MAT2322 Final Exam
Fall 2003

1. (10 points) Find the critical points of the function $f(x, y) = x^4 + y^4 - 4xy$ and use the second derivative test to classify them.

2. (10 points) Find the points on the cone $z^2 = x^2 + y^2$ which are closest to the point $(1, 1, 0)$.

3. (10 points) Find the mass of the solid bounded by the xz-plane, the yz-plane, the plane $z = 1$, and the plane $x + y + z = 2$, if the density of the solid is given by $\delta(x, y, z) = 12(z - 1)^2$.

4. (10 points) Consider the vector field $\vec{F} = (2x + aye^{-x})\vec{i} + e^{-x}\vec{y}$,

(a) For which value of a does $\vec{F}$ have a potential function?

(b) Find the potential function for the vector field in part (a).

(c) Calculate $\int_C \vec{F} \cdot d\vec{r}$, where C is given by:

$$x = \cos t, \ y = \sin t, \qquad \frac{\pi}{4} \leq t \leq \pi.$$

5. (10 points) Let C be the contour of the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 2)$, oriented counter-clockwise if viewed from above. Let $\vec{F}$ be the vector field

$$\vec{F}(x, y, z) = (e^{x^2} + z)\vec{i} + (\sin y + x)\vec{j} + (\ln z + y)\vec{k}.$$

Use Stokes' Theorem to evaluate the integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

6. (10 points) Consider a particle whose motion is given by the vector function

$$\vec{r}(t) = (4t^2 - 1)\vec{i} + 2\vec{j} + (3t^2 + 2)\vec{k}.$$

(a) Find the velocity, speed, and acceleration of the particle at the moment of time $t = 1$.

(b) This particle moved from the point $(3, 2, 5)$ to the point $(35, 2, 29)$ along its trajectory. Find the distance traveled by the particle.

7. (10 points) Let $\vec{F} = (x + \cos y)\vec{i} + (y + \sin z)\vec{j} + (z + e^x)\vec{k}$, and let W be the solid bounded by the planes $z = 0$, $y = 0$, $y = 2$ and the parabolic cylinder $z = 1 - x^2$. If the surface S, which is the boundary of W, is oriented towards the exterior of the solid, compute

$$\int_S \vec{F} \cdot d\vec{A}.$$

8. (10 points) Find the flux of the vector field $\vec{F} = x\vec{i} + y\vec{j} + 3\vec{k}$ through the surface S consisting of the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$. The surface S has the upward orientation.

9. (10 points) Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where

$$\vec{F} = (x + xy^2)\vec{i} + 2(x^2y - y^2 \sin y)\vec{j},$$

and C is the curve from the origin to (1, 1) along $y = x^2$, from (1, 1) to (0, 1) along $y = 1$ and from (0, 1) to the origin along $x = 0$ in that order.