

Name \_\_\_\_\_

Student Number \_\_\_\_\_

**University of Calgary  
Faculty of Engineering  
Final Examination**

**ENGG407  
Numerical Methods**

Friday December 15, 2006, RED GYM

3:30-6:30 pm

1. Examination is closed book. However, a single aid sheet is allowed which is provided by the student.
2. Programmable calculator with storage and text display features may be used during examinations.
3. Final exam counts for 45% of overall course grade.
4. It is necessary to earn a passing grade on the final exam in order to pass the course as a whole.
5. Exam consists of 10 questions. Write answers in the space provided below each question.
6. Total marks for the exam is 111. Marks value of each question is indicated.

<b>question</b>	<b>value</b>	<b>Mark</b>
<b>1</b>	10	
<b>2</b>	8	
<b>3</b>	14	
<b>4</b>	10	
<b>5</b>	10	
<b>6</b>	10	
<b>7</b>	10	
<b>8</b>	10	
<b>9</b>	13	
<b>10</b>	16	
<b>total</b>	<b>111</b>	

**EXAMINATION RULES AND REGULATIONS****STUDENT IDENTIFICATION**

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

**EXAMINATION RULES**

- (1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
- (2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
- (3) All inquiries and requests must be addressed to supervisors only.
- (4) Candidates are strictly cautioned against:
  - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
  - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
  - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
  - (d) leaving answer papers exposed to view;
  - (e) attempting to read other student's examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

- (5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.
- (6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
- (7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
- (8) The candidate is to write his/her name on each answer book as directed and is to number each book.
- (9) A candidate must report to a supervisor before leaving the examination room.
- (10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
- (11) If during the course of an examination a student becomes ill or receives word of a domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor

so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or University Counselling Services during normal working hours or consult their physician/counsellor in the community.

Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.

(12) Smoking during examinations is strictly prohibited.

RO 94-01

1)(10) Consider a specific numerical algorithm that uses a range of -100 to 100 to represent general numbers used in computation. Assume that the algorithm uses 8 bits to represent the numbers. A uniform distribution of numbers is used.

a)(3) What is the interval between successive numbers represented by this format? State your assumptions.

$$\frac{200}{2^8 - 1} \quad \left( \begin{array}{ccc} \text{start} & \dots & \text{end} \\ -100 & & 100 \end{array} \right)$$

0, ... 255  
254 intervals

$$\Delta x = 0.7843$$

b)(3) What is the representation for a number  $x = \sqrt{10}$ ? State your assumptions.

$$\frac{\sqrt{10} - (-100)}{\Delta x} = \frac{103.1623}{0.7843} = 131.5 \text{ intervals}$$

rounded up is 132.

$$132 = 128 + 4 \quad \begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \text{MSB} & & & & & & & \text{LSB} \end{array}$$

c)(4) Now consider using floating point numbers instead. Assume a floating point format that uses  $r$  bits for the exponent and  $n$  bits for the mantissa. Determine appropriate values for  $r$  and  $n$  such that the range of -100 to 100 is covered with maximum resolution. Assume that the total number of bits for the floating point representation is 12 including the sign bits of the mantissa and exponent. Explain your answer.

$$100 < 128 = 2^7$$

maximum value of exponent is 7 Hence  $2^r - 1 \geq 7$

$$\therefore r = 3$$

$$n = 12 - 2 - 3 = 7$$

2)(8) Consider a two dimensional Gaussian function given as

$$f(x, y) = 10e^{-5x^2 + 2xy - 10y^2}$$

a)(6) Determine the first order two dimensional Taylor expansion of  $f(x, y)$  expanded about the point  $x=0.1$  and  $y=0.2$ .

$$\begin{aligned} f(x, y) &= f(0.1, 0.2) + \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y \\ &= f_0 (1 + (-10x + 2y) \Delta x + (2x - 20y) \Delta y) \\ f_0 &= 10 e^{-5(0.1)^2 + 2(0.2) - 10(0.2)^2} = 6.63 \end{aligned}$$

$$f(x, y) = 6.63 (1 + (-4 - 1) \Delta x + (0.2 - 4) \Delta y)$$

$$f(x, y) = 6.63 - 3.97 \Delta x - 25.19 \Delta y$$

$$\Delta x = x - 0.1$$

$$\Delta y = y - 0.2$$

b)(2) Evaluate the Taylor expansion at a point  $x=0.11$  and  $y=0.19$ .

$$\Delta x = 0.01$$

$$\Delta y = -0.01$$

$$f(x, y) = 6.63 - 0.04 + 0.25 = 6.84$$

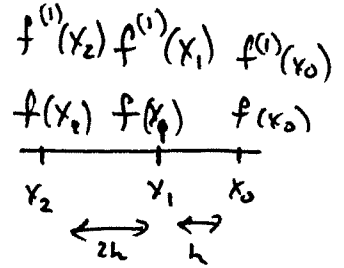
3)(14) Consider a sensor whose outputs  $y = f(x)$  and  $z = f^{(1)}(x)$  depend on a single independent variable  $x$ . From these outputs it is desired to obtain an estimate of

$\left. \frac{d^3 f(x)}{dx^3} \right|_{x=x_0}$ . The specific inputs into this estimate are the following measurements

$$\{f(x_0), f(x_0 - h), f(x_0 - 3h), f^{(1)}(x_0), f^{(1)}(x_0 - h), f^{(1)}(x_0 - 3h)\}$$

a)(5) List all the possible equations that can be used which are functions of

$\left. \frac{d^3 f(x)}{dx^3} \right|_{x=x_0}$  and the data values.



$$f(x_0 - h) = f(x_0) + f^{(1)}(x_0)(-h) + f^{(2)}(x_0) \frac{(-h)^2}{2} + f^{(3)}(x_0) \frac{(-h)^3}{6}$$

$$f(x_0 - 3h) = f(x_0) + f^{(1)}(x_0)(-3h) + f^{(2)}(x_0) \frac{(-3h)^2}{2} + f^{(3)}(x_0) \frac{(-3h)^3}{6}$$

$$f^{(1)}(x_1) = f^{(1)}(x_0) + f^{(2)}(x_0)(-h) + f^{(3)}(x_0) \frac{(-h)^2}{2}$$

$$f^{(1)}(x_2) = f^{(1)}(x_0) + f^{(2)}(x_0)(-3h) + f^{(3)}(x_0) \frac{(-h)^2}{2}$$

b) (4) Devise an algorithm to approximate  $\left. \frac{d^3 f(x)}{dx^3} \right|_{x=x_0}$  based on a subset of these

equations. Justify your selection of equations.

Take the samples closest to the  $x_0$  as these give lowest truncation error

$$f(x_1) - f(x_0) + h f^{(1)}(x_0) = \begin{bmatrix} -\frac{h^2}{2} & -\frac{h^3}{6} \end{bmatrix} \begin{bmatrix} f^{(2)}(x_0) \\ f^{(3)}(x_0) \end{bmatrix}$$

$$f^{(1)}(x_1) - f^{(1)}(x_0) = \begin{bmatrix} -h & \frac{h^2}{2} \end{bmatrix} \begin{bmatrix} f^{(2)}(x_0) \\ f^{(3)}(x_0) \end{bmatrix}$$

$$\begin{bmatrix} f(x_1) - f(x_0) + h f^{(1)}(x_0) \\ f^{(1)}(x_1) - f^{(1)}(x_0) \end{bmatrix} = \begin{bmatrix} -\frac{h^2}{2} & -\frac{h^3}{6} \\ -h & \frac{h^2}{2} \end{bmatrix} \begin{bmatrix} f^{(2)}(x_0) \\ f^{(3)}(x_0) \end{bmatrix}$$

c) (5) Potentially, the algorithm could be improved by using all the equations in a least square sense. Show how to derive such an algorithm.

$$M = AP$$

$$M = \begin{bmatrix} f(x_1) - f(x_0) + h f^{(1)}(x_0) \\ f^{(1)}(x_1) - f^{(1)}(x_0) \\ f(x_2) - f(x_0) + 3h f^{(1)}(x_0) \\ f^{(1)}(x_3) - f^{(1)}(x_0) \end{bmatrix}; A = \begin{bmatrix} -\frac{h^2}{2} & -\frac{h^3}{6} \\ -h & \frac{h^2}{2} \\ -\frac{(3h)^2}{2} & -\frac{(3h)^3}{6} \\ -3h & \frac{(3h)^2}{2} \end{bmatrix}$$

$$P = \begin{bmatrix} f^{(2)}(x_0) \\ f^{(3)}(x_0) \end{bmatrix}$$

4) (10) A process generates two output variables  $v(t)$  and  $w(t)$  dependent on one independent variable  $t$ . The process is governed by a pair of ordinary differential equations given as

$$A \frac{d^2 v(t)}{dt^2} + B \frac{dv(t)}{dt} + Cv(t) = g(t) + w(t)$$

$$E \frac{dw(t)}{dt} + Fw(t) = h(t)$$

where  $g(t)$  and  $h(t)$  are forcing functions dependent on  $t$  only and  $\{A, B, C, D, E, F\}$  are real constants that are known. Determine the recursive relation that will approximately simultaneously solve for  $v(t)$  and  $w(t)$  using the forward Euler method. Assume a fixed time increment of  $\Delta t$ .

$$x_1 = v(t)$$

$$x_2 = \frac{dv(t)}{dt}$$

$$x_3 = w(t)$$

$$A \frac{dx_2}{dt} + B x_2 + C x_1 = g + x_3$$

$$E \frac{dx_3}{dt} + F x_3 = h$$

$$\frac{dx_2}{dt} = -\frac{C}{A} x_1 - \frac{B}{A} x_2 + \frac{x_3}{A} + \frac{g}{A}$$

$$\frac{dx_3}{dt} = -\frac{F}{E} x_3 + \frac{h}{E}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{C}{A} & -\frac{B}{A} & \frac{1}{A} \\ 0 & 0 & -\frac{F}{E} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{g}{A} \\ \frac{h}{E} \end{bmatrix}$$

$$\bar{x}_{i+1} = \begin{bmatrix} 1 & \Delta t & 0 \\ -\frac{C}{A} \Delta t & 1 - \frac{B \Delta t}{A} & \Delta t / A \\ 0 & 0 & 1 - \frac{F \Delta t}{E} \end{bmatrix} \bar{x}_i + \begin{bmatrix} 0 \\ \Delta t g / A \\ \Delta t h / E \end{bmatrix}$$

Problem 5 solved in degrees

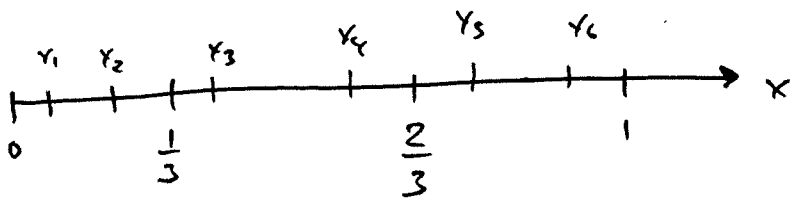
$$\begin{aligned}
 y_1 &= .0716 \\
 y_2 &= .2673 \\
 y_3 &= .4107 \\
 y_4 &= .6067 \\
 y_5 &= .75 \\
 y_6 &= .9458
 \end{aligned}$$

$$\frac{\text{Sum}}{6} = \frac{3.0524}{6} = .5087$$

algebraic

$$\begin{aligned}
 -\cos(x) + \frac{x^2}{2} \Big|_0^1 &= -1 + \cos\left(\frac{\pi}{180}\right) + \frac{1}{2} \\
 &= .4998
 \end{aligned}$$

5)(10) Numerically integrate the function  $f(x) = \sin(x) + x$  over the range of  $0 \leq x \leq 1$  using the two point Gauss Quadrature method. Assume that three equal width panels are used over this interval. Compare this numeric result with exact integration.



$$\begin{aligned}
 d &= \frac{\left(\frac{1}{3}\right)}{2} \left(1 - \frac{1}{3}\right) \\
 &= .0704
 \end{aligned}$$

$$\begin{aligned}
 x_1 &= .0704 & y_1 &= .1407 \\
 x_2 &= \frac{1}{3} - .0704 = .2629 & y_2 &= .5228 \\
 x_3 &= \frac{1}{3} + .0704 = .4037 & y_3 &= .7966 \\
 x_4 &= \frac{2}{3} - .0704 = .5963 & y_4 &= 1.1578 \\
 x_5 &= \frac{2}{3} + .0704 = .7771 & y_5 &= 1.4092 \\
 x_6 &= 1 - .0704 & y_6 &= 1.7310
 \end{aligned}$$

$$\begin{aligned}
 \text{Sum} &= 5.7582 \\
 \div 6 &= .9597
 \end{aligned}$$

$$-\cos(x) + \frac{x^2}{2} \Big|_0^1 = -\cos(1) + 1 + \frac{1}{2} = -\cos(1) + \frac{3}{2} = 0.9597$$

6)(10) It is desired to determine the best solution of a set of equations in the least squares sense. Suppose that a set of equations is given as

$$\begin{aligned} x &= 2 + y \\ x^2 + \exp(y) &= 3 \\ \sqrt{x^2 + y} &= xy \\ \sin(x) + y &= \cos(xy) \end{aligned}$$

With the additional constraint that x and y are real.

a)(8) Assuming an initial guess of  $x = x_0$  and  $y = y_0$ , devise a numerical algorithm that will determine the least square values of x and y. (Hint – consider the computer assignment 4 with the beacon position estimation)

$$\begin{aligned} A &= x - 2 - y \\ B &= x^2 + e^y - 3 \\ C &= \sqrt{x^2 + y} - xy \\ D &= \sin(x) + y - \cos(xy) \end{aligned} \quad J = \begin{bmatrix} 1 & -1 \\ 2x & e^y \\ \frac{-x}{\sqrt{x^2 + y}} - y & \frac{1}{2\sqrt{x^2 + y}} - y \\ \cos(x) + \sin(xy)y & 1 + \sin(xy)x \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = (J^T J)^{-1} J^T \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \underbrace{(J^T J)^{-1} J^T}_{\text{evaluated @ } x_i, y_i} \underbrace{\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}}_{\text{evaluated at } x_i, y_i}$$

b) (2) State how the algorithm would change if the constraint that  $x$  and  $y$  are real was removed such that  $x$  and  $y$  could be complex. (No derivations necessary just a sentence or two is sufficient.)

4 variables  ~~$x$~~   $x_{\text{real}}$   $x_{\text{imag}}$   
 $y_{\text{real}}$   $y_{\text{imag}}$

A, B, C, D function expanded into two equations each

8 equations 4 unknowns.

Still overdetermined structure.

7)(10) It is required to determine values of  $x$  and  $y$  that satisfy the two conditions

$$0 = x^2 + y + 1$$

$$0 = xy + e^{-jx}$$

The values of  $x$  and  $y$  are assumed to be complex. It is desired to solve this problem with the Newton Raphson method. Show the steps of one iteration with an initial guess of

$$x = 1 + j$$

$$y = 1$$

where  $j = \sqrt{-1}$ .

$$x = a + ib$$

$$y = c + id$$

$$e^{-jx} = e^{-j(a+ib)} = e^{-ja} e^b = (\cos(a) - j\sin(a)) \cdot e^b$$

$$x^2 + y + 1 \Rightarrow a^2 - b^2 + 2iab + c + id + 1$$

$$a^2 - b^2 + c + 1 = A$$

$$2ab + d = B$$

$$xy + e^{-jx} \Rightarrow (a+ib)(c+id) + e^b \cos(a) - j e^b \sin(a)$$

$$ac - bd + e^b \cos(a) = C$$

$$bc + ad - e^b \sin(a) = D$$

$$J = \begin{bmatrix} \frac{\partial A}{\partial a} & \frac{\partial A}{\partial b} & \frac{\partial A}{\partial c} & \frac{\partial A}{\partial d} \\ \frac{\partial B}{\partial a} & \frac{\partial B}{\partial b} & \frac{\partial B}{\partial c} & \frac{\partial B}{\partial d} \\ \frac{\partial C}{\partial a} & \frac{\partial C}{\partial b} & \frac{\partial C}{\partial c} & \frac{\partial C}{\partial d} \\ \frac{\partial D}{\partial a} & \frac{\partial D}{\partial b} & \frac{\partial D}{\partial c} & \frac{\partial D}{\partial d} \end{bmatrix} = \begin{bmatrix} 2a & -2b & 1 & 0 \\ 2b & 2a & 0 & 1 \\ c - e^b \sin(a) & -d + e^b \cos(a) & a & -b \\ d - e^b \cos(a) & c - e^b \sin(a) & b & a \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{i+1}$$

$$= \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_i$$

$$- J^{-1} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

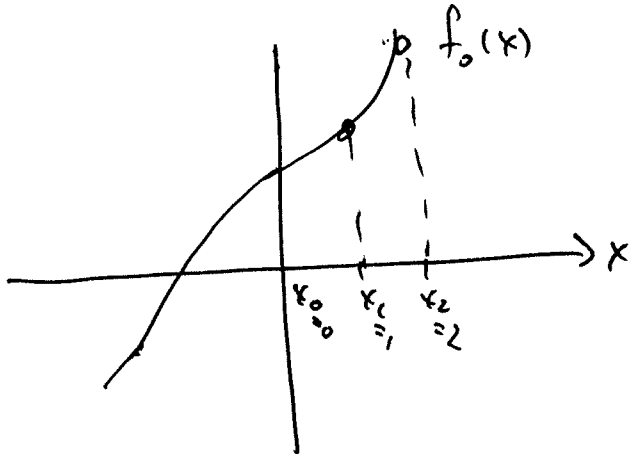
evaluated @  $a_i, b_i, c_i, d_i$

$$\begin{bmatrix} 2 \\ 2 \\ 1 + e^b \cos(a) \\ 1 - e^b \sin(a) \end{bmatrix}$$

8)(10) The roots of a function

$$f_0(x) = x^3 \sin(x) + 4x + 3$$

are to be determined using Muelers method. Initial function values are determined at  $x_0 = 0, x_1 = 1$  and  $x_2 = 2$ . Show the steps of one iteration explaining all the steps taken.



evaluate:

$$f_0(0) = 3$$

$$f_0(1) = 7.84$$

$$f_0(2) = 18.27$$

$$f_0 = 4.84$$

$$f_1 = 10.434$$

$$a = 2.797$$

$$b = 2.797 + 10.434 = 13.231$$

$$c = 18.27$$

$$z = \text{roots of } (c z^2 + b z + a)$$

largest magnitude root is  $-0.362 - j 0.148$

$$x_3 = \frac{1}{-0.362 - j 0.148} + x_2 = -2.36 + j 0.968 + x_2 = -1.36 + j 0.968$$

next iteration

$$x_0 = 1$$

$$x_1 = 2$$

$$x_2 = -0.36 + j 0.968 \text{ (rad)}$$

or  $-0.788 \text{ (deg)}$

For the degree answers

$$f_0(0) = 3$$

$$f_0(1) = 7.02$$

$$f_0(2) = 11.28$$

$$f_0 = \frac{7.02}{1} = 7.02$$

$$f_1 = 4.26$$

$$a = \frac{f_1 - f_0}{2} = \frac{2.74}{2} = 1.37$$

$$b = 1.37 + 4.26 = 5.63$$

$$c = 11.28$$

$$\text{roots } (c z^2 + b z + a)$$

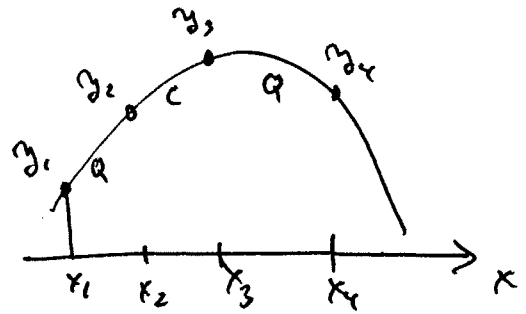
$$-0.3586 \text{ largest } | |$$

$$x_3 = -2.788 + x_2$$

$$x_3 = -1.788$$

9)(13) A function  $y = f(x)$  is sampled resulting in a data set consisting of ordered points  $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)\}$ , where  $x_1 < x_2 < x_3 < x_4$ . Design a three segment spline curve fit that has the following characteristics:

1. quadrature spline between the knots  $\{(x_1, y_1), (x_2, y_2)\}$
2. cubic spline between the knots  $\{(x_2, y_2), (x_3, y_3)\}$
3. quadrature spline between the knots  $\{(x_3, y_3), (x_4, y_4)\}$
4. zero curvature at the two end knots



a)(3) How many unknown parameters are there?

$$3 + 4 + 3 = 10$$

$$f_1 = a_1 x^2 + b_1 x + c_1$$

$$f_2 = a_2 x^3 + b_2 x^2 + c_2 x + d_2$$

$$f_3 = a_3 x^2 + b_3 x + c_3$$

b)(6) List all the equations available

$$f_1(x_1) = y_1$$

$$f_1(x_2) = y_2$$

$$f_1^{(2)}(x_1) = 0$$

$$f_3(x_3) = y_3$$

$$f_3(x_4) = y_4$$

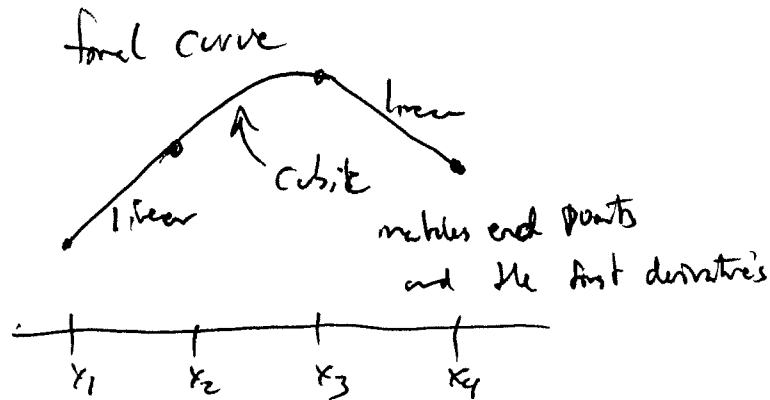
$$f_3^{(2)}(x_4) = 0$$

$$f_2(x_2) = y_2$$

$$f_2(x_3) = y_3$$

$$f_2^{(1)}(x_2) = f_1^{(1)}(x_2)$$

$$f_3^{(1)}(x_3) = f_2^{(1)}(x_3)$$



c)(4) Simplify the set of equations and determine the eventual solution of the spline coefficients. What is the final spline curve?

for  $f_1(x)$   $f_1^{(2)}(x) = 2a_1 \Rightarrow a_1 = 0$

$$b_1 x_1 + c_1 = y_1$$

$$b_1 x_2 + c_1 = y_2$$

$$\Rightarrow b_1(x_1 - x_2) = y_1 - y_2$$

$$b_1 = \frac{y_1 - y_2}{x_1 - x_2}, c_1 = y_1 - b_1 x_1$$

for  $f_3(x)$   $f_3^{(2)}(x) = 2a_3 \Rightarrow a_3 = 0$

$$b_3 x_3 + c_3 = y_3$$

$$b_3 x_4 + c_3 = y_4$$

$$b_3 = \frac{y_3 - y_4}{x_3 - x_4}, c_3 = y_3 - b_3 x_3$$

for  $f_2(x) = a_2 x^3 + b_2 x^2 + c_2 x + d_2$

$$f_2^{(1)}(x_2) = f_1^{(1)}(x_2) = c_1 = 3a_2 x_2^2 + 2b_2 x_2 + c_2$$

$$f_3^{(1)}(x_3) = f_2^{(1)}(x_3) = c_3 = 3a_2 x_3^2 + 2b_2 x_3 + c_2$$

$$f_2(x_2) = y_2$$

$$f_2(x_3) = y_3$$

**10) (16)** Short answer questions (one or two short sentences is sufficient)

a)(2) In numerical processing, integer and floating point number representations are the most common. Give an example where integer number representation has an advantage over floating point representation.

where you need uniform number density. Number anywhere in range is equally likely.

b)(2) It is required to find the root of a single variable nonlinear equation. However, the equation is very stiff and there is initially a large uncertainty over where the root is located. What root finding method should be used initially? Why?

bisection, NR would get lost.  
false position may take a long time to converge.

c)(2) In the bracketing method, the bracket happens to enclose a singularity. Will the method converge? Under what conditions will it converge?

As long as the bracket edge does not hit the singularity.  
As long as initial bracket contains the zero.

d)(2) Newton Raphson root finding method is applied to a single variable function with a singularity in the vicinity of the root. Can the method converge to the correct root? Under what conditions?

Yes provided the initial guess is between the root and singularity.

e)(2) The Newton Raphson method is used to solve a set of simultaneous equations. It is observed that when the Jacobian is calculated for a given iteration point is singular. What does this imply? Can the iteration be completed? Why or why not?

- zero gradient of jacobian.
- no iteration cannot be completed since  $J^{-1}$  does not exist

f)(2) In part "e", given that J is singular, what could be done to complete the iteration?

add random vector to  $F_i'$

g)(2) In numerically solving a differential equation, how does overall accuracy depend on step size?

Too large a step size, higher truncation error

**h)(2)** What is meant by propagated truncation error?

Successive iterations of the DEQ solver and the truncation error will generally increase.

Truncation error propagates from one segment to the next.

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