

a) Please circle True (T) or False (F) in the left margin.

- 1) T F Order qualifiers are those characteristics of a firm's product or service that cause consumers to actually purchase the product or service.
- 2) T F One of the reasons that measurement of productivity in services is more complicated than in manufacturing is due to the low degree of uniformity of the inputs in services.
- 3) T F The closer α is to 1 in simple exponential smoothing, the more responsive the forecasts are to changes in the actual demand.
- 4) T F In general, forecasts for annual demand are more accurate than forecasts for quarterly demand.
- 5) T F If you are the manager of a company that competes on the basis of speed and accuracy of delivery time, you should try to maximize the degree of utilization.
- 6) T F A multiple channel, infinite queue length system has an average inter-arrival time of 15 minutes and an average service time of 30 minutes. The minimum number of servers for this system to make sense (be stable) is 2.
- 7) T F For a given plant, it is desirable to have the actual output rate almost equal to the design capacity in order to minimize average cost per unit.
- 8) T F In concurrent engineering each department works independently of other departments in a sequential manner.
- 9) T F If a firm's target customer segment puts high value on low price, then the firm should design an efficient operations system.
- 10) T F If a firm's distinctive competency is flexibility, then its competitive priority should be quality.
- 11) T F If the demand for a product is stable and consistently high, in general, your decision should be to "buy" the product from an outside supplier, rather than make it "in-house".
- 12) T F A moving average forecast tends to be more responsive to changes in the data series when less data points are included while calculating the average.
- 13) T F A job shop process is characterized by very low volumes and very high varieties of goods or services produced.
- 14) T F The goal of queuing analysis is primarily to minimize the customer waiting time (and associated cost) by increasing capacity as much as possible.
- 15) T F In general, equipment flexibility requirement decreases over the life-cycle of a product.
- 16) T F In general, job shop manufacturing systems have lower unit costs than assembly line systems do because assembly line systems use costly specialized equipment.

b) Please circle your answer

- 1) Productivity can be improved by which of the following actions?
- I. Increasing output while keeping input at the same level
 - II. Reducing the amount of input while keeping output at the same level
 - III. Increasing the use of input, but increasing output even more
 - IV. Decreasing output, but decreasing the use of input even more
- a) I only
 - b) I, II, and III only
 - c) III and IV only
 - d) I, II, III and IV**
 - e) II, III, and IV only
- 2) Which of the following are generally associated with service operations?
- I. Intangible output
 - II. Low customer contact
 - III. Easy measurement of productivity
 - IV. Low uniformity of input
- a) I only
 - b) I and III
 - c) I and IV**
 - d) III only
 - e) II and IV
- 3) Suppose that the most recent actual data for demand is observed in February 2008. Using the double exponential smoothing technique, the forecast for August 2008 is equal to
- a) $F_{JULY08} + T_{JULY08}$
 - b) $F_{FEB08} + 6 * T_{FEB08}$**
 - c) $F_{MARCH08} + 5 * T_{MARCH08}$
 - d) $F_{FEB08} + 5 * T_{FEB08}$

- 4) Suppose that actual demand shows a decreasing trend. Then simple exponential smoothing technique would
- always overestimate the demand
 - always underestimate the demand
 - sometimes overestimate and sometime underestimates the demand
 - exactly predict the demand.
- 5) If you want to increase the responsiveness of n-period weighted moving average technique, you should
- increase n
 - decrease n
 - put more weight to the most recent data points
 - put less weight to the most recent data points
- I only
 - I and IV
 - II and III
 - I, II and III
 - III and IV
- 6) Effective capacity = 80 units per day
 Design capacity = 100 units per day
 Utilization = 48%
 Given the above information, the efficiency is:
- 20%
 - 35%
 - 40%
 - 60%
 - 80%
- 7) Suppose a bank has 4 tellers, and arriving customers join a single line with no limit on the queue length. The average number of people waiting in line at the bank during weekday lunch hour (12noon-1pm) is 10. During this hour, 120 people, on average, arrive to the bank. The tellers are busy 50% of the time. The average number of people in the system during the lunch hour is:
- 10
 - 12
 - 12.5
 - 14

- 8) An appointment system is effective in terms of reducing waiting time in a system primarily because:
- It increases the arrival rate
 - It reduces the randomness in the arrival process**
 - It reduces the randomness in the service process
 - It increases the utilization of the server
- 9) Continuous processes:
- are industrial processes that create products that “flow”, like sugar
 - have low unit cost
 - are costly to shut down and restart
 - produce items in small runs
- I, II and III**
 - I, II and IV
 - I, III and IV
 - II, III and IV
- 10) Job shops:
- are facilities that are costly to shut down and re-start
 - typically require specialized equipment
 - typically don't require a qualified workforce
 - produce items in high volume at a low unit cost
- I and II
 - III and IV
 - I and III
 - None of the above**
- 11) Which of the following are possible approaches to Service Design:
- Production Line Approach
 - Personalized Approach
 - Batch Processing
 - Self-service Approach
- I, II and III
 - I, II and IV**
 - I, III and IV
 - II, III and IV
- 12) Which of the following forecasting techniques would be most desirable to use during the growth phase of product life cycle?
- Linear trend
 - Simple Exponential
 - Moving Average
 - Double Exponential
- I and II
 - III and IV
 - I and III
 - I and IV**

- 13) Simple exponential smoothing is being used to forecast demand. The last forecast of 65 turned out to be five units less than actual demand. If the next forecast is 66, then it implies that the value of smoothing constant, alpha, is equal to
- a) 0.01
 - b) 0.1
 - c) 0.2
 - d) 0.15
- 14) If you are only told that the MAD of a forecasting model is high, what can you infer?
- a) Most forecasts have been lower than the actual demand
 - b) Most forecasts have been higher than the actual demand
 - c) Forecasts generally have been accurate
 - d) Forecasts generally have been inaccurate
- 15) Which of the following products is most likely to be made in a job shop environment?
- a) Graphite pencils
 - b) Custom furniture
 - c) Television sets
 - d) Cigarettes
- 16) The approach employed by McDonald's in delivering its services is most appropriately termed as
- a) Low customer contact approach
 - b) Self-service approach
 - c) Production line approach
 - d) Personal attention approach

Please very briefly answer the following questions

- I. Order the following manufacturing processes in terms of: i) Product variety (increasing order) and ii) Manufacturing volume (increasing order), that they are most capable to handle (just fill in the spaces in the tables below):
- a) Repetitive
 - b) Job shop
 - c) Continuous
 - d) Batch

Lowest variety	Low variety	Medium variety	High variety

Lowest volume	Low volume	Medium volume	High volume

Variety: Continuous, Repetitive, Batch, Job shop

Volume: Job shop, Batch, Repetitive, Continuous

- II. What is the primary advantage of net present value analysis over breakeven analysis approach? If there is a significant level of uncertainty associated with future demands, do you think breakeven or net present value is the correct technique to use? If not, what alternative technique you think would be more appropriate?

Net present value takes into account time value of money (BEA does not). When uncertainty is there NPV or BEA is not good, real options approach should be used.

a) Please circle True (T) or False (F) in the left margin.

- 1) T F In the basic EOQ model, if the annual holding cost per unit were to double, the optimal order quantity would also double.
- 2) T F In the Newsvendor model, if the salvage value per unit increases, the optimal order quantity decreases.
- 3) T F The bullwhip effect refers to the phenomenon that variance of orders is greater than that of sales, and this distortion increases as one moves upstream.
- 4) T F If your system is manually operated, it will require less inspection.
- 5) T F In the basic EOQ model, the purchase cost per unit does not influence the optimal order size.
- 7) T F A zero safety stock always implies that both the lead time and the demand rate are constant.
- 8) T F A retailer will order more when it is integrated with its supplier, compared to when they are disintegrated.

- 9) T F ABC classification system is based only on the unit value of a product.
- 10) T F If the sample size increases (everything else remaining the same), the distance between the control limits for a mean chart will decrease.
- 11) T F If the mean of a sample is above the upper control limit in a mean chart it implies that the process variability is increasing.
- 12) T F In an inventory system with uncertain demand, the annual service level increases as the fixed cost for placing an order increases (all else remaining the same).
- 13) T F Demand and supply randomness are the two key sources of uncertainty in a supply chain.
- 14) T F A p -chart is a control chart used to monitor the proportion of defects in a process and is used only when the number of occurrences per unit of measure can be counted.
- 15) T F One of the primary goals of six-sigma methodology is to achieve a process capability level of 3.
- 16) T F In general, control charts for attributes provide more information than those for variables.
- 17) T F A periodic review system normally requires more safety stock than a continuous review system (for the same service level).

b) Please circle your answer

1. Suppose that a retailer is operating at a Fill Rate of 98%. Which of the following operations strategies can reduce the need for safety stocks for the retailer, assuming that the fill rate will remain the same?
- I. Increasing the lead time
 - II. Decreasing the variability in lead time
 - III. Increasing ordering cost
 - IV. Increasing the demand variability
- a. II and III
 - b. II, III and IV
 - c. I, III and IV
 - d. IV only
 - e. I, II and IV

2. Which of the following is not a characteristic of a dependent demand item?
- Little need for safety stock
 - Demand for such an item is constant**
 - Need to be stocked just prior to production
 - Large quantities used at specific times
 - None of the above
3. Which of the following statements about the basic EOQ Model are true:
- If the ordering cost were to double, the optimal order quantity would increase by 50%
 - If annual demand were to double, the optimal order quantity would increase by 41.4%
 - If the holding cost/unit/time were to decrease by half, the optimal order quantity would decrease by 50%
 - If the ordering cost were to double, the total optimal cost (ordering + holding) would also double
- I and II
 - I and III
 - II and IV
 - II only**
 - I, II, and III
4. Innovative products are characterized by which of the following?
- High demand uncertainties
 - Difficulty in forecasting
 - Low profit margins
- I and III
 - II and III
 - I and II**
 - I, II and III
5. In the basic EOQ model, suppose that annual inventory holding cost = \$2/unit, ordering cost = \$20/order and demand = 8000 units/year. However, your supplier will only accept orders between 200 and 300 units, inclusive. What should be your optimal order size?
- Optimal order size = 100 units
 - Optimal order size = 200 units
 - Optimal order size = 300 units**
 - Optimal order size = 400 units
6. Quality improvement programs are examples of:
- Internal failure costs
 - External failure costs
 - Appraisal costs
 - Prevention costs**
 - None of the above

7. Suppose that the lower and upper tolerance limits for volume of a particular liquid product to be bottled are 410mL and 590mL, respectively. The manufacturer is considering implementing six-sigma quality control program. Then it has to reduce the standard deviation of its production system under the following value:

- a. 15 mL
- b. 30 mL
- c. 45 mL
- d. 90 mL
- e. None of the above

8. A plot below the lower control limit on the range chart:

- I. Should be ignored since lower variation is desirable
 - II. May be an indication that process variation has decreased from expected standard deviation
 - III. Should be investigated for assignable causes
- a. I and II
 - b. I and III
 - c. II and III
 - d. II only
 - e. I, II, and III

9. Suppose that demand in the Newsvendor model is normally distributed with mean 100 and standard deviation 20. If the critical fractile = 0.5, then the optimal order quantity is

- a) 120
- b) 110
- c) 100
- d) 90
- e) 80

10. A1: Constant Demand A2: Random Demand
 B1: Constant Leadtime B2: Random Leadtime
 C1: Constant Purchase Cost C2: Variable Purchase Cost (Quantity discount)
 D1: Single Period D2: Multiple Periods

Which of the following model has the correct set of assumptions given above?

- a) EOQ : A1,B1,C1,D1
- b) Quantity Discount Model : A1,B1,C2,D2
- c) Probabilistic Model : A2,B2,C1,D1
- d) Newsvendor Model : A2,B2,C1,D2
- e) None of the above

11. ABC analysis classifies inventory items into three classes based upon:
- (a) unit price
 - (b) the number of units on hand
 - (c) annual demand
 - (d) annual dollar volume**
 - (e) none of the above
12. The basic independent demand inventory model attempts to answer which of the following questions:
- I. What should be the optimal product characteristics?
 - II. How many units should be ordered?
 - III. When should the order be placed?
 - IV. What price should one pay for the item?
- (a) I and III
 - (b) II and III**
 - (c) II and IV
 - (d) II, III, and IV
 - (e) None of the above
13. If the traditional three-sigma limits on a control chart are replaced by two-sigma limits, which of the following will be true?
- (a) Type I error decreases
 - (b) Center line increases
 - (c) Type I error increases**
 - (d) Center line decreases
 - (e) None of the above

14. Which of the following is likely to happen when there is an increase in process variability?
- (a) \bar{X} and R charts both reveal the increase
 - (b) \bar{X} -chart reveals the increase, R-chart does not reveal the increase
 - (c) \bar{X} -chart does not reveal the increase, R-chart reveals the increase**
 - (d) \bar{X} and R charts cannot be used to identify such variability in process
 - (e) None of the above
15. Mean lead time demand for sugar in a supermarket is 600 kg., and it is known from previous experience that the standard deviation of lead time demand is 52 kg. If the acceptable stock-out risk during lead time is approximately 4%, the amount of safety stock (SS) and reorder point (ROP) are given by
- (a) SS = 52, ROP = 652
 - (b) SS = 52, ROP = 624
 - (c) SS = 91, ROP = 624
 - (d) SS = 91, ROP = 691**
 - (e) SS = 24, ROP = 624

Please very briefly answer the following questions

- a) When should one use control charts for variables and when control charts for attributes? Within the control charts for attributes, when should one use p chart and when c chart?

Variables – when data measured; Attributes – when data counted

P chart – for proportion of defectives ; c chart – for number of defects per unit

- b) In order to achieve strategic fit, how would you match the following four types of supply chains to the four cells of the table below (the cells marked A, B, C and D):

- Risk-hedging Supply Chain
- Efficient Supply Chain
- Agile Supply Chain
- Responsive Supply Chain

		Demand Uncertainty	
		Low	High
Supply Uncertainty	Low	A	B
	High	C	D

A – efficient B – responsive C – risk-hedging D – agile

QUESTION 4 (13 Marks)

Suppose you/your group is the owner of a company that produces CD players. The present production rate is 1,000 CD players/day and the selling price is \$210/unit. It requires 200 workers working 8 hours/day to produce the CD players and they are paid \$20/hour. The material cost is \$100/unit and overhead cost is \$50,000/day.

(a) What is the unitless multi-factor (labor + material + overhead) productivity ratio?

(b) What is the labor productivity ratio (both unitless and in terms of units/hour)?

You/your group are presently thinking about buying a new expensive machine that is quite complex to handle but will improve the quality of the product. The production rate will increase to 1,200 CD players/day, the selling price would be \$248/unit, material cost will reduce to \$90/unit and the overhead cost will be \$100,000/day (to pay for the depreciation of the machine). It will still require 200 workers working 8 hours per day (now to produce 1,200 units), however the labour rate will be \$30/hour.

(c) What will be the unitless multi-factor (labor + material + overhead) productivity ratio with the new machine?

(d) What will be the labor productivity ratio (both unitless and in terms of units/hour) with the new machine?

(e) Based on the analysis, what is your decision – buy the machine or not? *Explain.*

QUESTION 4

$$\textcircled{1} \textcircled{2} \text{ (a) } MFP = \frac{1000(210)}{50000 + 100(1000) + 200(8)(20)} = 1.1538$$

$$\text{(b) } LP \text{ (unitless)} = \frac{1000(210)}{200(8)(20)} = 6.5625$$

$$\textcircled{4} \text{ } LP \text{ (units/hr)} = \frac{1000}{200(8)} = 0.625$$

$$\textcircled{2} \text{ (c) } MFP = \frac{1200(248)}{200(8)(30) + 100000 + 90(1200)} = 1.1625$$

$$\textcircled{1} \text{ (d) } LP \text{ (unitless)} = \frac{1200(248)}{200(8)(30)} = 6.2$$

$$\textcircled{4} \text{ } LP \text{ (units/hr)} = \frac{1200}{200(8)} = 0.75$$

(e) Buy the m/c since MFP increases.

\square

QUESTION 2 (8 Marks)

Mr. Ratchet, the local auto mechanic, finds that it usually takes him 2 hours to diagnose and fix a typical problem.

- (a) What is his daily productivity (assume an 8-hour day)?

$$\frac{4 \text{ units}}{8 \text{ hrs}} = 0.5 \text{ units/hr} \quad \boxed{4}$$

- (b) Mr. Ratchet believes he can purchase a small computer trouble-shooting device which will allow him to find and fix a problem in the incredible (to him) time of 1 hour. He will, however, have to spend an extra hour each morning adjusting the computerized diagnostic device. What will be the impact on his daily productivity if he purchases the device?

$$\frac{7 \text{ units}}{8 \text{ hrs}} = 0.875 \text{ units/hr} \quad \boxed{3}$$

$$\text{Improvement} = 75\% \quad \boxed{1}$$

QUESTION 7 (14 Marks)

Goodyear Tire and Rubber Company is the ninth largest tire manufacturer in the world. Here are sales revenues for 1991 through 2001:

Year	Period	Demand	St Or Ft	Tt	TAft or FITt	Abs. Error	Trend	Abs. Error
1991	1	\$4,865.90	\$4,865.90	\$201.50			\$5,106.06	
1992	2	\$5,067.40	\$5,067.40	\$201.50	\$5,067.40		\$5,161.68	
1993	3	\$5,515.60	\$5,318.24	\$221.24	\$5,268.90	\$246.70	\$5,217.30	\$298.30
1994	4	\$5,728.80	\$5,577.34	\$236.38	\$5,539.48	\$189.32	\$5,272.92	\$455.88
1995	5	\$5,497.70	\$5,750.52	\$211.10	\$5,813.72	\$316.02	\$5,328.54	\$169.16
1996	6	\$5,197.70	\$5,808.83	\$149.99	\$5,961.62	\$763.92	\$5,384.16	\$186.46
1997	7	\$5,094.40	\$5,785.94	\$80.83	\$5,958.82	\$864.42	\$5,439.78	\$345.38
1998	8	\$5,108.80	\$5,715.18	\$20.20	\$5,866.77	\$757.97	\$5,495.40	\$386.60
1999	9	\$5,550.60	\$5,698.42	\$5.41	\$5,735.37	\$184.77	\$5,551.02	\$0.42
2000	10	\$5,738.90	\$5,710.84	\$8.22	\$5,703.83	\$35.07	\$5,606.64	\$132.26
2001	11	\$5,860.00	\$5,747.25	\$19.49	\$5,719.06	\$140.94	\$5,662.26	\$197.74

- (a) If a manager is interested in forecasting *the next four years*, why might moving average and simple exponential smoothing techniques not be desirable?

(b) From Excel's Data Analysis Tool the linear-trend equation is $T_t = 5050.44 + 55.62t$. Use the linear-trend and double-exponential smoothing ($\alpha = 0.2$ and $\beta = 0.4$) methods to provide forecasts for the years 2002 through 2005.

(c) Based on MAD (calculated using years 1993-2001) which method, linear-trend or double-exponential smoothing, provides the better forecasts? *Explain.*

(d) Linear-trend and double-exponential smoothing are two alternative methods used for forecasting time series data with trend. State a reason why you may prefer double-exponential smoothing over the linear-trend method.

(e) Could there be another component besides trend present in the time series?

QUESTION 7

(a) Since there is a trend in the data and because the techniques will only project one period into the future.

(b)

<u>Year</u>	<u>DES</u>	<u>Linear Trend</u>
2002	\$5,766.75	\$5,717.88
2003	\$5,786.24	\$5,773.50
2004	\$5,805.73	\$5,829.12
2005	\$5,825.23	\$5,884.74

(c) For DES : $MAD = \$388.79$

For Linear Trend : $MAD = \$241.36$

⇒ The linear trend method provides the better forecasts since it has a lower MAD.

(d) DES updates estimates of the average and trend components every period.

(e) There may be a cyclical component, possibly 6 or 7 year cycle, but it is difficult to determine with limited data.

2. (b) A battery company distributes a large-capacity battery that has been used for some industrial processes. They know that electric cars are now in production and they think that a jump (a permanent, one-time increase) has occurred in their demand. They are forecasting by simple exponential smoothing, since there has been no trend. The battery costs \$25 to produce. For November of 2000, the exponentially smoothed forecast was 150. The recent demands of November = 230 and December = 250.

- (i) Which is more appropriate, $\alpha = 0.05$ or $\alpha = 0.30$. Why?
- (ii) Incorporate November and December into the exponentially smoothed average, using simple exponential smoothing and the α you selected in part (i). Compute the forecasts for December and January.
- (iii) What are the forecasts for February and March based on the data available at the end of December?
- (iv) The company is using forecasts as a basis for deciding on production quantities. What difficulties do you foresee in using this method in this situation? What should they do to improve forecasts?

b) (i) $\alpha = 0.30$ should be used in order to react to the change.

(ii) $F_{DEC} = 0.3(230) + 0.7(150) = 174$
 $F_{JAN} = 0.3(250) + 0.7(174) = 196.8$

(iii) $F_{FEB} = F_{MAR} = 196.8$

(iv) The total impact of the change is not yet reflected in the forecast.

Hence, the production order quantities are probably too small. One would have hoped to have planned ahead by foreseeing this increase in demand. A better forecast would probably be to take the average of the two recent demands. That is, 240.

QUESTION 5 (13 Marks)

An e-retailer of PCs is able to collect the data for customer demand (in mil \$) from its website. However, due to shortage in inventory, it is not able to fulfill all the demand. The sales and demand for 2000 are given below.

Month	Demand (in mil \$)	Sales (in mil \$)
Jan	20	18
Feb	24	24
March	27	25
April	31	25
May	37	32
June	47	40
July	53	45

- (a) Which data, sales or demand, should be used if the e-retailer wants to forecast its *demand* for August 2000 – October 2000?
- (b) Use a 3-month moving average to forecast demand for April 2000 – July 2000.

(c) Use exponential smoothing with $\alpha = 0.25$ to forecast demand for April 2000 – July 2000. Assume that the initial forecast for March is the unweighted average of demands for January 2000 – March 2000.

(d) Use exponential smoothing with $\alpha = 0.1$ to forecast demand for April 2000 – July 2000. Assume that the initial forecast for March is equal to the actual demand for March 2000.

(e) Compare the performance of the methods of parts (b), (c) and (d) using the mean squared error as the performance criterion. Which method would you recommend?

(f) Based on your recommended method what should be the demand forecast for August 2000 – October 2000?

QUESTION 5

(a) The forecast should be based on demand data. $\boxed{2}$

(b)	Apr.	23.7	(c)	Apr.	24.5	(d)	Apr.	27.0
	May	27.3		May	26.1		May	27.4
$\boxed{2}$	June	31.7	$\boxed{2}$	June	28.8	$\boxed{2}$	June	28.4
	July	38.3		July	33.4		July	30.2

(e) $MSE(b) = 199.1 \rightarrow$ recommend $MA(3)$

$$MSE(c) = 291.7 \quad \boxed{3}$$

$$MSE(d) = 324.8$$

(f) For all 3 months it should be 45.7. $\boxed{2}$

Question 3: (12 points) The Dean of the BCom Program of McGill University wants to forecast the student enrollment for next year's fall term based on the following historical data:

<u>YEAR</u>	<u>ENROLLMENT</u>
1998	450
1999	480
2000	500
2001	525

a) (5 points) Forecast year 2002 and 2003 enrollments using the exponential smoothing method with $\alpha=0.4$.

b) (7 points) Forecast year 2002 and 2003 enrollments using the trend-adjusted-exponential smoothing (double-exponential) method with $\alpha=0.4$ and $\beta=0.2$.

a) $F_t = A_t$
 $F_{98} = 450 \rightarrow F_{99} = 450$
 $F_{00} = 450 + 0.4(480 - 450) = 462$
 $F_{01} = 462 + 0.4(500 - 462) = 477.2$
 $F_{02} = 477.2 + 0.4(525 - 477.2) = 496.3 \approx \underline{\underline{496}}$ (3 pts)

$F_{03} = F_{02} = 496$ in E.S.

b) $FIT_{98} = 450$ $T_{98} = 480 - 450 = 30$ (2 pts)

- $F_{99} = 450 + 0.4(480 - 450) = 450$

$T_{99} = 30 + 0.2(480 - 450) = 30$

$FIT_{99} = 480$

- $F_{00} = 480$

$T_{00} = 30$

$FIT_{00} = 510$

- $F_{01} = 510 + 0.4(500 - 510) = 506$ (3 pts)

$T_{01} = 30 + 0.2(500 - 510) = 28$

$FIT_{01} = 534$

- $F_{02} = 534 + 0.4(525 - 534) = 530.4$

$T_{02} = 28 + 0.2(525 - 534) = 26.2$

$FIT_{02} = 556.6 \approx \underline{\underline{557}}$

- $FIT_{03} = 530.4 + 26.2 + 26.2 = 582.8 \approx \underline{\underline{583}}$ (2 pts)

Question 3. (20 points) Intelligent Manufacturing Co is considering three alternatives for obtaining a machine part to be used in a new product. It can buy the product from an outside supplier or produce it in-house using a numerically controlled (NC) lathe, or produce it using a machining (MC) center.

For each alternative, the annual fixed cost, the variable cost, and the annual production capacity is given in the table below.

Alternative	Fixed Cost (\$)	Variable Cost (\$)	Production Capacity
Buy	0	200	Unlimited
NC Lathe	80,000	75	2,500
MC Center	200,000	15	5,000

Assume that the machine part generates a revenue of \$300 per unit.

- a) (6 points) Calculate the break-even point for each alternative.
- b) (6 points) Determine the annual output level for which Intelligent Co. would be indifferent to i) Buy and NC Lathe alternatives, ii) Buy and MC Center alternatives, and iii) NC Lathe and MC Center alternatives.
- c) (4 points) Based on your results in a) and b), determine the range of annual demand for which each alternative is best. (Consider annual demand volumes from 0 to 5,000 units)
- d) (4 points) If there is a significant level of uncertainty associated with annual demand, do you think cost-volume analysis (as above) is the correct analysis technique to use? Why or why not? If not, what alternative technique you think would be more appropriate?

23.a) Buy: $Q_{BEP} = 0$

NC Lathe: $Q_{BEP} = \frac{80,000}{300-75} = 355.55$

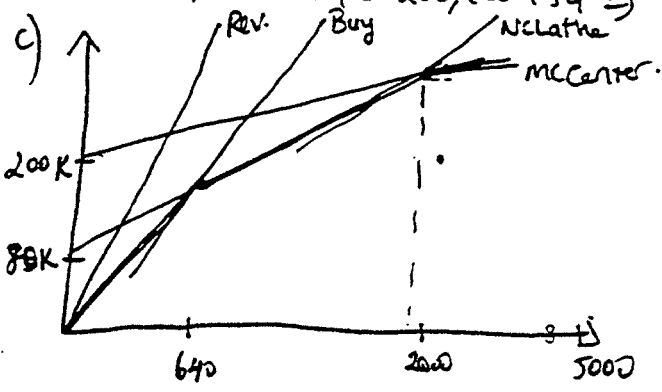
MC Center: $Q_{BEP} = \frac{200,000}{300-15} = 701.75$

d) No. B/E assumes revenues, costs known with certainty
 9/20 more probabilistic approx like Decision trees!

6) $80,000 + 75Q = 200Q \Rightarrow Q = 640$ unit (Buy - NC Lathe)

$200,000 + 15Q = 200Q \Rightarrow Q = \frac{200,000}{200-15} = 1081.08$ (Buy - MC Center)

$80,000 + 75Q = 200,000 + 15Q \Rightarrow Q = \frac{120,000}{60} = 2,000$ (NC Lathe - MC Center)



Demand	Option
0 - 640	Buy
640 - 2000	NC Lathe
2000 - 5000	MC Center

Question 4. (15 points). The General Store at the University of Western Ontario is an auxiliary bookstore located near the dormitories that sells academic supplies, toiletries, sweatshirts and T-shirts, magazines, packaged food items, canned soft drinks and fruit drinks. The manager of the store has noticed that several pizza delivery services near campus make frequent deliveries. As such, the manager is considering selling pizza at the store. She could buy pre-made pizzas (yum-yum) and heat them in an oven. The cost of the oven and freezer would be \$27,000. The frozen pizzas cost \$3.75 to buy from a distributor and to prepare (including labor and a box). To be competitive with the local delivery services, the manager believes she should sell the pizzas for \$8.95 apiece.

- Determine how many pizzas would have to be sold to break-even?
- If the General Store sells 20 pizzas per day, how many days would it take to break-even?
- What volume of output will be required to achieve a profit of \$5,000?
- The manager of the store anticipates that once the local pizza delivery services start losing business they will react by cutting prices. If after a month (30 days) the manager has to lower the price of a pizza to \$7.95 to keep demand at 20 pizzas per day, what will the new break-even point be? What is the total break-even volume (i.e. the number of pizzas already sold plus the new break-even point)?

4. (a)
$$\frac{27000}{8.95 - 3.75} = 5,192.30 \text{ pizzas}$$

(b)
$$\frac{5192.3}{20} = 259.6 \text{ days}$$

(c)
$$Q(8.95 - 3.75) - 27000 = 5000$$

$$Q = 6153.85 \text{ pizzas}$$

(d) Revenue for first 30 days

$$= 30(20)(8.95 - 3.75) = \$3120$$

→ \$23,880 fixed cost not recouped after 30 days.

New
$$\frac{23880}{7.95 - 3.75} = 5,685.7 \text{ pizzas}$$

Total BE = 600 + 5685.7 = 6285.7 pizzas
 volume

- (c) The range of volume that would make each alternative attractive.
- (d) If the variable cost per unit of the supplier decreases (all other things remain the same), what is the effect (increases or decreases) on the answer of part (c)?
- (e) The NPV (net present value) of the project if the forecasted demand for the model is 3,000 units/year for three years (the life cycle for the model is 3 years). Assume, just for part (e), that either option will require an initial investment of \$1,000,000 as development expenditure at time 0, in addition of any other cost, and the annual interest rate for NPV calculation in MCI is 10%. Note that the selection of option in this part should be based on the answer to part (c).

QUESTION 3

a) option #1 : $\frac{500000}{2000-1750} = 2000 > 1500$

$$1500(2000-1750) + (Q_B - 1500)(2000-1725) - 500000 = 0$$
$$\Rightarrow Q_B = 1954.5$$

option #2 : $Q_B = 0$

b) ↓ for option #1 $\boxed{2}$

c) $Q(2000-1850) = 1500(250) + (Q-1500)(225) - 500000$

$$\Rightarrow Q = 4,300$$

$\boxed{4}$ For volumes $< 4,300$; use option #2
For volumes $> 4,300$; use option #1

(d) increases $\boxed{1}$

(e) use option #2 since $3,000 < 4,300$.

$$NPV = \frac{3000(150)}{1.1} + \frac{3000(150)}{(1.1)^2}$$

$$\boxed{4} + \frac{3000(150)}{(1.1)^3} - 1000000 = \$119,083.40$$

2

Question 2. (15 points) Canadian Vending Inc. (CVI) supplies vended food to McGill University. Because students often kick the machines out of anger and frustration (not due to O.M. of course!), management has a constant repair problem. The machines break down on an average every 20 minutes. CVI estimates that downtime costs the company \$25/hour per machine.

From past experience, CVI estimates that one worker can service machines at an average rate of 5 per hour, two workers working together can service 7 per hour, and a team of three workers can do 8 repairs per hour. Each maintenance worker gets paid at \$4/hr. Assume Poisson arrivals and exponential service times.

a) (10 points) What is the optimal maintenance crew size for serving the machines?

a) (5 points) Under your proposed system in a), what is the probability that a new breakdown will wait for the maintenance crew? If the new breakdown indeed waits for a maintenance crew, how long on average would it wait?

Q2. $\lambda = 3/\text{hr}$

Team Size	μ
1	5
2	7
3	8

Cost Labor: 4\$/hr
Capacity: 25\$/hr.

a) Size = 1.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{9}{5(5-3)} = 0.9 \text{ m/cs.}$$

$$L_s = 0.9 + \frac{\lambda}{\mu} = 1.5 \text{ m/c}$$

$$\text{Cost} = \begin{cases} 1.5 \times 25 = 37.5 \\ 1 \times 4 = 4 \end{cases} \Rightarrow 41.5/\text{hr.}$$

Size = 2

$$L_q = \frac{9}{7(7-3)} = \frac{9}{28} = 0.3214$$

$$L_s = 0.3214 + \frac{3}{7} = 0.75 \text{ m/c.}$$

$$\text{Cost} = \begin{cases} 0.75 \times 25 = 18.75 \\ 2 \times 4 = 8 \end{cases} \Rightarrow \underline{\underline{26.75/\text{hr.}}}$$

Size = 3

$$L_q = \frac{9}{8(8-3)} = 0.225$$

$$L_s = 0.225 + \frac{3}{8} = 0.6$$

$$\text{Cost} = \begin{cases} 0.6 \times 25 = 15 \\ 3 \times 4 = 12 \end{cases} \Rightarrow 27$$

Optimal Team Size = 2.

b) $P_w = 1 - P_0 = \frac{\lambda}{\mu} = \underline{\underline{0.428}}$.

$$W_q = \frac{1}{\mu\mu - \lambda} = \frac{1}{7 \times 7 - 3} = \underline{\underline{0.25 \text{ hrs.}}}$$

check!

$$W_q = L_q / \lambda = 0.3214 / 3 = 0.1071$$

$$\left. \right] P_w = 0.428 \checkmark$$

Question 4. (25 points) Consider a job shop with two workers. Currently, worker A is trained in operation A, and worker B is trained in operation B. Parts requiring *only* operation A arrive at a rate of 10 per hour, and parts requiring *only* operation B arrive at a rate of 11 per hour. Both workers have an average service time of 4 minutes. Assume that interarrival times and service times are exponentially distributed.

The manager of the job shop is considering training the workers so that they can perform both operations. In the proposed configuration, there will be a single waiting line instead of the current practice of separate queues in front of each worker. Although each worker would be more flexible after the training, his average service time would remain the same. The manager found out from the accounting department that the unit cost of carrying work-in-process inventory (i.e., parts *waiting* for the required operation) in the job shop is \$120 per year. The training session for each worker would cost \$250. The manager thinks that the proposed system will be in effect for 3 years before a major structural change in the job shop.

a) (16 points) Calculate the annual cost of each configuration. (Ignore the training costs)

b) (9 points) Assuming an annual interest rate of 20%, should the manager implement the proposed change?

$$\mu = \frac{1}{4}(60) = 15$$

NOTE: The table for waiting lines is provided in the Appendix.

(a) Current system: 2 separate single channel (model 1) waiting lines

$$L_q - A = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{15(15 - 10)} = 1.33\bar{3}$$

$$L_q - B = \frac{11^2}{15(15 - 11)} = 2.01\bar{6}$$

$$\text{Annual Cost} = 120(L_q - A + L_q - B) = 402$$

Please use this page if you need more space to answer Question 4.

Proposed system : 1 waiting line,
2 servers (model 3)
 $\lambda = 21, \mu = 15, m = 2$

$$L_q = 1.345 \text{ (from table)}$$

$$\text{Annual cost} = 120 L_q = 161.4$$

$$(b) \text{ Annual Savings} : 402 - 161.4 = 240.6$$

$$\text{Training cost} : 250 (2) = 500$$

$$NPV = -500 + \frac{240.6}{(1.2)} + \frac{240.6}{(1.2)^2} + \frac{240.6}{(1.2)^3}$$

$$= 6.8 > 0$$

\therefore Implement.

Question 5. (10 points). The director of a large engineering staff is considering the rental of several computers that will permit the staff to access a special information service. On the basis of a survey, the director finds that the department will generate an average demand of 8/hr for the service. The engineers estimate that an average service request will take 15 minutes. A computer terminal for this service rents for \$10/hr. When the average salary of the engineering staff is considered, the cost of keeping an engineer idle (i.e. waiting for a terminal) is \$30/hr. For a quick and dirty analysis, the director assumes that arrivals are Poisson and service times are exponential. Furthermore, the engineering staff is large enough to assume an infinite population.

- (a) How many terminals should the director rent in order to minimize total costs?
- (b) Under your proposed system what is the probability that an engineer will wait for a terminal? If the engineer indeed waits, on average how much would he/she wait?

5. (a) $\lambda = 8, \mu = 4.$

m	L_q	$(\$30 L_q)$ Cost Wait	$\$10 m$	TC
3	0.889	26.67	30	56.67
4	0.174	5.22	40	45.22 *
5	0.040	1.20	50	51.20

(b) $W_a = \frac{1}{4(4) - 8} = 0.125 \text{ hrs} = 7.5 \text{ min} \quad \boxed{2}$

$P_w = \frac{W_q}{W_a} = \frac{.174/8}{.125} = 17.4\% \quad \boxed{2}$

QUESTION 3 (14 Marks)

Customers arrive to a department store at the rate of 45 per hour. The manager of the department store has conducted customer surveys and determined it costs the store approximately \$15 for every hour a customer spends in the checkout process. A typical customer spends on average 4 minutes at the checkout cashier. Assume Poisson arrivals and exponential service times. The department store pays \$96/day (1 day = 8 hours) for cashier clerks.

- (a) Determine the optimal number of cashiers to have on duty in order to minimize the department store's total cost. When more than one cashier is used, customers will still wait in one queue.

For the optimal number of cashiers, determine the following:

- (b) Utilization of the system and the average number of customers in service.

(c) Probability that there is one or more customers in the system.

(d) Average waiting time for a customer *not immediately served*.

(e) Probability that an arrival will not have to wait for service.

QUESTION 3

$$(a) \lambda = 45/\text{hr} \Rightarrow m \text{ must be } \geq 4.$$

$$\mu = 15/\text{hr}$$

$$\begin{aligned} TC(m=4) &= L_s * 15 + 4 * 12 \\ &= (1.528 + 3) * 15 + 48 \\ &= \$115.92/\text{hr} \end{aligned}$$

$$\begin{aligned} TC(m=5) &= (.354 + 3) * 15 + 60 \\ &= \$110.31/\text{hr} \end{aligned}$$

$$\begin{aligned} TC(m=6) &= (1.099 + 3) * 15 + 72 \\ &= \$118.49/\text{hr} \end{aligned}$$

\Rightarrow opt. # of cashiers = 5.

$$(b) \rho = \frac{45}{5(15)} = 0.6$$

$$r = \frac{\lambda}{\mu} = 3$$

$$(c) 1 - P_0 = 1 - .047 = .953$$

$$(d) W_a = \frac{1}{5(15) - 45} = \frac{1}{30} \text{ hr} = 2 \text{ min.}$$

$$(e) 1 - P_w = 1 - \frac{W_q}{W_a} = 1 - \frac{.354/45}{1/30} = .764$$

$$(\text{=} P_0 + P_1 + P_2 + P_3 + P_4)$$

Question 2. (10 points)

The Call-Us Plumbing Supply Company stocks thousands of plumbing items sold to regional plumbers, contractors and retailers. One of the crucial items is the #3925 valve, which sells at a constant rate of 10,000 units per year. The current policy of the company is to order 400 valves from the supplier each time an order is placed. It costs the company \$5.50 to place an order, and the annual inventory holding cost is 20% of the item cost. Recently, the supplier of the #3925 valve has offered the following quantity discounts:

<u>Order Quantity</u>	<u>Price Per Valve</u>
1-399	\$2.20
400-699	\$2.00
700 or more	\$1.80

Thus, with the current ordering policy of 400 the company is able to purchase the valves at the price of \$2.00 per item. However, the managers are not sure whether this is the best buying policy.

- (a) What is the minimum cost order quantity for the valves?
- (b) What would be the annual savings of the policy you are suggesting over the current buying policy of the firm?

2. (a)	<u>Order Qty</u>	<u>EOQ</u>	$\rightarrow \sqrt{\frac{2DS}{H}}$
	1-399	500	
	400-699	524.4	
	≥ 700	552.77	

$$TC_{399} = 2.20(10000) + \frac{399(.44)}{2} + \frac{10000}{399}(5.50)$$

$$\boxed{8} \quad = \$22,225.62$$

$$TC_{524} = 2.00(10000) + \frac{524(.4)}{2} + \frac{10000}{524}(5.50)$$

$$= \$20,209.76$$

$$TC_{700} = 1.80(10000) + \frac{700(.36)}{2} + \frac{10000}{700}(5.50)$$

$$= \$18,204.57$$

$$(b) \quad TC_{400} = 2(10000) + \frac{400(.40)}{2} + \frac{10000}{400}(5.50)$$

$$= \$20,217.50$$

$$\boxed{2} \quad \text{Savings} = \$20,217.50 - \$18,204.57$$

$$= \$2,012.93$$

Question 4 (5+2+3= 10 points)

BAKMON, a Montreal bakery, must decide how many cakes to order for the upcoming week. Bakmon purchases the cakes from a large manufacturer in town. Bakmon pays \$10/cake to the manufacturer and sells them to customers for \$16/cake. Cakes can not be carried over to the next week, so all left-over cakes are sold at half-price at the end of the week. Analysis of recent weekly demand shows the following:

Demand	89	90	91	92
Probability	0.2	0.3	0.4	0.1

- (a) How many cakes should Bakmon order so as to maximize the expected profit? What is the expected profit for Bakmon at the optimal order quantity?
- (b) Suppose Bakmon decides to improve the service level being provided to customers. Specifically, they want to provide a 100% cycle service level. What should be the order quantity so as to provide 100% service level? Will the expected profit increase or decrease under this policy compared to part (a)?
(HINT: There is no need for any calculation in this part)
- (c) Suppose the manager of Bakmon incurs a penalty of \$20 for each customer that could not find a cake due to stock-out. However, they continue to use the order quantity calculated in part (a). Calculate the expected profit for Bakmon in this case?

PART A $p=\$16/\text{unit}$ $c=\$10/\text{unit}$ $s=8/\text{unit}$ $c_u=p-c=\$6$ $c_o=c-s=\$2$ $CR=c_u/(c_u+c_o)=0.75$

quantity demanded	probability	cumulative probability
89	0.2	0.2
90	0.3	0.5
91	0.4	0.9
92	0.1	1

Optimal order quantity is 91 units and the expected profit is \$539.6

demand	probability	revenue	salvage revenue	cost	profit	probability*profit
89	0.2	1424	16	910	530	106
90	0.4	1440	8	910	538	215.2
91	0.3	1456	0	910	546	163.8
92	0.1	1456	0	910	546	54.6

539.6

PART B The order quantity should be 92 units. the expected profit will be higher in this case compared to the expected profit with order quantity 91 units.

PART C If we include the penalty cost of \$20/excess demand, the expected profit is \$537.6

demand	prob.	revenue	salvage revenue	cost	penalty cost	profit	probability*profit
89	0.2	1424	16	910	0	530	106
90	0.4	1440	8	910	0	538	215.2
91	0.3	1456	0	910	0	546	163.8
92	0.1	1456	0	910	20	526	52.6
							537.6

Question 5 (5+2+2+5=14 points)

JSK Inc. has a retail store in Montreal. Currently the focus of the management is on a particular product with barcode number: SKUN162342. The retail store purchases the product for \$2/unit. Placing an order costs about \$40. The retailer uses an interest rate of 40% for internal purposes. The weekly demand is normally distributed with a mean of 200 units and a standard deviation of 100 units. Lead time required for replenishment is also a random variable and normally distributed with a mean of 4 weeks and a standard deviation of 1.73 weeks. The retailer wants to provide a 97% cycle service level. Assume that 1 year has 50 weeks.

- a) What is the total annual (purchase + holding + ordering) cost for this particular item for JSK Inc.?
- b) Suppose that there is a penalty cost of \$3/customer in case of a stock-out. What is the annual penalty cost for the item in this case?
- c) Suppose that there is a penalty cost of \$500/cycle if there is a stock-out during a cycle. What is the annual penalty cost for the item in this case?

JSK Inc. has been informed that one of the major competitors in the market is closing its store in the neighborhood. The weekly demand for the competitor is normally distributed with a mean of 600 units and a standard deviation of 75 units. Manager believes that all the customers of the competitor will now come to JSK Inc. (i.e., now JSK's demand would be the sum of the original demand and the competitors' demand) Keeping in mind this increase in demand, JSK have decided to provide an annual service level of 98%.

- d) What is the ROP under this situation? What is the probability of having a stock-out during a cycle?

(a)

$$S = \$40/\text{order} \quad i = 0.40 \quad c = 1/\text{unit} \quad t = 1/\text{unit}$$

$$d = 200/\text{week} \quad \sigma_d = 100/\text{week} \quad LT = 4 \quad \sigma_{LT} = 1.73$$

$$\mu_{LTD} = d \cdot LT = 200 \cdot 4 = 800 \quad \sigma_{LTD} = \sqrt{LT\sigma_d^2 + d^2\sigma_{LT}^2} = \sqrt{4(100)^2 + (1.73)^2(200)^2} = 400$$

$$Q = \sqrt{\frac{2DS}{i(c+t)}} = \sqrt{\frac{2 \cdot 200 \cdot 50 \cdot 40}{0.40(1+1)}} = 1000 \text{ units}$$

$$F(z) = 0.97 \Rightarrow z = 1.88$$

$$s.s = z\sigma_{LTD} = 1.88 \cdot 400 = 752$$

$$TC = \frac{D}{Q}S + H\left(\frac{Q}{2} + ss\right) + (c+t)D = \frac{200 \cdot 50}{1000}40 + 0.8\left(\frac{1000}{2} + 752\right) + 2 \cdot 200 \cdot 50 = \$21401.6$$

b)

$$z = 1.88 \Rightarrow \text{Table} \Rightarrow E[z] = 0.012 \Rightarrow E[n] = E[z]\sigma_{LTD} = 4.8 \Rightarrow E[N] = E[n]\frac{D}{Q} = 48$$

$$\text{penalty cost} = 48 \cdot 3 = \$144/\text{year}$$

$$c) F(z) = \text{CSL} = 0.97$$

$$\text{penalty cost} = 500 \cdot (1 - 0.97) = \$15/\text{year}$$

d)

$$d_1 = 200/\text{week} \quad \sigma_{d1} = 100/\text{week}$$

$$d_2 = 600/\text{week} \quad \sigma_{d2} = 75/\text{week}$$

$$d_T = 200 + 600 = 800$$

$$\sigma_T = \sqrt{\sigma_{d1}^2 + \sigma_{d2}^2} = \sqrt{100^2 + 75^2} = 125$$

$$LT = 4 \quad \sigma_{LT} = 1.73$$

$$\mu_{LTD} = d_T \cdot LT = 800 \cdot 4 = 3200 \quad \sigma_{LTD} = \sqrt{LT\sigma_T^2 + d_T^2\sigma_{LT}^2} = \sqrt{4(125)^2 + (1.73)^2(800)^2} = 1408$$

$$Q = \sqrt{\frac{2DS}{i(c+t)}} = \sqrt{\frac{2 \cdot 800 \cdot 50 \cdot 40}{0.40(1+1)}} = 2000 \text{ units}$$

$$\text{ASL} - \text{FR} = 0.98 = 1 - \frac{E[N]}{D} \Rightarrow E[N] = 800 \Rightarrow E[n] = 40 \Rightarrow E[z] = 0.028$$

$$E[z] = 0.028 \Rightarrow z = 1.52$$

$$s.s = 1.52 \cdot 1408 = 2140.16$$

$$\text{ROP} = 3200 + 2140.16 = 5340.16 \text{ units}$$

$$(d) \quad z = 1.52 \Rightarrow F(z) = 0.9357$$

Question 2 (20 points) Western Jeans Company manufactures jeans in two independently run but *identical* production plants. The major input to the manufacturing facility of Western Jeans Company is denim. The amount of denim used daily by each of the production plants is normally distributed with an average of 3,000 yards and a standard deviation of 600 yards. Western Jeans purchases denim from a textile mill. The leadtime required to receive an order of denim from the textile mill to each plant is a constant 4 days. Western Jeans estimates the ordering cost to be \$800/order and the holding cost to be \$10/yard/year for each plant. Assume 300 working days in a year.

a) (6 points) Western Jeans believes that a ⁹⁶99% service level is necessary to avoid denim shortage at the facilities so that manufacturing stoppage at the plants is minimized. What is the optimal reorder point (R) and order quantity (Q) for each plant that provides this service level? Calculate the annual *inventory holding* cost of this policy for each plant. (Since the plants are identical, you need not calculate these quantities separately). Calculate the *total* inventory holding cost for Western's 2-plant manufacturing system.

b) (10 points) Western Jeans is planning to consolidate (merge) the two plants into one central manufacturing plant. Clearly, the new plant will take over all the manufacturing responsibility. It is perceived that the leadtimes, ordering and holding cost structure will remain unaffected from the consolidation. If Western wants to maintain the same level of service, what would the optimal reorder point (R) and order quantity (Q) be for the consolidated plant? Calculate the annual *inventory holding* cost of this policy.

c) (4 points) Compare the inventory holding costs of the 2-plant system in a) and the consolidated system in b). What economic factor(s) can you attribute the difference to?

Note: The Standard Normal Probability Distribution Table is provided in the Appendix.

a)

$$\lambda = 3,000 \quad \sigma_D = 600 \quad S = 800 \quad H = 10 \quad 96\% \text{ SL}$$

$$L = 4 \text{ days}$$

$$Q = \sqrt{\frac{2 \cdot S \cdot D}{H}} = \sqrt{\frac{2 \cdot 800 \cdot 3,000 \cdot 300}{10}} = \underline{\underline{12,000 \text{ yds.}}}$$

$$96\% \text{ SL} \Rightarrow z = 1.75 \text{ (or } 1.80)$$

LT-Demand $\mu = 3,000 \times 4 = 12,000$

$$\sigma = \sqrt{600^2 \times 4} = 1,200 \text{ (1)}$$

$$ROP = 12,000 + (1.75)(1,200) = 12,000 + \sqrt{2,100} = \underline{\underline{14,100}}$$

Holding Cost: $H \left(\frac{Q}{2} + SS \right) = 10 \left(\frac{12,000}{2} + 2,100 \right) = 81,000 \text{ \$/yr.}$

Total Inv. H Cost = $81,000 \times 2 = \underline{\underline{162,000}}$

Please use this page if you need more space to answer Question 2.

b) Consolidated

$$\bar{d} = 6,000$$

$$\sigma_{\bar{d}} = \sqrt{2} \sigma_d = \sqrt{2}(600)$$

$$L=4, S=800 H=10 \quad 96\%$$

$$3 \left\langle Q = \sqrt{\frac{2 \cdot 800 \cdot 6,000 \times 300}{10}} = \underline{\underline{16,970.56}}$$

$$\text{LT Demand: } z=1.75$$

$$\mu = 6,000 \times 4 = 24,000$$

$$2 \text{ pts. } \sigma = \sqrt{2}(600)(\sqrt{4}) = 1200\sqrt{2} = 1,697.05$$

$$2 \left\langle R = 24,000 + (1.75)(1,697.05) = 24,000 + \underline{\underline{2,969.85}} = \underline{\underline{26,969.85}}$$

$$1 \left\langle \text{Holding Cost} = H \left(\frac{Q}{2} + SS \right) = 10 \left(\frac{16,970.56}{2} + 2,969.85 \right) \\ = 114,551.3 \text{ /yr.}$$

$$c) \text{ Inv. Cost } 162,000 \rightarrow 114,551 \left(\frac{162,000}{\sqrt{2}} \right)$$

i) Economies of Scale in purchasing/setup smaller

$$2 \left[\underline{Q^* \text{ increase by a factor of } \sqrt{2} \text{ not } 2} \right] \quad \begin{matrix} Q^{\text{con}} \neq 2Q \\ \sqrt{2}Q \end{matrix}$$

ii) Risk Pooling (decrease in ^{uncertainty in} overall demand/usage)

$$2 \left[\underline{\sigma \text{ increase by a factor of } \sqrt{2} \text{ not } 2} \right]$$

$$SS^{\text{con}} \neq 2SS \\ = \sqrt{2}$$

QUESTION 3 (10 Marks)

A local retailer has noticed that the demand for a certain product is normally distributed with a mean of 600 units per week and a standard deviation of 150 units per week. The lead time is constant at 4 weeks. Assume that one year equals 52 weeks. It costs the retailer \$115.38 to place an order and the inventory holding cost is 25 cents/dollar/year. The purchase cost is \$5 per unit. The retailer wants to minimize its inventory costs.

- (a) What will be the optimal order size and average number of orders per year?
- (b) What will be the reorder point for a cycle (lead time) service level of 97%? For a reorder point with a cycle service level of 97%, what will be the corresponding annual service level (fill rate)?

(c) What will be the total annual inventory cost of the system [ordering + holding (cycle inventory and safety stock) + purchase] for cycle service level of 97% and annual service level of 97%?

(d) Now suppose the inventory holding cost suddenly has decreased to 20 cents/dollar/year (all other parameters remain the same as before). What will be the reorder point if the retailer aims for a cycle (lead time) service level of 97%. What will be the reorder point if the retailer aims for an annual service level (fill rate) of 97%?

- (e) Now suppose inventory holding cost is again 25 cents/dollar/year; however the lead time is not constant but is normally distributed with a mean of 2 weeks and a standard deviation of 1 week and the retailer aims for a fill rate of 97% (all other parameters remain the same as before). What will be the reorder point?

3. $d = 600 / \text{week} \Rightarrow D = 31200 / \text{year}$

$\sigma_d = 150 / \text{week}$

$LT = 4$

$S = \$115.38 / \text{order}$

$H = \$1.25 / \text{unit/year}$

a) $EOQ = \sqrt{\frac{2SD}{H}} = 2340 \text{ units (approx)} \quad [0.5]$

Average no. of orders/year = $\frac{D}{Q} = 13.33 \quad [0.5]$

b) $CSL = 0.97 \Rightarrow z = 1.88$

$\sigma_{LTD} = \sqrt{LT} * \sigma_d = 300.$

$ROP = 4 * 600 + 1.88 * 300 = 2964 \text{ units} \quad [1.5]$

~~ASL = 1 - 0.97 \Rightarrow $1 - \frac{E(z) \sigma_{LTD}}{Q} = 0.97$ \Rightarrow $E(z) = 0.234 \Rightarrow z = 0.36$ \Rightarrow $ROP = 4 * 600 + 0.36 * 300 = 2508 \text{ units}$~~ (*) (see next page)

c) $TC = \frac{SD}{Q} + H(\frac{Q}{2} + SS) + pD \quad [SS = 564]$

$= \$159705.90 / \text{year.}$

[For $ASL = 0.97$, $E(z) = 0.23$
 $\Rightarrow z = 0.36 \Rightarrow SS = 108$]

~~$TC = \frac{SD}{Q} + H(\frac{Q}{2} + SS) + pD = \$157844.04 / \text{year.}$~~

d) ROP for the CSL will remain the same
 $= 2964 \text{ units.} \quad [1]$

$t = 0.2 * 5 = \$1 / \text{unit/year.}$

~~$ASL = 0.9$~~ $EOQ = 2683.23 \text{ units} / \sigma_{LTD} = \sqrt{LT} * \sigma_d$ [LT = 4]

$ASL = 0.97 \Rightarrow E(z) = 0.2683$ ~~212.73~~
= 300

$\Rightarrow z = 0.3$

$ROP \text{ for } ASL = 4 * 600 + 0.3 * 300 = 2490 \text{ units}$
 ~~$= 2208.48 \text{ units}$~~

$$e) \sigma_{LTD} = \sqrt{LT \cdot \sigma_d^2 + d^2 \cdot \sigma_{LT}^2}$$

$$= \cancel{670.82} 636.4 \quad \boxed{1}$$

$$\sigma_d = 150$$

$$\sigma_{LT} = 1$$

$$LT = 2$$

$$d = 600$$

$$EOQ = 2340$$

$$ASL = 0.97 \Rightarrow E(z) = \overset{0.110}{\cancel{0.704}} \Rightarrow z = 0.84$$

$$\Rightarrow ROP = Q \cdot 600 + 0.84 \cdot \overset{636.4}{\cancel{670.82}}$$

$$= \cancel{2990.32} = 1735 \quad \boxed{1}$$

(*) (from previous page) if $ROP = 2964$ units, then $z = 1.88$

$$\text{For } z = 1.88, E(z) = 0.012$$

$$\Rightarrow ASL = 1 - \frac{E(z) \sigma_{LTD}}{Q} = 1 - \frac{0.012 \cdot 300}{2340}$$

$$= 0.9985 \quad \boxed{1.5}$$

Question 8 (4+4+2=10 points)

Six samples have been taken from a machine that is used to fill 2-litre bottles of water, where each sample consists of five different observations. Sample means are given in the table below. It is known from previous experience that the machine's output has a standard deviation of 0.01 litre.

Sample	Mean
1	2.005
2	2.001
3	1.998
4	2.002
5	1.995
6	1.999

- (a) Suppose that the upper control limit has been computed as 2.009 based on the given data. What is the corresponding Type I error (alpha risk)? Is the process in control in this case?
- (b) Suppose that the management desires to reduce their alpha risk to about 2 percent. Based on this decision, calculate the new upper and lower control limits for the data given above. According to the new limits, is the process in control?
- (c) Compare the two limits in (a) and (b) in terms of detecting non-random variations.

$$\text{a) } \mu = 2.0 \text{ litres, } \sigma = .01 \text{ litre, } n = 5 \quad UCL = 2.0 + z \frac{0.01}{\sqrt{5}} = 2.009 \Rightarrow z = 2$$

Using the table, we find that the area under the Normal curve between $z = 0$ and $z = 2$ corresponds to a probability of 0.4772. Hence alpha risk is $= 1 - 2*(0.4772) = 0.0456$, which corresponds to 4.56%.

$$LCL = 2.0 - 2 \frac{0.01}{\sqrt{5}} = 1.991 \quad \text{The limits are } (1.991, 2.009), \text{ therefore the process is in control.}$$

b) An alpha risk of 2% corresponds to an area of $0.02/2 = 0.01$ on each side of the distribution, we are looking for a z^* value which yields a probability of 0.49 under the normal curve from $z = 0$ to z^* (or equivalently a z^* that will yield a probability of 0.99 from $-\infty$ to z^*). Using the table, we find that the closest value is $z^* = 2.33$. Then,

$$UCL = 2.0 + 2.33 \left(\frac{0.01}{\sqrt{5}} \right) = 2.01042 \quad \text{and} \quad LCL = 2.0 - 2.33 \left(\frac{0.01}{\sqrt{5}} \right) = 1.98958 \quad \text{The process is still in control.}$$

c) Wider limits make it more difficult to detect non-random variations, so when the management decides to reduce their alpha risk to 2 percent, they are in fact decreasing the probability of making errors as to concluding that non-randomness is present when only randomness is present (but also increasing the probability of making errors as to concluding that the process is in control when it is not).

Question 8. (8 points)

As a quality analyst, you want to control the quality of a packaging process. You know from past experience that whenever the process is under control, package weight is normally distributed with a standard deviation of 2 ounces. Each day last week, you have randomly selected four packages and weighed each:

<u>Day</u>	<u>Weight (Ounces)</u>			
Monday	23	22	23	24
Tuesday	22	20	20	22
Wednesday	20	19	20	21
Thursday	18	19	20	19
Friday	20	20	20	20

- (a) Based on the samples above, construct 3- σ control limits on the mean weight and range.
- (b) Suppose that after constructing the control limits, you have observed the packaging process for another 3 days and have obtained the following information:

<u>Day</u>	<u>Weight (Ounces)</u>			
Monday	22	21	22	23
Tuesday	23	21	19	21
Wednesday	18	21	23	22

Based on these results can you conclude that the process is in control? *Why or why not?*

- (c) Refer to part (a). What is the probability of a type I error?

8. (a)

<u>Day</u>	<u>\bar{x}</u>	<u>R</u>	
m	23	2	$\bar{\bar{x}} = 20.6$
T	21	2	$\bar{R} = 1.6$
W	20	2	
Th	19	2	
F	20	0	

$$UCL_{\bar{x}} = 20.6 + 3 \left(\frac{2}{\sqrt{4}} \right) = 23.6$$

$$LCL_{\bar{x}} = 20.6 - 3 \left(\frac{2}{\sqrt{4}} \right) = 17.6$$

$$UCL_R = 2.28 (1.6) = 3.648$$

$$LCL_R = 0$$

(b)

<u>Day</u>	<u>\bar{x}</u>	<u>R</u>	
m	22	2	} out of control. (015 control limits)
T	21	4	
W	21	5	

↑
in control
(inside control limits)

$$(c) \quad z(1 - \frac{\alpha}{2}) = 3.00$$

$$1 - \frac{\alpha}{2} = .9987$$

$$\alpha = .0026$$

Question 9 (7 points)

A medical facility does MRIs for sports injuries. Occasionally a test yields inconclusive results and must be repeated (i.e., retested). Using the following 8 sample results of $n = 200$ observations each, calculate the two-sigma control limits for the appropriate control chart. State whether or not the process is in control?

	Sample							
	1	2	3	4	5	6	7	8
Number of retests	1	2	2	0	2	1	2	0

Since the tests can be divided into two groups as those with conclusive results and the remaining with inconclusive results, a p -chart should be used.

$$\bar{p} = \frac{10}{8(200)} = 0.00625$$

Control limits are calculated as follows:

$$\bar{p} \pm 2\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.00625 \pm 2\sqrt{\frac{0.00625(1-0.00625)}{200}}$$

$$UCL = 0.017395$$

$$LCL = -0.0049 \text{ but effective } LCL = 0 \text{ since it can not be negative.}$$

	Sample							
	1	2	3	4	5	6	7	8
Number of retests	1	2	2	0	2	1	2	0
Fraction	0.005	0.01	0.01	0	0.01	0.005	0.01	0

According to a two-sigma limit p -control chart, the process is in control.

Question 7. (8 points)

A large national producer of cookies and baked goods uses a control chart to monitor the number of chocolate chips in its chocolate chip cookies. One cookie is sampled each hour. The results of the last 12 hours were:

<u>Hour</u>	<u>Number of Chips per Cookie</u>	<u>Hour</u>	<u>Number of Chips per Cookie</u>
1	7	7	3
2	4	8	6
3	3	9	3
4	3	10	2
5	5	11	4
6	4	12	4

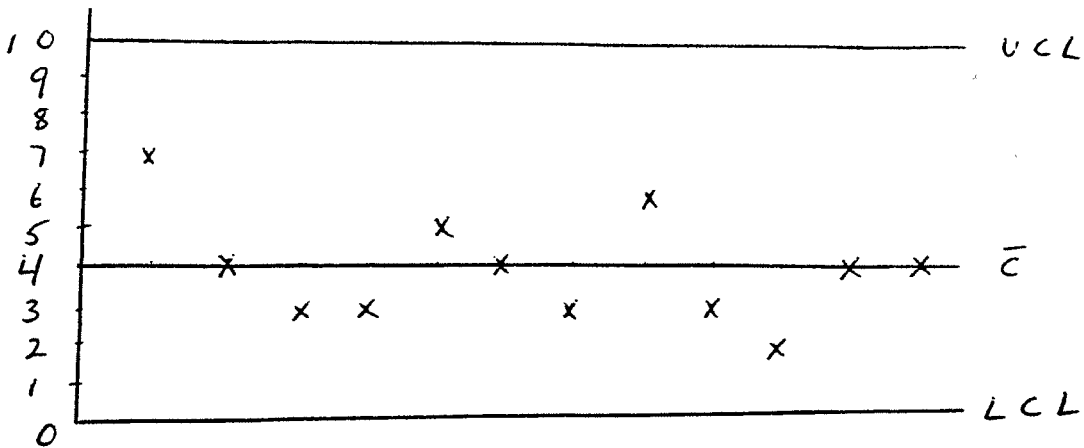
- (a) Construct the appropriate control chart based on the 12 observations above. What is the centerline and what are the upper and lower control limits? Is the process in control?
- (b) The company would like to have an average of six chips per cookie as a target. What are the upper and lower control limits for the appropriate control chart? Would you change your conclusion in part (a)?
- (c) Refer to part (a). What would the control limits be in order to control the probability of a type I error at 3.5%?

$$7. (a) \bar{c} = \frac{\sum \text{chips}}{\sum \text{cookies}} = \frac{48}{12} = 4$$

3

$$UCL = 4 + 3(2) = 10$$

$$LCL = 4 - 3(2) = -2 \Rightarrow 0$$

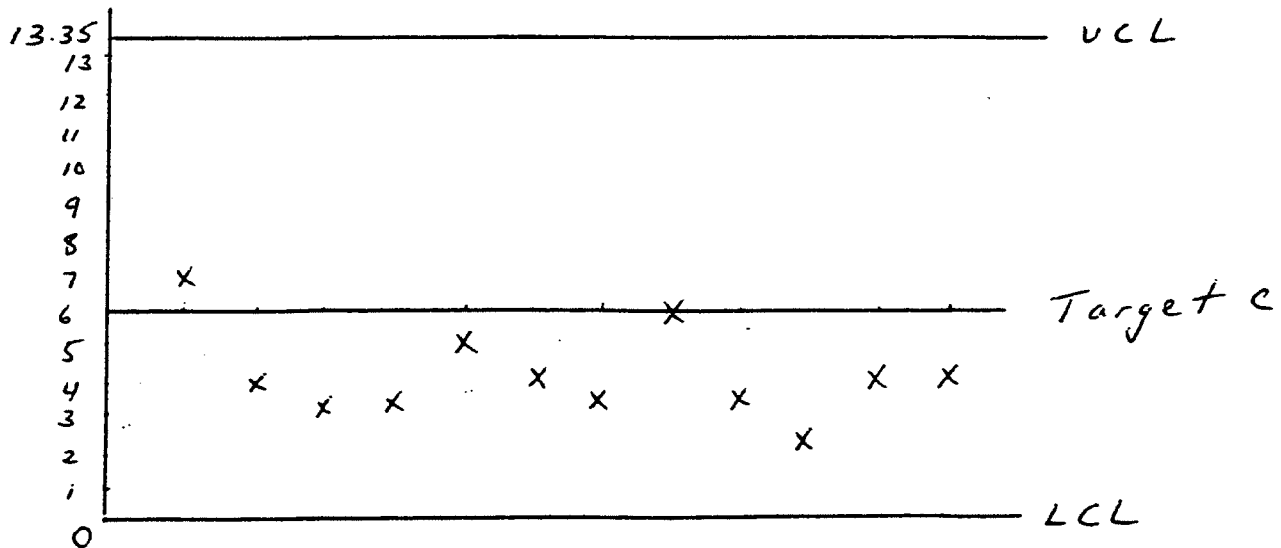


The process is in control.

$$(b) UCL = 6 + 3\sqrt{6} = 13.35$$

$$LCL = 6 - 3\sqrt{6} \Rightarrow 0$$

3



Although the process seems in control, an investigation should be made because most points are on one side.

$$2 (c) 4 \pm 2.11(2) \Rightarrow UCL = 8.22 \quad Z(1 - \frac{0.035}{2}) = 2.11$$

$$LCL = 0 - 8$$