Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Discrete Mathematics for Computing MAT1348A

Practice Exam

15 April 2015

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Instructions:

- This is a three-hour *closed-book* exam; no notes are allowed. Calculators are *not* permitted.
- Completely turn off all electronic devices.
- The exam consists of 21 questions on 17 pages. Page 17 provides additional work space. Do not detach it.
- Questions 1-5 are multiple-choice. You must enter the letter corresponding to each correct answer in the table preceding Question 1. No partial marks will be given for other work.
- Questions 6-9 are true/false. You must circle the correct response. You need not justify your answers.
- Questions 10-17 are short-answer. Write the final answer in the appropriate answer box, and briefly justify your answer where required.
- Questions 18-21 are long-answer. You must clearly show all relevant steps in your solution to receive full marks. Clearly indicate the final answer.
- Be sure to read carefully and follow the instructions for the individual problems.
- For rough work, you may use the back pages. Do not use scrap paper of your own.
- Use proper mathematical notation and terminology.
- If you require clarification, raise your hand.
- Good luck!

Seat number:________

Last name:______________________________

First name:______________________________

Student number:__________________________

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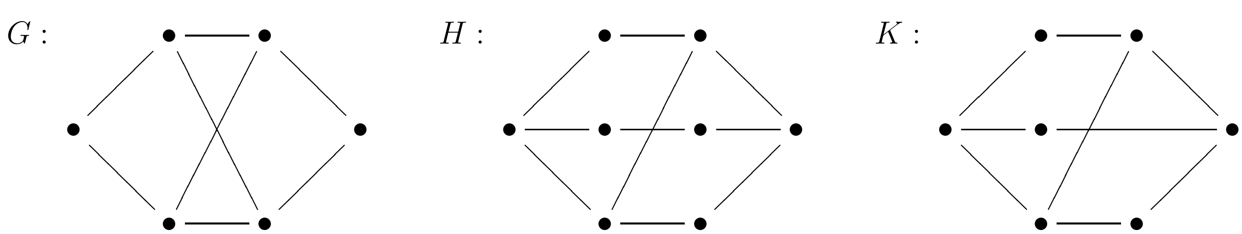
Questions 1–5 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4	5
Answer					

1. How many binary strings of length 10 start with 01 and contain at most four 0s?

   A. 12 **B.** 176 **C.** 93 **D.** 255 **E.** 32 **F.** 194
   G. None of the above.

[2pts] 2. Which of the following graphs are bipartite?



A. Only G **B.** Only H **C.** Only K **D.** Only G and K
E. Only G and H **F.** All of them. **G.** None of them.

3. Let $A = \{0, 1, 2, 3, 4\}$ and $B = \{0, 2, 4, 6, 8\}$. The number of onto (surjective) functions from A to B is:

   A. 1 **B.** 5 **C.** 5! **D.** 2^5 **E.** 5^5 **F.** 5^2

   G. None of the above.

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4. Let A, B, C be arbitrary subsets of the universal set $\mathcal{U}$. Which of the following statements is necessarily true?

   A. $A \cap B \subseteq A \cap C$ implies $B \subseteq C$

   B. $A \cup B \cup C = \mathcal{U}$ implies $(A \cup B) - (A \cup C) = \emptyset$

   C. $A \subseteq B \cup C$ implies $A - B \subseteq C - B$.

   D. $A \cap B \subseteq C$ implies $(C - A) \cap (C - B) = \emptyset$.

   E. None of the above.

5. From an urn containing balls numbered 1–10, we randomly draw three different balls, and record the sum of the three numbers. What is the smallest number of times we need to repeat this procedure to guarantee that the same sum shows at least twice?

   A. 22 **B.** 23 **C.** 27 **D.** 28 **E.** 120 **F.** 121

   G. None of the above.

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6. Which of the following is logically equivalent to $(\neg b \wedge a) \vee (\neg a \wedge c) \vee (\neg b \wedge c)$?

   A. $\neg(a \wedge c) \leftrightarrow (c \vee b)$ **B.** $\neg(a \wedge \neg b) \rightarrow (\neg a \wedge \neg c)$ **C.** $(a \wedge b) \leftrightarrow (c \vee \neg a)$
   D. $\neg(a \wedge b) \leftrightarrow (a \vee c)$ **E.** $\neg(c \wedge a) \rightarrow (a \wedge b)$

True/false questions — circle T (true) or F (false). You need not justify your answers.

7. For each of the statements below, determine whether it is true or false. *Circle each correct answer.*

For any set A, $A \in \mathcal{P}(A)$	T	F
For all sets A and B, $\mathcal{P}(B) \subseteq \mathcal{P}(A)$ implies $B \subseteq A$	T	F
For all sets A and B, $A \cup B \in \mathcal{P}(B)$ implies $A \subseteq B$	T	F
$\{\emptyset, \{\emptyset\}\} \subseteq \mathcal{P}(\emptyset)$	T	F
The relation $\{(n, n+k) \mid n \in \mathbb{N}, k \in \mathbb{N}^+\}$ is transitive	T	F
There exists a relation on $\mathbb{Z}$ that is both symmetric and anti-symmetric.	T	F

8. Consider the following function:

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}^+; \quad f(x, y) = (xy + x, 2^y)$$

Which of the following statements about f are true? *Circle each correct answer below.*

f is one-to-one (injective).	T	F
f is onto (surjective).	T	F
f is bijective.	T	F

9. For each of the statements below, determine whether it is true or false. *Circle each correct answer below.*

Every proposition is logically equivalent to a proposition containing only the connectives $\neg$ and $\to$.	T	F
If the complete truth tree of a proposition P has no inactive paths, then P is a tautology.	T	F
If the set of premises of an argument is inconsistent, then the argument is valid.	T	F
For any two functions $f, g : \mathbb{N} \to \mathbb{N}$, if f and g are bijective, then so is $g \circ f$.	T	F
For any two finite sets A, B, if $\|A\| < \|B\|$, then every function $f : B \to A$ is onto but not one-to-one.	T	F

Short-answer questions — write your final answer in the answer box. Wherever indicated, you must briefly justify your answers to receive full marks.

10. In the following question, you do not have to justify your answers.

(a) Give a precise definition of a *transitive* relation.

Answer:

(b) Give a precise definition of a *symmetric* relation.

Answer:

(c) Give an example of a relation on the set $A = \{0, 1, 2, 3\}$ which is symmetric but not transitive.

Answer:

(d) Give an example of a relation on the set $B = \{0, 1\}$ which is symmetric but not transitive.

Answer:

11. From a group of 12 men and 15 women, a committee consisting of 6 people is chosen. In how many ways is this possible if the committee must contain at least one, but no more than four women? *Your answer may include unevaluated factorials, binomial coefficients, powers, products, or sums.*

Answer:

Justification:

12. Find a proposition in DNF equivalent to $(a \to b) \leftrightarrow (c \wedge \neg a)$.

Answer:

Justification:

13. How many integers between 200 and 1000 (inclusive) are divisible by 8 or by 12?

Answer:

Justification:

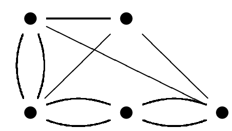
14. Give an example (draw a figure) of simple graphs G and H such that all of the following conditions are met:

- G and H are not isomorphic
- G and H both have 6 vertices
- G and H both have 6 edges
- G and H have the same degree sequence

Answer:

(No justification is needed.)

15. Does the following graph admit an Euler trail?



Answer: YES NO

Justification:

16. Determine the coefficient of x^{11} in the expansion of $(2x^2 - \frac{3}{x})^{28}$. *Your answer may include unevaluated factorials, binomial coefficients, powers, products, or sums.*

Answer:

Justification:

17. Consider the following argument:

    French fries are healthy, unless you put mayonnaise on them.
    French fries are tasty only if you put mayonnaise on them.
    Therefore, for French fries to be tasty it is necessary that they be unhealthy.

    (a) Translate this argument into propositional logic. Clearly define the propositional variables you use.

    (b) Use a truth tree to determine the validity of the argument.

18. Use **Mathematical Induction** to prove that for all integers $n \geq 1$,

$$1 + \frac{1}{2} + \cdots + \frac{1}{n} \leq \frac{n}{2} + 1.$$

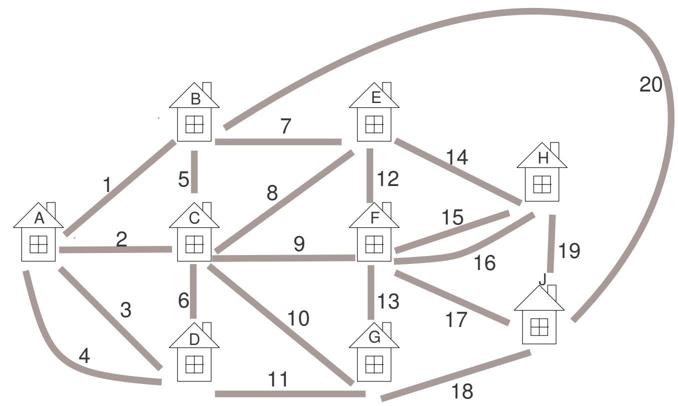
19. Define a binary relation $\mathcal{R}$ on the set $\mathbb{Z}$ as follows:

$$x\mathcal{R}y \qquad \text{if and only if} \quad x + y = 2k \text{ for some integer } k.$$

(a) Prove that $\mathcal{R}$ is an equivalence relation.

(b) Describe the equivalence classes of $\mathcal{R}$. How many distinct equivalence classes are there?

20. Consider the following village (the grey lines indicate roads between houses).



(a) Is it possible to take a walk through the village in such a way that you use every road exactly once? Cite appropriate theorems from graph theory to support your answer.

(b) Is it possible for such a walk to start and end at the same house?

(c) Suppose road number 17 is closed. Can we take a walk using all the remaining roads exactly once?

21. Let $f, g : \mathbb{Z} \to \mathbb{Z}$ be arbitrary functions.

    (i) Suppose that $g \circ f$ is one-to-one (injective). Is g then necessarily one-to-one as well? Prove or give a counterexample.

    (ii) For any subset $S \subseteq \mathbb{Z}$, denote as usual $f(S) = \{f(x) : x \in S\}$.

    Prove that for any $A, B \subseteq \mathbb{Z}$, we have

$$f(A \cup B) = f(A) \cup f(B).$$

Additional work space. Do not detach this page.