$$\int \underset{\text{int}}{x}\ \underset{\text{diff}}{\ln x}\, dx = \frac{1}{2}x^2\ln x - \int \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}\right)dx \qquad \text{6 marks}$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\, dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

$$\int_1^2 x\ln x\, dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2\Big|_1^2$$

$$= (2\ln 2 - 1) - \left(0 - \frac{1}{4}\right)$$

$$= 2\ln 2 - 1 + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$$

$$\int e^{\sqrt{x}}\,dx \qquad \text{Let } \sqrt{x} = u \quad du = \frac{1}{2\sqrt{x}}\,dx \qquad \text{6 marks}$$

$$dx = 2u\,du$$

$$\int \underset{\text{diff}}{2u}\ \underset{\text{int}}{e^u}\,du = 2ue^u - \int 2e^u\,du$$

$$= 2ue^u - 2e^u + C$$

$$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$\int \sin 3x \cos 2x \, dx$ **3 marks**

$$\sin 3x \cos 2x = \frac{1}{2}\left[\sin(3x-2x) + \sin(3x+2x)\right]$$

$$= \frac{1}{2}\left[\sin x + \sin 5x\right]$$

$$\int \frac{1}{2}\left[\sin x + \sin 5x\right] dx = -\left[\frac{1}{2}\cos x + \frac{1}{10}\cos 5x\right] + C$$

$\int \tan^3 x \, dx = \int (\sec^2 x - 1)\tan x \, dx$ **7 marks**

$$= \int \left(\sec^2 x \tan x - \tan x\right) dx$$

$$= \frac{1}{2}\sec^2 x + \ln|\cos x| + C$$

Note: $\int \sec^2 x \tan x \, dx = \int \sec x \,(\sec x \tan x)\, dx$

let $u = \sec x$ then $du = \sec x \tan x \, dx$

$$= \int u \, du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}\sec^2 x + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + C$$

$$\int x\sqrt{\frac{1+x}{1-x}}\,dx$$

$$x\sqrt{\frac{1+x}{1-x}} = x\,\frac{\sqrt{1+x}\,\sqrt{1+x}}{\sqrt{1-x}\,\sqrt{1+x}} = \frac{x\,(1+x)}{\sqrt{1-x^2}}$$

$$= \frac{x}{\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + C$$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx =$$

simple substitution

trigonometric substitution

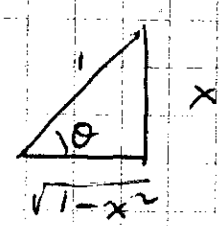
$x = \sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\cancel{(\cos\theta)}(\sin^2\theta)}{\cancel{\cos\theta}}\,d\theta = \int \sin^2\theta\, d\theta$$

$$= \int \frac{1-\cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$



$$= \frac{1}{2}\left(\theta - \frac{2}{2}\cos\theta\sin\theta\right) + C$$

$$= \frac{1}{2}(\theta - \cos\theta\sin\theta) + C$$

$$= \frac{1}{2}\left(\sin^{-1}x - (\sqrt{1-x^2})(x)\right) + C$$

$$\frac{x+4}{2x(x-2)^2(x^2+x+1)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{Dx+E}{(x^2+x+1)} + \frac{Fx+G}{(x^2+x+1)^2}$$

5marks

$$\int \frac{x^2}{x^2-x-2}\,dx$$

$$\begin{array}{r|l} & 1 \\ x^2-x-2 & x^2 \\ & \underline{x^2-x-2} \\ & x+2 \end{array}$$

$$\frac{x^2}{x^2-x-2} = 1 + \frac{x+2}{x^2-x-2}$$

$$\frac{x+2}{x^2-x-2} = \frac{x+2}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$= \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$A(x+1)+B(x-2) = x+2$$

When $x=-1$, $-3B=1$ $B=-\frac{1}{3}$
When $x=2$, $3A=4$ $A=\frac{3}{4}$

or $\begin{cases} A+B=1 \\ A-2B=2 \end{cases}$

$$\int \frac{x^2}{x^2-x-2}\,dx = \int 1 + \left(\frac{3}{4}\right)\left(\frac{1}{x-2}\right) - \frac{1}{3}\left(\frac{1}{x+1}\right)dx$$

$$= x + \frac{3}{4}\ln|x-2| - \frac{1}{3}\ln|x+1| + C$$