (C) 150 N·m
 (D) 170 N·m
- (17) Which of the following statements is incorrect for beam bending?
- (A) In symmetric beam bending the applied load passes through the axis of symmetry of the beam cross section.
 (B) The top and bottom surfaces of the beam experience maximum deformation.
 (C) At neutral axis normal strain is the maximum.
 (D) Neutral axis passes through the centroid of the beam cross section.

- (18) For the composite shaft under torsion shown below, which of the following statements is correct?



- (A) Maximum shear stresses of the thin-walled cylinder and rod are equal.
 (B) Internal torques of the thin-walled cylinder and rod are equal.
 (C) Shear stress is zero in the rod.
 (D) Maximum twisting angle of the shaft occurs at the interface between the thin-walled cylinder and rod.
- (19) For a simply supported beam subjected to a uniformly distributed load, as shown below, which of the following statements is incorrect?



- (A) Maximum slope occurs at the center of the beam.
 (B) Shear force is not constant along the beam.
 (C) Bending stress is zero at the supports.
 (D) Shear stress is zero at the central section of the beam.
- (20) In buckling analysis of a strut/column, which of the following statements is incorrect?
- (A) Fixed-fixed strut mode is better than fixed-free strut mode for buckling resistance.
 (B) Buckling failure can be described as failure due to elastic instability.
 (C) Critical buckling load is proportional to the Young's modulus of strut material.
 (D) The larger the slenderness ratio, the higher the critical buckling stress is.

Question 2 (20%)

A pin-jointed structure is subjected to a uniformly distributed load 1 kN/m and a concentrated load 2 kN , as shown in Figure 2. The Young's modulus of member BC material $E = 200 \text{ GPa}$ and yield stress $\sigma_y = 300 \text{ MPa}$.

- (1) Pin A has a diameter of 4 mm ; calculate the shear stress of the pin.
- (2) Determine the internal force of member BC .
- (3) Is member BC safe against buckling?
- (4) Is member BC safe against yielding?

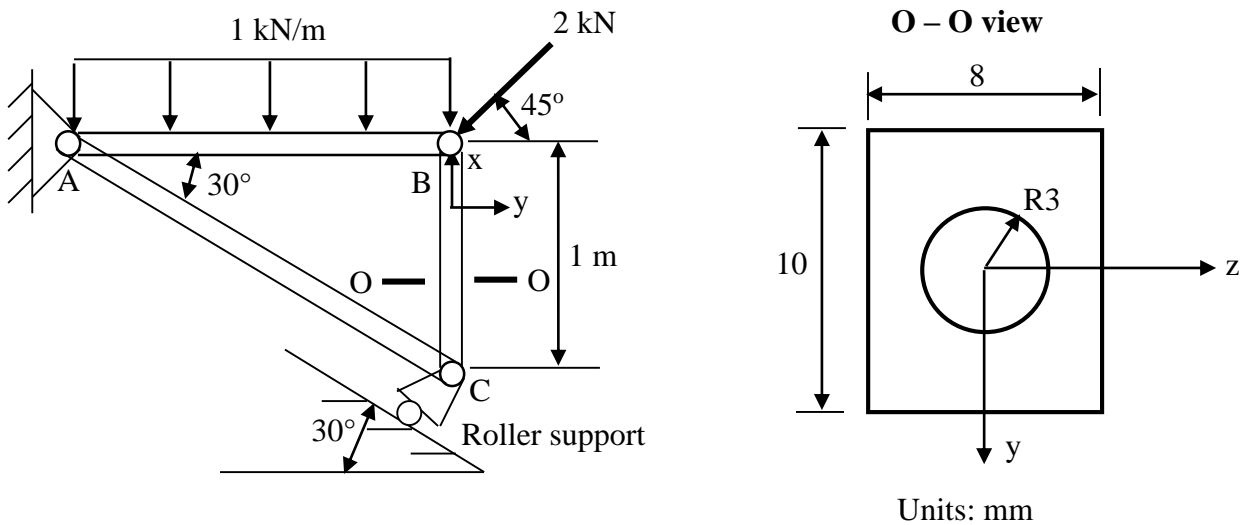
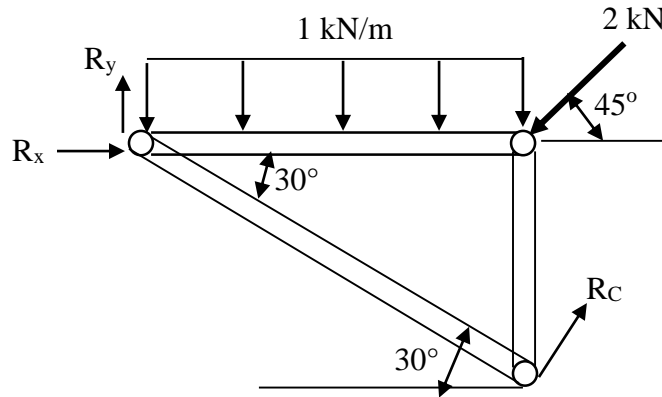


Figure 2

Solution:

(1)



$$\frac{1}{\sin 30^\circ} R_C = \frac{1}{2(\tan 30^\circ)^2} + 2 \cos 45^\circ \frac{1}{\tan 30^\circ}, R_C = 1.975 \text{ kN}$$

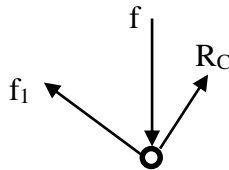
$$R_x + R_C \sin 30^\circ = 2 \cos 45^\circ, R_x = 0.4267 \text{ kN}$$

$$R_y + R_C \cos 30^\circ = 2 \sin 45^\circ + \frac{1}{\tan 30^\circ}, R_y = 1.4357 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{0.4267^2 + 1.4357^2} = 1.5 \text{ kN}$$

$$\tau = \frac{R}{\pi \times 2^2} = \frac{1.5 \times 10^3}{\pi \times 2^2} = 119.43 \text{ MPa}$$

(2)



$$f_1 \cos 30^\circ = R_C \sin 30^\circ, f_1 \sin 30^\circ + R_C \cos 30^\circ = f$$

$$f = 2.28 \text{ kN}$$

(3) Pin-ended strut

$$I_z = \frac{8 \times 10^3}{12} - \frac{\pi \times 3^4}{4} = 603 \text{ mm}^4$$

$$P_c = \frac{\pi^2 EI_z}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 603}{(1000)^2} = 1189 \text{ N} = 1.189 \text{ kN}$$

Fixed-fixed strut

$$I_y = \frac{10 \times 8^3}{12} - \frac{\pi \times 3^4}{4} = 363 \text{ mm}^4$$

$$P_c = \frac{\pi^2 EI_y}{(0.5L)^2} = \frac{\pi^2 \times 200 \times 10^3 \times 363}{(0.5 \times 1000)^2} = 2863 \text{ N} = 2.863 \text{ kN}$$

$$f = 2.28 \text{ kN} > 1.189 \text{ kN Unsafe!}$$

Member *BC* will be buckled.

$$(4) A = 8 \times 10 - \pi \times 3^2 = 51.74 \text{ mm}^2$$

$$\sigma = \frac{f}{A} = \frac{2.28 \times 1000}{51.74} = 44 \text{ MPa} < \sigma_y = 300 \text{ MPa. Safe!}$$

Member *BC* will not yield.

Question 3 (15%)

A copper tube having inner and outer diameters 55 mm and 60 mm, respectively, is used as a sleeve for a steel bolt having diameter 20 mm, as shown in Figure 3. The tube is fixed to a rigid wall, as shown. At room temperature (20°C), the rigid nut holds the assembly in a snug position such that the axial forces in the bolt and sleeve are negligible. Determine the axial and hoop stresses of the tube at temperature 180°C. Given that the Young's modulus $E_{copper} = 110 \text{ GPa}$ and $E_{steel} = 200 \text{ GPa}$; the thermal expansion $\alpha_{copper} = 16 \times 10^{-6} / ^\circ\text{C}$ and $\alpha_{steel} = 12 \times 10^{-6} / ^\circ\text{C}$; the Poisson's ratio of copper $\nu = 0.3$.

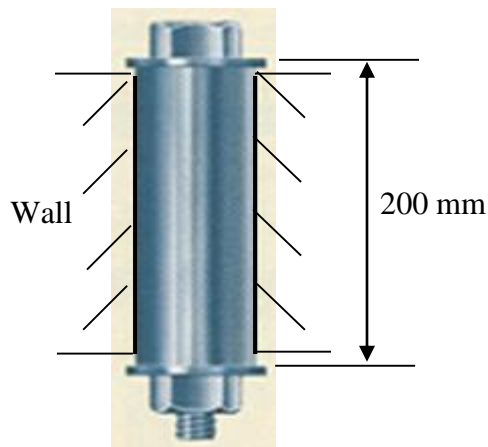
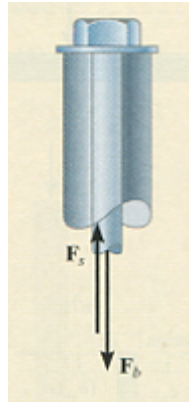


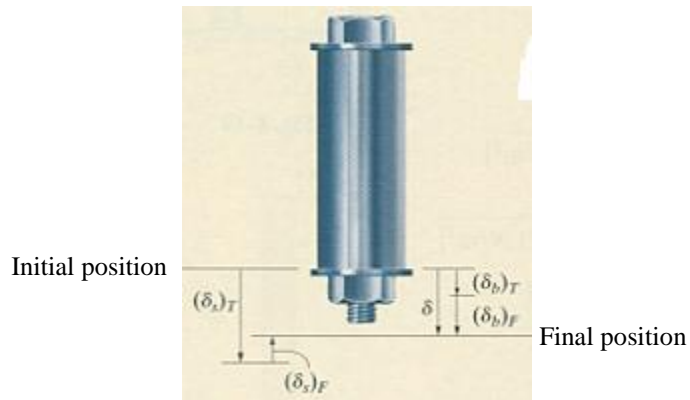
Figure 3

Solution:



$$F_s = F_b = F$$

$$\varepsilon_{xs} = \varepsilon_{xb}$$



The bolt is in tension and the tube is in compression.

For the bolt,

$$\varepsilon_{xb} = \frac{\sigma_{xb}}{E_b} + 12 \times 10^{-6} (180 - 20) = \frac{F}{\frac{\pi \times 10^2}{200 \times 10^3}} + 12 \times 10^{-6} (180 - 20) = 1.6 \times 10^{-8} F + 1.92 \times 10^{-3}$$

For the tube (sleeve),

$$\sigma_{xs} = -\frac{F}{2 \times \pi \times 30 \times 2.5} = -2.12 \times 10^{-3} F$$

$$\varepsilon_{ys} = 0, 0 = \frac{\sigma_{ys}}{E_s} - \nu \left(\frac{\sigma_{xs}}{E_s} \right) + 16 \times 10^{-6} (180 - 20)$$

$$0 = \frac{\sigma_{ys}}{110 \times 10^3} - 0.3 \left(\frac{\sigma_{xs}}{110 \times 10^3} \right) + 16 \times 10^{-6} (180 - 20)$$

$$\sigma_{ys} = 0.3\sigma_{xs} - 281.6$$

$$\varepsilon_{xs} = \frac{\sigma_{xs}}{E_s} - \nu \left(\frac{\sigma_{ys}}{E_s} \right) + 16 \times 10^{-6} (180 - 20)$$

$$\varepsilon_{xs} = \frac{\sigma_{xs}}{110 \times 10^3} - 0.3 \left(\frac{0.3\sigma_{xs} - 281.6}{110 \times 10^3} \right) + 16 \times 10^{-6} (180 - 20) = 8.27 \times 10^{-6} \sigma_{xs} + 3.328 \times 10^{-3}$$

$$\varepsilon_{xs} = -8.27 \times 10^{-6} \times 2.12 \times 10^{-3} F + 3.328 \times 10^{-3} = -1.75 \times 10^{-8} F + 3.328 \times 10^{-3}$$

$$\varepsilon_{xs} = \varepsilon_{xb}$$

$$1.6 \times 10^{-8} F + 1.92 \times 10^{-3} = -1.75 \times 10^{-8} F + 3.328 \times 10^{-3}$$

$$F = 4.2 \times 10^4 \text{ N}$$

$$\sigma_{xs} = -2.12 \times 10^{-3} F = -2.12 \times 10^{-3} \times 4.2 \times 10^4 = -89.04 \text{ MPa}$$

$$\sigma_{ys} = 0.3\sigma_{xs} - 281.6 = 0.3(-89.04) - 281.6 = -308.31 \text{ MPa}$$

Question 4 (15%)

A steel stepped shaft consisting of three parts having the same length is subjected to two external torques, as shown in Figure 4. Part *AB* is solid and has diameter 80 mm; part *BC* is also solid and has diameter 100 mm; part *CD* is hollow and has inner and outer diameters 40 mm and 80 mm, respectively. The end *A* is fixed. Determine the ratio T_1/T_2 so that the twist angle of the free end *D* is zero.

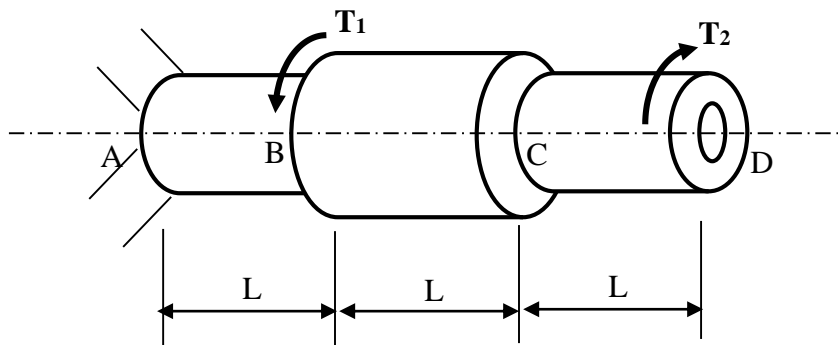
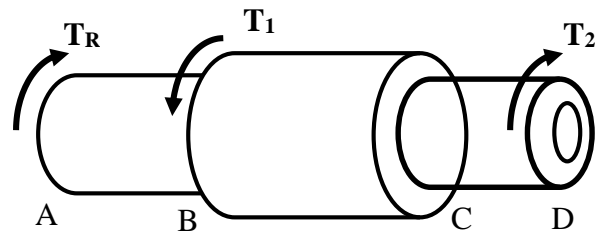


Figure 4

Solution:



$$T_R + T_2 = T_1, T_R = T_1 - T_2$$

For part AB,

$$\theta_{AB} = \frac{T_R L}{GJ_{AB}} = \frac{(T_1 - T_2)L}{GJ_{AB}}$$

For part BC,

$$\theta_{BC} = \frac{(T_1 - T_R)L}{GJ_{BC}} = \frac{T_2 L}{GJ_{BC}}$$

For part CD,

$$\theta_{CD} = \frac{T_2 L}{GJ_{CD}}$$

$$\theta_D = \theta_{AB} - \theta_{BC} - \theta_{CD} = 0$$

$$\frac{(T_1 - T_2)L}{GJ_{AB}} = \frac{T_2 L}{GJ_{BC}} + \frac{T_2 L}{GJ_{CD}} \cdot \frac{T_1 - T_2}{J_{AB}} = \frac{T_2}{J_{BC}} + \frac{T_2}{J_{CD}}$$

$$J_{AB} = \frac{\pi \times 40^4}{2}, J_{BC} = \frac{\pi \times 50^4}{2}, J_{CD} = \frac{\pi \times 40^4}{2} - \frac{\pi \times 20^4}{2}$$

$$\frac{T_1 - T_2}{40^4} = \frac{T_2}{50^4} + \frac{T_2}{40^4 - 20^4}$$

$$T_1/T_2 = 2.48$$

Question 5 (15%)

An aluminum beam having a symmetric cross section is loaded as shown in Figure 5, with the cross section details provided. The concentrated load P is applied through a rigid handle.

- (1) Draw shear force and bending moment diagrams.
- (2) Determine the maximum tensile and compressive normal stresses of the beam.
- (3) Compute the maximum shear stress of the beam.

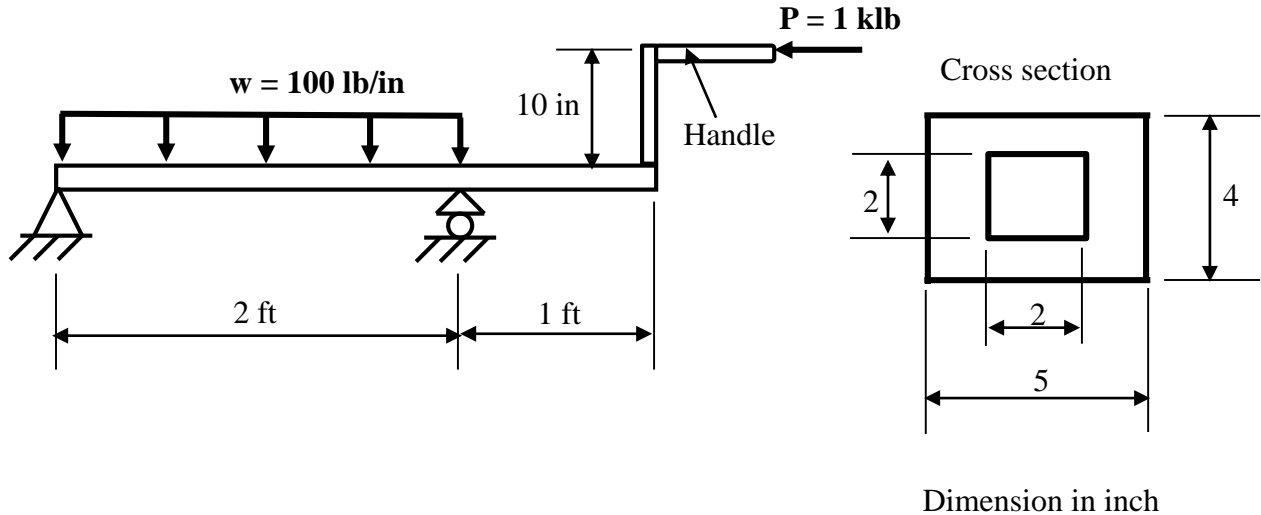
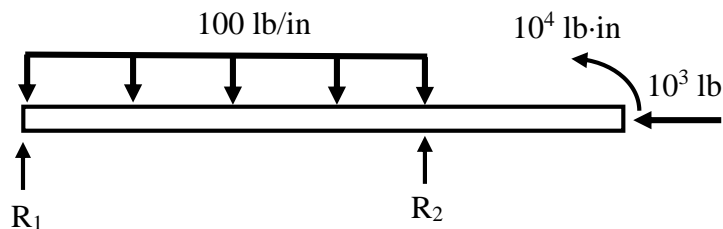


Figure 5

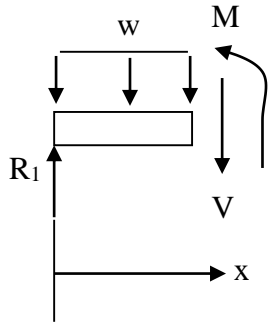
Solution:

(1)



$$R_1 + R_2 = 100 \times 24 = 2400 \text{ lb}$$

$$24R_2 + 10^4 = 100 \times 24 \times 12, R_2 = 783.33 \text{ lb}, R_1 = 1616.67 \text{ lb}$$

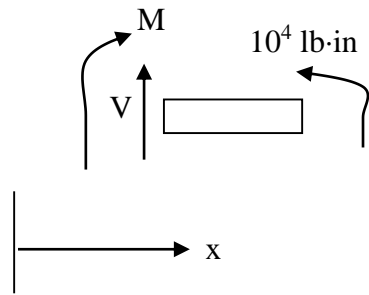


$$V + wx = R_1, \quad V = R_1 - wx = 1616.67 - 100x, \quad \text{Let } V = 0, \quad x = 16.17 \text{ in}$$

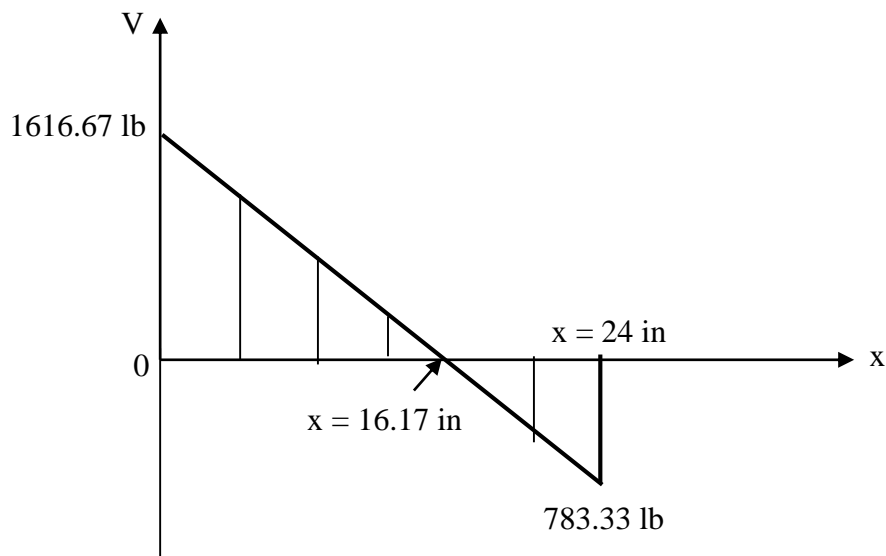
$$M + wx \cdot \frac{x}{2} = R_1 x$$

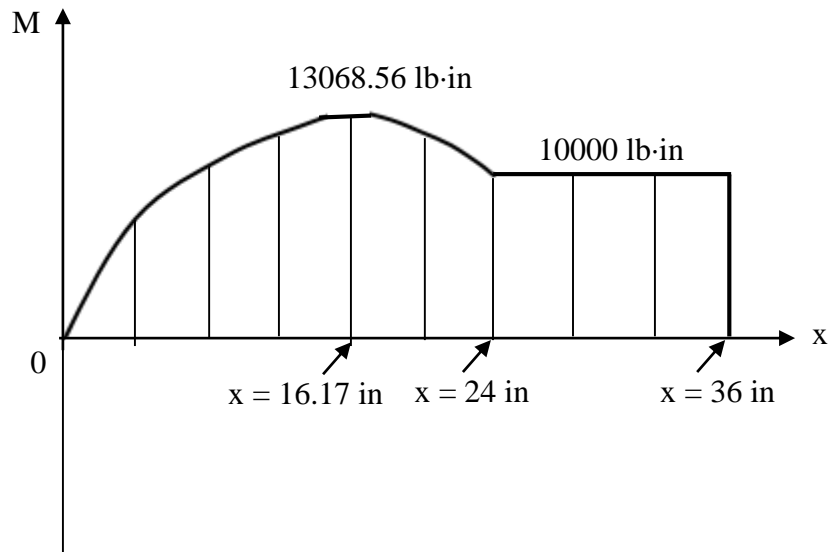
$$M = R_1 x - \frac{wx^2}{2} = 1616.67x - 50x^2$$

$$\text{At } x = 16.17 \text{ in, } M_{\max} = 1616.67 \times 16.17 - 50 \times 16.17^2 = 13068.56 \text{ lb}\cdot\text{in}$$



$$V = 0 \text{ and } M = 10^4 \text{ lb}\cdot\text{in}$$





(2)

$$I = \frac{5 \times 4^3}{12} - \frac{2 \times 2^3}{12} = 25.33 \text{ in}^4$$

$$A = 4 \times 5 - 2 \times 2 = 16 \text{ in}^2$$

$$\sigma_{\max} = \frac{13068.56 \times 2}{25.33} = 1031.86 \text{ psi (Tensile)}$$

$$\sigma_{\max} = \frac{13068.56 \times (-2)}{25.33} = -1031.86 \text{ psi (Compressive)}$$

$$\sigma = \frac{F}{A} = \frac{-1000}{16} = -62.5 \text{ psi}$$

$$\sigma_{\max T} = 1031.86 - 62.5 = 969.36 \text{ psi (Tensile)}$$

$$\sigma_{\max C} = -1031.86 - 62.5 = -1094.36 \text{ psi (Compressive)}$$

(3)

$$\begin{aligned} \tau(y) &= \frac{V \cdot Q'(y)}{I \cdot b} \\ &= \frac{1616.67(1 \times 5 \times 1.5 + 2 \times 1 \times 1.5 \times 0.5)}{25.33 \times 3} = 191.48 \text{ psi} \end{aligned}$$

Question 6 (15%)

A horizontal steel beam is fixed at one end A by pin-joint and is pin-jointed with a vertical brass rod at the other end B , as shown in Figure 6. The end C of the rod is fixed by pin-joint. A uniformly distributed load w and a point load P are applied on the beam and rod, as shown. The second moment of area of the beam $I = 2 \times 10^6 \text{ mm}^4$ and the Young's modulus of steel $E_{steel} = 210 \text{ GPa}$. The brass rod has diameter 5 mm and the Young's modulus of brass $E_{brass} = 110 \text{ GPa}$. Determine the magnitude of the load w so that the brass rod is extended by 2 mm.

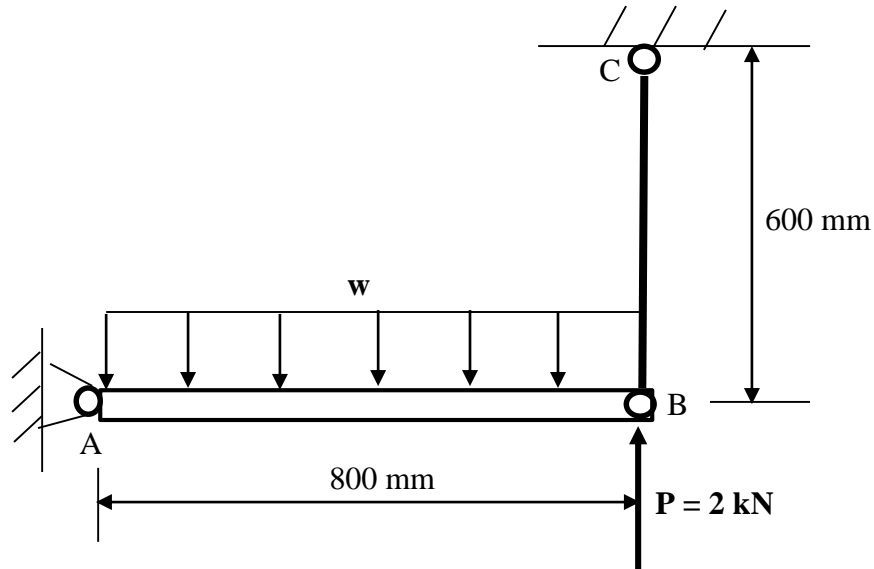
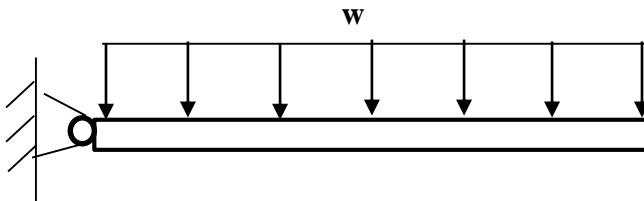
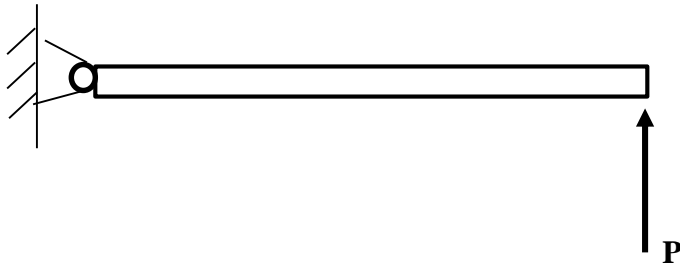


Figure 6

Solution:



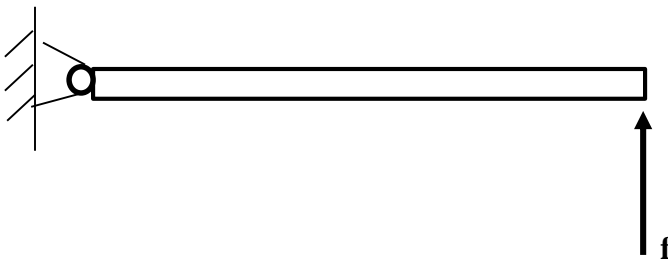
$$v_w = \frac{wL^4}{8EI} = \frac{800^4 w}{8 \times 210 \times 10^3 \times 2 \times 10^6} = 0.122w$$



$$v_p = \frac{PL^3}{3EI} = \frac{2000 \times 800^3}{3 \times 210 \times 10^3 \times 2 \times 10^6} = 0.813 \text{ mm}$$

For the brass rod,

$$f = \frac{602 - 600}{600} \times 110 \times 10^3 \times \pi \times 2.5^2 = 7295.83 \text{ N}$$



$$v_f = \frac{PL^3}{3EI} = \frac{7295.83 \times 800^3}{3 \times 210 \times 10^3 \times 2 \times 10^6} = 2.965 \text{ mm}$$

$$0.122w - 0.813 - 2.965 = 2 \text{ mm}$$

$$w = 47.36 \text{ N/mm}$$