CVG2140 – Solution to Assignment No. 8 (Combined Stresses)

Problem 1. Determine the principal stresses and their directions at three surface locations, *A*, *B*, and *C*, of the brass cantilever beam with rectangular cross-section shown in Fig. 1. The beam is subjected to its own weight as well as a concentrated load of 12 kN at its free end. The material properties of brass are as follows: density ρ = 8,500 kg/m^3, Young's modulus *E* = 100 GPa, and yield stress σ_y = 450 MPa.

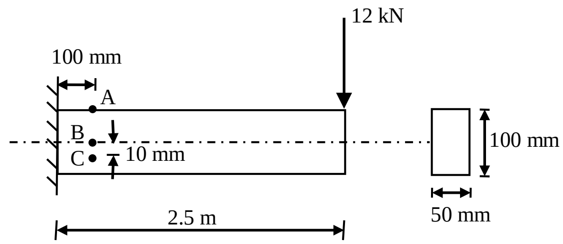
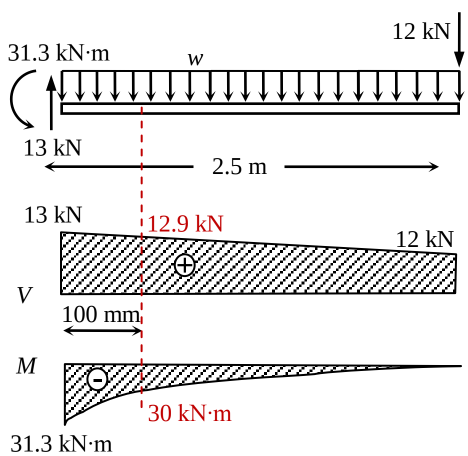


Fig. 1

The weight per linear meter of the cantilever beam is:

$$w = \rho g A = 8500 \times 9.81 \times 0.1 \times 0.05 = 0.417 \text{ kN/m}$$



At the three locations:

$$\sigma_x = -\frac{My}{I} = -\frac{12My}{bh^3}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{VQ}{Ib} = \frac{12VQ}{b^2h^3}$$

	Q (mm^3)	σ_x (MPa)	τ_{xy} (MPa)	σ_1 (MPa)	θ_{p1}	σ_2 (MPa)	θ_{p2}
A	0	360	0	360	0°	0	90°
B	62.5×10^3	0	-3.87	3.87	45°	-3.87	135°
C	60×10^3	-72	-3.71	0.19	92.9°	-72.19	2.9°

Problem 2. Three loads are applied to the short rectangular post shown in Fig. 2(a). The cross-sectional dimensions of the post are shown in Fig. 2(b). Determine:

(a) The normal and shear stresses and points *H* and *K*;

(b) The principal stresses and maximum in-plane shear stress at point *H*, and show the orientation of theses stresses in an appropriate sketch.

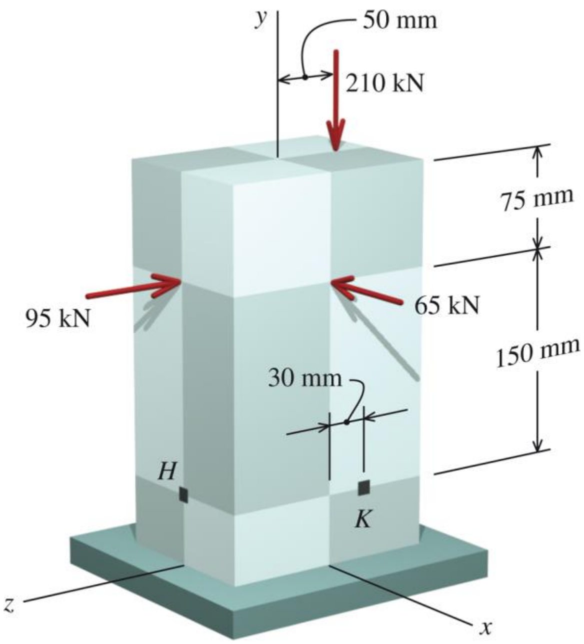


Fig. 2(a)

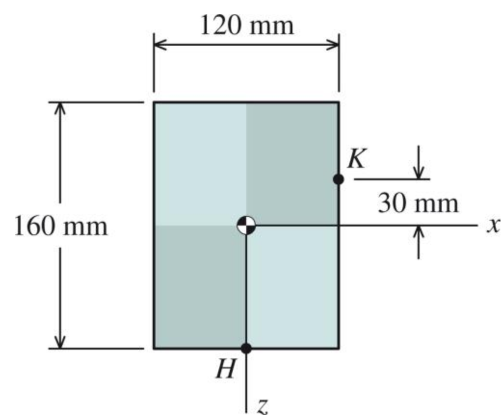


Fig. 2(b)

Section properties:

$$A = 120 \times 160 = 19,200 \text{ mm}^2$$

$$I_x = \frac{120 \times 160^3}{12} = 40.96 \times 10^6 \text{ mm}^4$$

$$I_z = \frac{160 \times 120^3}{12} = 23.04 \times 10^6 \text{ mm}^4$$

Equivalent forces at the plane containing *H* and *K*:

$$N_y = 210 \text{ kN}$$

$$V_x = -65 \text{ kN}$$

$$V_z = -95 \text{ kN}$$

$$M_x = -(210)(0.05) - (95)(0.15) = -24.75 \text{ kN}\cdot\text{m}$$

$$M_z = (65)(0.15) = 9.75 \text{ kN}\cdot\text{m}$$

Axial stress at *H* and *K* due to N_y:

$$\sigma_y = \frac{210 \times 10^3}{19,200} = 10.94 \text{ MPa (compression)}$$

Shear stress at *H* and K due to V_x:

$$\tau_{xy}^{H}=\frac{\left(65\times10^{3}\right)\times\left(160\times60\times30\right)}{\left(23.04\times10^{6}\right)\times\left(160\right)}=5.08\ \text{MPa}$$

$$\tau_{xy}^{K}=0\ \text{MPa}$$

Shear stress at *H* and K due to V_z:

$$\tau_{yz}^{H}=0\ \text{MPa}$$

$$\tau_{yz}^{K}=\frac{\left(95\times10^{3}\right)\times\left(120\times50\times55\right)}{\left(40.96\times10^{6}\right)\times\left(120\right)}=6.38\ \text{MPa}$$

Axial stress at *H* and *K* due to M_x:

$$\sigma_{y}^{H}=\frac{24.75\times10^{6}\times80}{40.96\times10^{6}}=48.34\ \text{MPa (tension)}$$

$$\sigma_{y}^{K}=\frac{24.75\times10^{6}\times30}{40.96\times10^{6}}=18.13\ \text{MPa (compression)}$$

Axial stress at *H* and *K* due to M_z:

$$\sigma_{y}^{H}=\frac{9.75\times10^{6}\times0}{23.04\times10^{6}}=0\ \text{MPa}$$

$$\sigma_{y}^{K}=\frac{9.75\times10^{6}\times60}{23.04\times10^{6}}=25.39\ \text{MPa (tension)}$$

(a) Summary of stresses at *H*:

$$\sigma_{x}^{H}=0\ \text{MPa}$$

$$\sigma_{y}^{H}=-10.94+48.34+0=37.4\ \text{MPa (tension)}$$

$$\tau_{xy}^{H}=5.08\ \text{MPa}$$

Summary of stresses at *K*:

$$\sigma_{z}^{K}=0\ \text{MPa}$$

$$\sigma_{y}^{K}=-10.94-18.13+25.39=-3.68\ \text{MPa (compression)}$$

$$\tau_{yz}^{K}=6.38\ \text{MPa}$$

(b) Principal stresses and maximum in-plane shear stress at *H*:

$$\sigma_{1,2}^{H} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{0+37.4}{2} \pm \sqrt{\left(\frac{0-37.4}{2}\right)^2 + 5.08^2}$$
$$= 18.7 \pm 19.4 \text{ MPa}$$

Therefore, σ_1 = 38.1 MPa and σ_2 = -0.7 MPa

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} = \frac{-2\times 5.08}{(0-37.4)} = 0.27166$$

θ_p = 7.6° (counterclockwise from the *x*-axis)

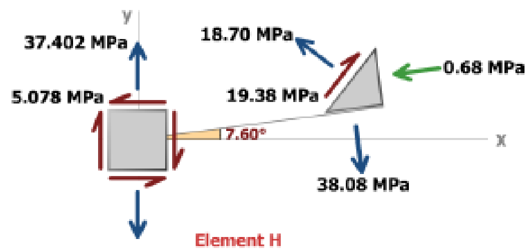
By substituting this value into the transformation equation:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{0+37.4}{2} + \frac{0-37.4}{2}\cos 15.2^{\circ} + (-5.08)\sin 15.2^{\circ}$$
$$= -0.7 \text{ MPa}$$

Therefore, σ_1 = 38.1 MPa is acting at 7.6°+90°=97.6° (CCW) and σ_2 = -0.7 MPa is acting at 7.6° (CCW)

$$\tau_{\max}^{H} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.4 \text{ MPa (maximum in-plane shear stress)}$$

$$\sigma_{avg}^{H} = \frac{\sigma_x + \sigma_y}{2} = \frac{0+37.4}{2} = 18.7 \text{ MPa}$$ (normal stress on planes of maximum in-plane shear stress)



Problem 3. The state of stress at a point is shown in Fig. 3. By using Mohr's circle, determine:

a) The principal stresses and their orientation;

b) The maximum in-plane shear stress and average normal stress, specifying the corresponding orientation; and,

c) The equivalent state of stress if the coordinate system is oriented 30º clockwise from the element shown in Fig. 3.

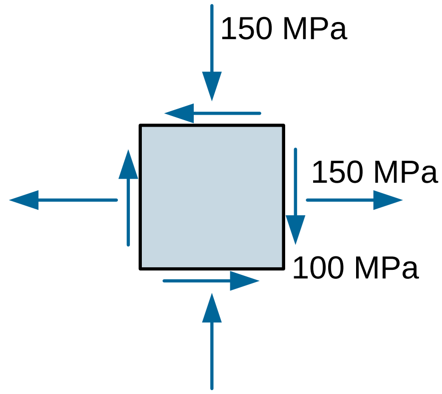


Fig. 3

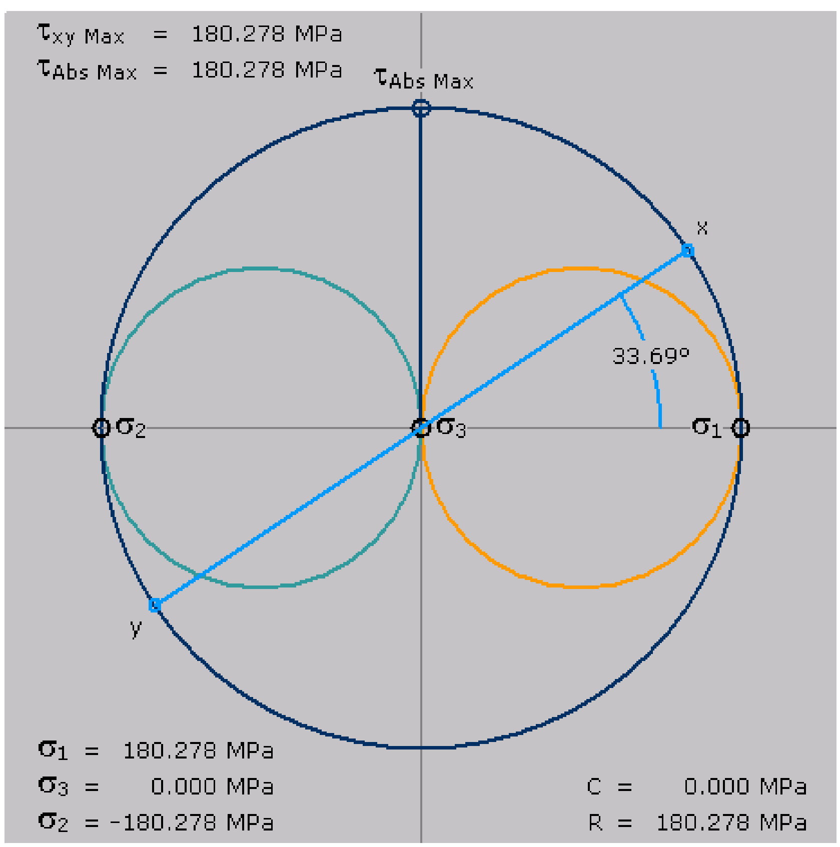
From Fig. 3:

$\sigma_x = 150$ MPa, $\sigma_y = -150$ MPa, $\tau_{xy} = -100$ MPa

Therefore,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 - 150}{2} = 0 \text{ MPa}$$

$$R = \tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{150 + 150}{2}\right)^2 + 100^2} = 180.28 \text{ MPa}$$

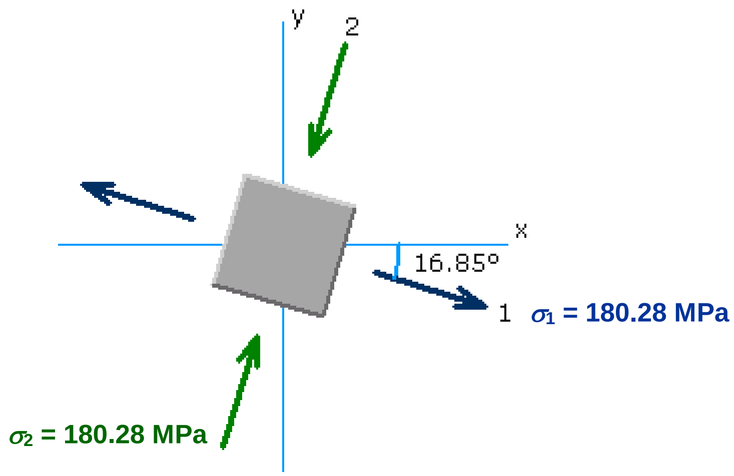


a) $2\theta_p = \tan^{-1}\left(\frac{100}{150}\right) = 33.69^{\circ}$

θ_p = -33.69°/2 = -16.85° (negative, because the rotation from x to σ_1 is clockwise)

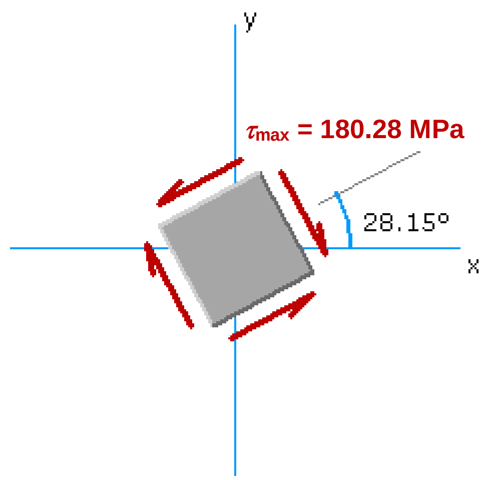
σ_1 = 0 + 180.28 = 180.28 MPa acting at -16.85°

σ_2 = 0 – 180.28 = -180.28 MPa acting at -16.85°+90°=73.15°

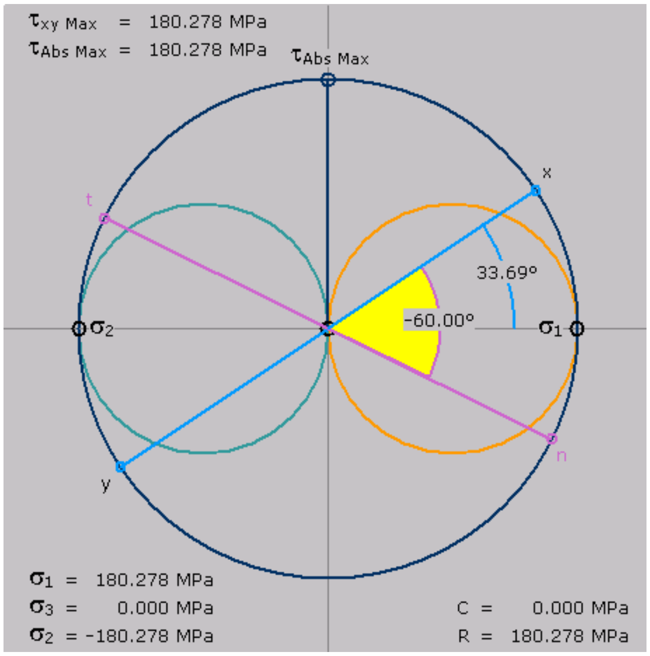


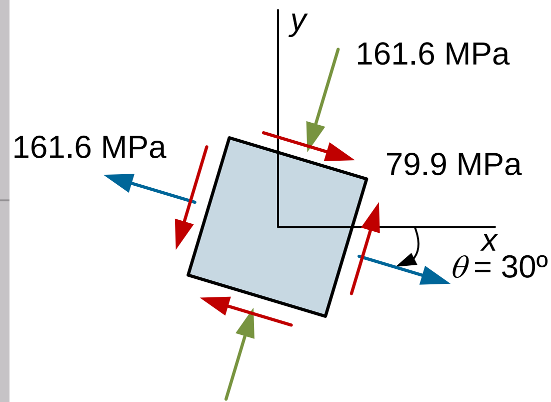
b) τ_{max} = R = 180.28 MPa acting at -16.85°+ 45° = 28.15°

σ_{avg} = C = 0 MPa



c)





$\sigma_{x'}$ = R×cos(60°-33.69°) = 180.28×cos(26.31°)= 161.6 MPa

$\sigma_{y'}$ = - R×cos(60°-33.69°) = -161.6 MPa

$\tau_{x'y'}$ = R×sin(60°-33.69°) = 180.28×sin(26.31°) = 79.9 MPa