Econ 302: Assignment 2–Solution

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1. a. In the long run the equilibrium price $p^\star$ is the minimum value of AC. From $C(y_i)$,

$$MC(y_i) = 3y_i^2 - 16y_i + 36$$

$$AC(y_i) = y_i^2 - 8y_i + 36$$

$p^\star$ is determined when $MC(y_i) = AC(y_i)$,

$$3y_i^2 - 16y_i + 36 = y_i^2 - 8y_i + 36$$

$$y_i^\star = 4$$

$$p^\star = AC(y_i^*) = 20.$$

So the long run free entry equilibrium per-firm quantity produced by each firm in the industry is 4.

b. Given $p^\star = 20$ and market demand, the equilibrium quantity in the industry

$$Y^* = 2420 - 20 = 2400$$

So the number of firms in the long run free entry equilibrium is

$$n = \frac{Y^*}{y_i^*} = \frac{2400}{4} = 600.$$

c. The price-taking assumption is sensible under the assumption that there is free entry and exit and the number of firms is large. In the equilibrium, each firm's quantity produced constitutes a very small share of the market quantity, and each firm is not able to influence the market outcome by his choice of quantity.

2. a.

$$MR = 10 - 2y$$

$$MC = 2$$

$$MR = MC \implies y_m = 4, p_m = 10 - y_m = 6$$

$$\begin{aligned}\pi_m &= p_m y_m - C(y_m) \\ &= 16\end{aligned}$$

$$\begin{aligned}\epsilon(y_m) &= \frac{p(y_m)}{y_m.p'(y_m)} \\ &= -\frac{6}{4} \\ &= -1.5\end{aligned}$$

b.

$$MR = MC \implies 10 - 2y = 4 \implies y_m = 3, P_m = 7$$

$$\pi_m = 3 \times 7 - 4 \times 3 = 9$$

$$\begin{aligned}\epsilon(y_m = 3) &= \frac{p(y_m)}{y_m.p'(y_m)} \\ &= \frac{7}{3 \times (-1)} \\ &= -\frac{7}{3}\end{aligned}$$

c. The monopolist with a larger MC operates at the higher level of elasticity. Because when MC increases, the monopolist reduces output Y_m and increases price P_m. $|\epsilon(y_m)| = \frac{p(y_m)}{y_m.p'(y_m)}$ increases since $p'(y_m)$ is a constant for linear demand functions.

d. For a linear demand $P = a - by$, we have

$$MR = a - 2by$$

$$MR = MC \Longrightarrow a - 2by = c \Longrightarrow$$

$$y_m = \begin{cases} \frac{a-c}{2b} & \text{if } a > c \\ 0 & \text{if } a \leq c \end{cases}$$

$$\begin{aligned} |\epsilon(y_m)| &= |\frac{p(y_m)}{y_m.p'(y_m)}| \\ &= \frac{a - by_m}{by_m} \\ &= \frac{\frac{a+c}{2}}{\frac{a-c}{2}} \\ &= \frac{a+c}{a-c} > 1 \text{when } a > c \end{aligned} \quad (1)$$

3. a. Since $MC = 0$,
   The monopolist should offer 4 units to low-demand consumers with price $\frac{1}{2} \times 8 \times 4 = 16$, and offer 10 units to high-demand consumers with price $\frac{1}{2} \times 10 \times 10 = 50$.

   b. The optimal quantity x^l in the low-demand package should satisfy

$$P^l(x^l) = \frac{1}{2}P^h(x^l)$$

$$8 - 2x^l = \frac{1}{2}(10 - x^l)$$

$$x^l = 2$$

   at the price of area $A^{'}$=$4 * 2 + \frac{1}{2} \times 4 \times 2 = 12$.
   For the high-demand consumers, it is optimal for the firm to offer $x^h = 10$.

In order to prevent the high-demand consumers to take the low-demand package, the firm should charge a price equals the consumers' surplus of high-demand consumers minus area $B^{'}$, which satisfies

area $B^{'} = \frac{1}{2} \times (4+2) \times 2 = 6$.

The price for high-demand consumers: $\frac{1}{2} \times 10 \times 10 - 6 = 44$.

The reason these packages are optimal is as follows. In order to induce the high-demand consumer to choose the high-demand package, the firm needs to leave consumer's surplus of area $B^{'}$ to the high-demand consumer. For the firm, by choosing $x^l < 4$, the firm can reduce the area $B^{'}$ and gain $\triangle B$ from the high-demand package but as the same time will incur a loss from the low-demand package, which is $\triangle A$. When $P^l(x^l) = \frac{1}{2}P^h(x^l)$, the marginal change in $\triangle B$ is equal to the marginal change in $\triangle A$, which gives the optimal x^l.

c. The third degree price discrimination is better from the economy's view. In this special case, since the individual demand is known to the monopolist, the third degree price discrimination leads to the same outcome as the first-degree price discrimination, which is an efficient outcome. However, the second degree price discrimination still results in DWL of area $\triangle A$.