Department of Physics

PCS 125 – Waves and Fields

Please insert in the circle the initial of your last name

Please Print Clearly:

Student:____________________________ ____________________________

Last Name *First Name*

Student Number: ____________________ **Student Section:** _______________

Mid -Term Test – February 26, 2010

Check the name of your professor:

Dr. Beauchemin		Dr. Carvalho		Dr. Hu		Dr. Tavakkoli		Dr. Yuan	

Read the following attentively

- **Duration of the test: 1 hour and 30 minutes (90 minutes).**
- **The test consists of 1 full answer question (12 marks) and 12 multiple-choice questions (2 marks each). Answer ALL questions.**
- **For each multiple choice question:**
  - **You need to show, in the space provided in this booklet, a justification for your answer. If no work or correct argument is shown, the resultant mark may be zero, even if the answer in the bubble sheet is correct.**
  - **The work shown in your exam booklet may be checked, <u>at least for one</u> of the questions, selected randomly.**
  - **<u>Transfer your choices to the bubble sheet only when you're sure of them.</u>**
  - **Some answers may be rounded off.**
- **For the full answer question, show, in the space provided, all your work. For part marks to be awarded, the steps leading to the answers in each part of the problem have to be organized, unambiguous and clear.**
- **Pagers and cell phones must be silenced and placed out of reach. Hats and earphones are not allowed. If a phone rings, its owner is responsible for disrupting the exam and the phone will be confiscated or the owner will be asked to leave the room resulting in a zero on the test.**
- **Sharing of pens, erasers and calculators is not permitted.**
- **Talking to another student or glancing over another student's paper is not permitted and may result in a charge of academic misconduct.**
- **Formula sheet provided with the test booklet. No other sheets allowed.**
- **The only calculators allowed are either the Sharp EL-546 or Casio FX-991. If you are found to have another type of calculator (even if not programmable) <u>it will be confiscated</u> for the duration of the test.**

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Student signature (no signature, no mark!)

MULTIPLE CHOICE QUESTIONS (each worth 2 marks)

1. The position of an air-track cart that is oscillating on a spring is given by

$$x = (12.4\ \text{cm})\cos[(6.35\ \text{rad/s})t].$$

At what value of t after $t = 0$ is the cart first located at $x = +8.47$ cm?

(A) 4.34 s **(B) 0.379 s** **(C) 0.129 s** **(D) 7.39 s** **(E) 0.629 s**

2. Consider the following three statements:

(1) As a pendulum swings through its cycle, the mass is moving fastest and has its greatest acceleration at the bottom of the swing.
(2) If you double the mass of a pendulum, the period of vibration does not change.
(3) If pendulum A is twice as long as pendulum B, the period of oscillation of A will be twice as long as the period of B, if they have the same mass.

(A) All three are true. **(B) Only (1) is true.** **(C) Only (1) and (2) are true.**
(D) Only (2) is true. **(E) Only (3) is true.**

3. The superposition of two waves on a rope,
$y_1 = 2.0 \sin[\, \pi\, (x/2 - \omega t)\,]$

and

$y_2 = 2.0 \sin[\, \pi\, (x/2 - \omega t - 1/2)\,]$,

where y_1, y_2, and x are in meters and t is in seconds, will result in a wave travelling at a velocity of 12 m/s with a frequency of

(A) 3.00 Hz **(B) 0.955 Hz** **(C) 1.22 Hz** **(D) 3.82 Hz**

(E) cannot be determined

4. A mass on a spring has an angular oscillation frequency of 2.56 rad/s. The spring constant is 27.2 N/m, and the system's kinetic energy is 4.16 J when $t = 1.56$ s. What is the oscillation amplitude? Assume that the mass is at its positive maximum displacement from the equilibrium position when $t = 0$.

(A) 63.1 cm **(B) 83.7 cm** **(C) 47.7 cm** **(D) 73.4 cm** **(E) 55.3 cm**

5. A speaker generates a continuous tone of 440 Hz. The sound travels through two tubes of different length before being detected by a microphone. If the microphone detects a maximum in sound intensity, what is the minimum difference in length between the two tubes if the speed of sound in air is 339 m/s?

(A) 0.10 m **(B) 0.39 m** **(C) 0.77 m** **(D) 1.11 m** **(E) 1.54 m**

6. Two identical strings have the same length and are under the same tension. The linear mass density of string B is 3 times that of string A. Which statement is correct for all *n* harmonics on the two strings, $n = 1, 2, 3\ldots$?

(A) $f_{n,B} = 1/9\, f_{n,A}$
(B) $f_{n,B} = 1/\sqrt{3}\, f_{n,A}$
(C) $f_{n,B} = \sqrt{3}\, f_{n,A}$
(D) $f_{n,B} = 1/3\, f_{n,A}$.
(E) $f_{n,B} = 9\, f_{n,A}$

7. For the transverse wave described by $y = 0.15\sin\left[\frac{\pi}{16}(2x - 64t)\right]$ (SI units), determine the absolute value of the maximum transverse acceleration of the particles of the medium.

(A) 7.54 m/s^2 **(B)** 15.1 m/s^2 **(C)** 23.7 m/s^2 **(D)** 47.4 m/s^2
(E) None of the above

8. Four wave functions are given below. Rank them in order of the magnitude of the wave speeds, from least to greatest.

I. $y(x,t) = 5\sin(4x - 20t + 4)$
II. $y(x,t) = 5\sin(3x - 12t + 5)$
III. $y(x,t) = 5\cos(4x + 24t + 6)$
IV. $y(x,t) = 14\cos(2x - 8t + 3)$

(A) IV, II, I, III
(B) IV = II, I, III
(C) III, I, II, IV
(D) IV, I, II=III
(E) III, IV, II, I

9. Transverse sinusoidal waves are travelling at a speed of 500m/s on a 1-m long piano string fastened at both ends. If the only two points where the vibrational amplitude is zero are the two ends of the string, find the frequency of vibration of the string.

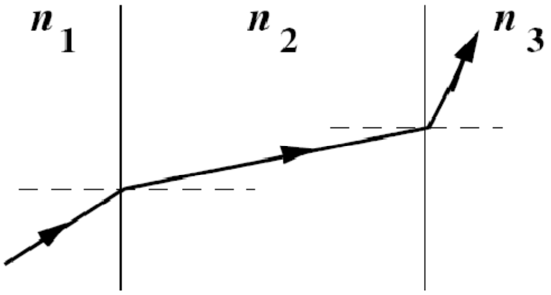
(A) 250 Hz
(B) 500 Hz
(C) 1000 Hz
(D) 2000 Hz
(E) 2500 Hz

10. A light ray is propagating in a crystal where its wavelength is 520 nm. It strikes the interior surface of the crystal with an incidence angle of 30^0 and emerges into the surrounding air at 60^0 to the surface normal. The light's frequency is

(A) 1.1×10^{14} Hz
(B) 2.9×10^{14} Hz
(C) 5.8×10^{14} Hz
(D) 3.3×10^{14} Hz
(E) 1.0×10^{15} Hz

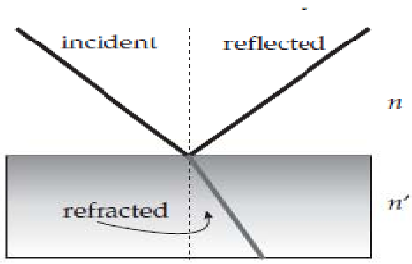
11. The figure shows the path of a portion of a ray of light as it passes through three different transparent materials with refractive index n_1, n_2 and n_3 respectively. (**Note:** *Angles in the figure are drawn to scale*). What can be concluded concerning the refractive indices of these three materials?

(A) $n_1 < n_2 < n_3$
(B) $n_1 > n_2 > n_3$
(C) $n_1 < n_3 < n_2$
(D) $n_2 < n_1 < n_3$
(E) $n_3 < n_1 < n_2$



12. A light ray is partially reflected and partially refracted at a boundary between two media, the upper one having index of refraction *n*, the lower one having index of refraction *n'* as shown in the figure. The reflected ray is perpendicular to the refracted ray if and only if

(A) $n' = n \tan \theta_{incident}$
(B) $n' = n \cot \theta_{incident}$
(C) $n' = n \sec \theta_{incident}$
(D) $n' = n \dfrac{\sin\theta_{incident}}{\sin\theta_{reflection}}$
(E) $n' = n \dfrac{\sin\theta_{refraction}}{\sin\theta_{reflection}}$



FULL ANSWER QUESTION (12 marks)

a) (5 marks)

A 400 kg mass is attached to a spring which stretches 6.0 cm when the mass is hung from it. The mass spring system is set in simple harmonic motion with an amplitude of 2.7 cm. Let the equilibrium position be y=0 and positive displacements be above the equilibrium position .

i) What is the spring constant?
What is the frequency of the motion?

$$k = \frac{40.0 \times 9.8}{0.06} = 65.3 \times 10^2 \text{ N/m}$$

$$\omega = \sqrt{\frac{k}{m}} = 12.78 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 2.03 \text{ Hz}$$

ii) With the condition that, at t=0 the mass is at $y = -1.6$ cm and going down, write the appropriate equation for the SHM of this system (with the explicit phase constant).

The general equation of SHM is $y = A\cos(\omega t + \phi)$

where $\omega = 12.78$ rad/s.

From the initial condition we know that

$-1.6 = 2.7\cos(\phi) \Rightarrow \phi = 2.2$ rad

Thus

$y = A\cos(\omega t + \phi) = 2.7\cos(12.8t + 2.2)$ cm

b) (4 marks)

Assume now that the SHM motion described in part a) is the source of a wave on a horizontal string kept at a tension of 3.0 N. Neglect damping of the mass/spring. An observer measures the distance between two consecutive crests of the wave to be 3.5 m.

What is the speed of the wave and the mass per unit length of the string?

Since the distance between two consecutive crests is 3.5 m we know that the wavlength is $\lambda = 3.5$ m. The frequency of the wave is the same as the frequency of the mass spring. Thus the speed of the wave is $v = \lambda f = 3.5 \times 2.03 = 7.1$ m/s. Since

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{T}{v^2} = \frac{3.0}{7.1^2} = 59.5 \text{ g/m}$$

c) (3 marks)

Assume now that standing waves are set up in this same string (with same linear mass density and same tension) but with a different vibrator.
This standing wave is described by y=5.4 sin (kx) cos (8.0πt).

Determine the length of string required for this standing wave to have three loops.

From the equation for the standing wave we can extract the value of the frequency

$2\pi f = 8.0\pi \Rightarrow f = 4.0\text{ Hz}$

$v = \lambda f \Rightarrow \lambda = \dfrac{7.1}{4.0} = 1.775\text{ m}$. For three loops it is required that

$L = \dfrac{3}{2}\lambda = 2.66\text{m}$