

**FACULTY OF ENGINEERING AND COMPUTER SCIENCE  
FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS  
ENGR 213 - SAMPLE**

**Special instructions: Do all problems**  
**Only Faculty approved calculators are allowed**  
**ALL PROBLEMS CARRY THE SAME WEIGHT**

**PROBLEM No. 1.** Solve the following equation by the separation of variables method.

$$(y^2 + xy)dx + x^2dy = 0$$

(Hint: use the idea of homogeneous functions)  $\Rightarrow$

**PROBLEM No. 2.** Find the solution to the following equations by the exact differentials method:

(a)  $(x + y)^2 dx + (2xy + x^2 - 1)dy = 0$

(b)  $(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0$

**PROBLEM No. 3.** Solve the following linear differential equations using the integrating factor method:

(a)  $xy' + y = \frac{2}{x^3}$

(b)  $y' + 5y = e^{-2x}$

**PROBLEM No. 4.** Solve the following Bernoulli equation,

$$x^2 y' - 2xy = y^4$$

**PROBLEM No. (5)** Give the general solutions of the following differential equations:

(a)  $y^{(4)} + y'' - 12y = 0$

(b)  $y^{(7)} - 6y^{(6)} + 20y^{(5)} - 56y^{(4)} + 112y^{(3)} - 160y'' + 192y' - 128y = 0$

Note:

$$m^7 - 6m^6 + 20m^5 - 56m^4 + 112m^3 - 160m^2 + 192m - 128 = (m - 2)^3(m^2 + 4)^2$$

**PROBLEM No. 6.** Give the general solutions of the following differential equations:

(a)  $y''+6y'+8y=\sin 3x$  /

(b)  $y''''+2y''+y=x^2$  ↓

**PROBLEM No. 7.** The Space Shuttle lands in Kennedy Space Center. The spacecraft touches down at  $t=0$  with a velocity of 100 m/sec. The spacecraft chute is deployed at  $t=4$  sec. Between the touch down and the deployment of the chute ( $0 \leq t \leq 4$ ) the velocity of the spacecraft is constant. After the deployment of the chute the velocity is governed by the equation:

$$\frac{dV}{dt} = -0.002V^2$$

Determine when the velocity of the spacecraft reaches 20 m/sec.

**PROBLEM No. 8.** Solve the differential equation by the method of variation of parameters:

$$y''+10y'+25y=e^x \int$$

**PROBLEM No. 9.** Solve the following system of differential equations using the method of your preference:

$$\frac{dx}{dt} = 2(x-y)$$

$$\frac{dy}{dt} = y-x$$

**PROBLEM No. 10.** Use the power series method to solve the differential Equation:

$$y''+x^2y=0$$

(write the first 6 terms of the power series solution)