CVG2140 – Solution to Assignment No. 7 (Shear & Torsion)

Problem 1:

If the beam in Fig. 1 is subjected to a shear of *V* = 15 kN, determine: (a) the web's shear stress at *A* and *B*, and (b) the location and magnitude of the maximum shear stress in the cross section.

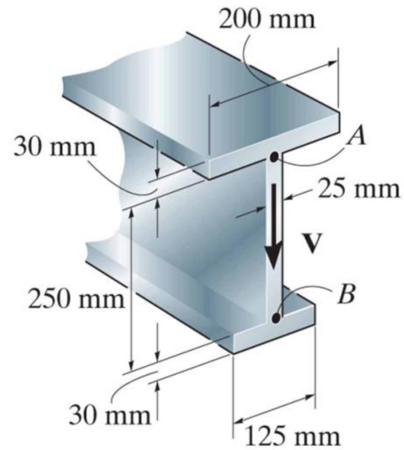


Fig. 1

The location of the neutral axis is given by:

$$\bar{y} = \frac{(15\times125\times30)+(155\times25\times250)+(295\times200\times30)}{(125\times30)+(25\times250)+(200\times30)} = 174.7 \text{ mm from the bottom}$$

The moment of inertia with respect to the neutral axis (i.e., bending axis) is given by:

$$I = \frac{125\times30^3}{12} + (125\times30\times159.7^2)$$
$$+\frac{25\times250^3}{12} + (25\times250\times19.7^2)$$
$$+\frac{200\times30^3}{12} + (200\times30\times120.3^2)$$
$$= 218.2\times10^6 \text{ mm}^4$$

The web's shear stress at A is given by:

$$Q_A = 200\times30\times(295-174.7) = 721.8\times10^3 \text{ mm}^3$$

$$\tau_A = \frac{VQ_A}{I_{NA}b} = \frac{15\times10^3\times721.8\times10^3}{218.2\times10^6\times25} = \underline{1.98 \text{ MPa}}$$

The web's shear stress at B is given by:

$$Q_B = 125\times30\times(174.7-15) = 598.9\times10^3 \text{ mm}^3$$

$$\tau_B = \frac{VQ_B}{I_{NA}b} = \frac{15\times10^3\times598.9\times10^3}{218.2\times10^6\times25} = \underline{1.65 \text{ MPa}}$$

The maximum shear stress, which occurs at the N.A., is given by:

$$Q_{\max} = \left[200\times30\times(295-174.7)\right] + \left[25\times105.3\times52.65\right] = 860.4\times10^3 \text{ mm}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{I_{NA}b} = \frac{15\times10^3\times860.4\times10^3}{218.2\times10^6\times25} = \underline{2.36 \text{ MPa}}$$

Problem 2:

Determine the absolute maximum shear stress in the I-beam shown in Fig. 2, as well as the maximum shear stress in the cross section that passes through points *C* and *D*.

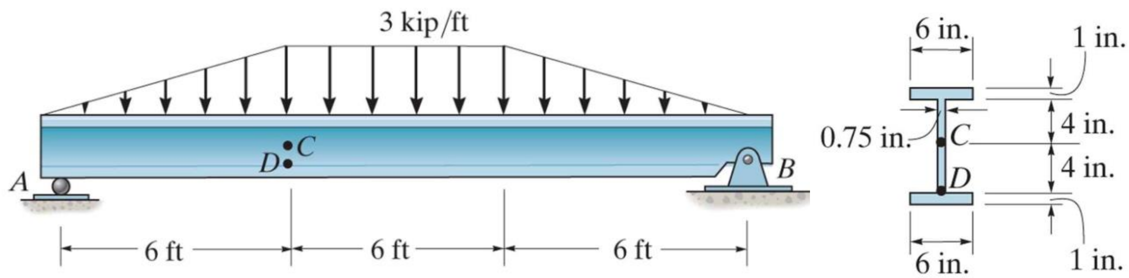
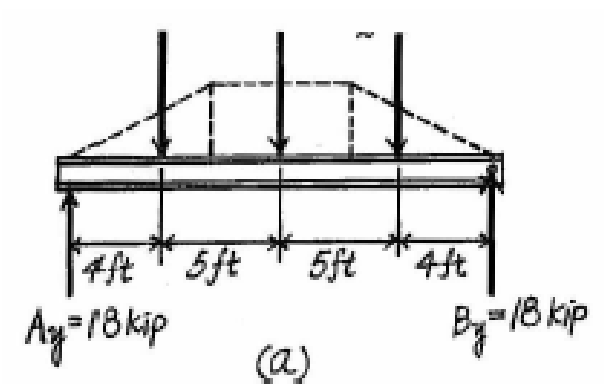
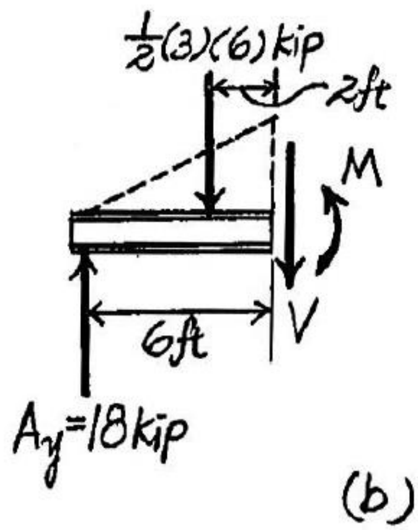


Fig. 2





The shear force acting at the section where *C* and *D* are located can be determined from Fig. (b):

$$\sum F_y = 18 - \frac{1}{2}(3)(6) - V = 0 \quad \Rightarrow \quad V = 9 \text{ kip}$$

The moment of inertia of the beam's cross section about the neutral axis is:

$$I = \frac{6 \times 10^3}{12} - \frac{5.25 \times 8^3}{12} = 276 \text{ in}^4$$

The first moments of area Q_C and Q_D can be calculated with reference to Fig. (c):

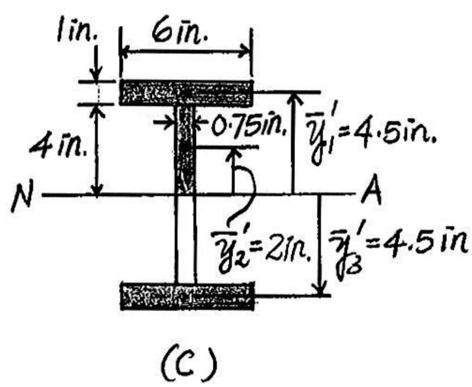
$$Q_C = (6 \times 1 \times 4.5) + (0.75 \times 4 \times 2) = 33 \text{ in}^3$$

$$Q_D = 6 \times 1 \times 4.5 = 27 \text{ in}^3$$

The shear stresses at *C* and *D* are respectively:

$$\tau_C = \frac{VQ_C}{Ib} = \frac{9 \times 33}{276 \times 0.75} = \underline{1.43 \text{ ksi}}$$

$$\tau_D = \frac{VQ_D}{Ib} = \frac{9 \times 27}{276 \times 0.75} = \underline{1.17 \text{ ksi}}$$



Problem 3:

The solid shaft in Fig. 3 has a diameter of 40 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum. Determine also the angle of twist between points *A* and *E*. ($L_{AB} = L_{CD}$ = 1 m, L_{BC} = 0.5 m, L_{DE} = 0.4 m, G = 75 GPa)

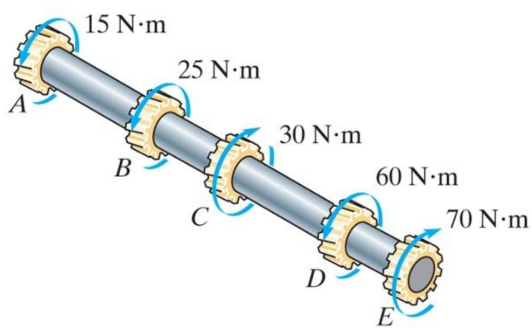
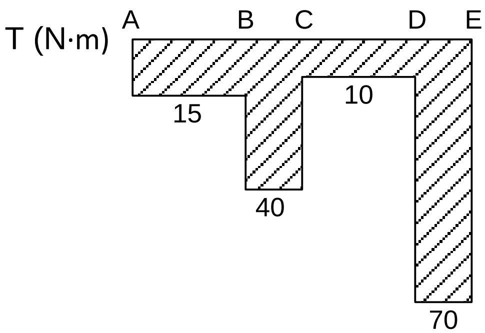


Fig. 3

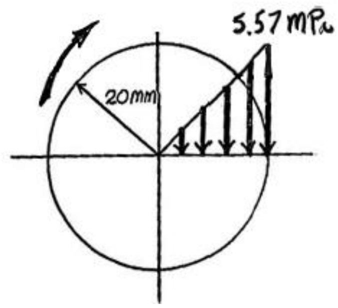
The internal torques developed in each segment of the shaft are shown in the following torque diagram:



Since segment DE is subjected to the greatest torque, the absolute maximum shear stress occurs here. Therefore,

$$\tau_{\max} = \frac{T_{DE} r}{J} = \frac{70 \times 0.02}{\frac{\pi}{32} 0.04^4} = \underline{5.57 \text{ MPa}}$$

The shear stress distribution along the radial line is shown in the following figure:



The angle of twist between points *A* and *E* is given by:

$$\phi_{A/E} = \frac{1}{GJ}\left(T_{AB}L_{AB} + T_{BC}L_{BC} + T_{CD}L_{CD} + T_{DE}L_{DE}\right)$$

$$= \frac{1}{75 \times 10^9 \times \left(\frac{\pi}{32} 0.04^4\right)}\left[(15 \times 1) + (40 \times 05) + (10 \times 1) + (70 \times 04)\right]$$

$$= 0.00387 \text{ rad} = 0.222^\circ$$

Problem 4:

The steel shaft in Fig. 4 is made from two segments: *AC* has a diameter of 0.5 in, and *CB* has a diameter of 1 in. If it is fixed at its ends *A* and *B* and subjected to a torque of 500 lb·ft at *D*, determine the maximum shear stress in the shaft. $G_{st} = 10.8\times10^3$ ksi.

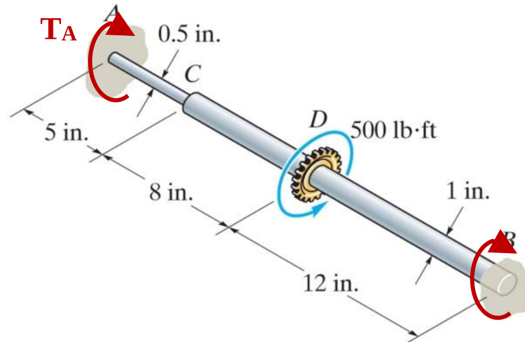


Fig. 4

Equilibrium:

$$T_A + T_B - 500 = 0 \qquad (1)$$

Compatibility condition:

$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A \times 5}{G\times\frac{\pi}{32}0.5^4} + \frac{T_A\times 8}{G\times\frac{\pi}{32}1^4} = \frac{T_B\times 12}{G\times\frac{\pi}{32}1^4} \qquad (2)$$

$$88T_A = 12T_B$$

Solving equations (1) and (2) yields:

T_A = 60 lb·ft, T_B = 440 lb·ft

The shear stress in each segment is:

$$\tau_{AC}^{\max} = \frac{T_A r_{AC}}{J_{AC}} = \frac{60\times12\times0.25}{\frac{\pi}{32}0.5^4} = 29.3 \text{ ksi}$$

$$\tau_{CD}^{\max} = \frac{T_B r_{DB}}{J_{DB}} = \frac{440\times12\times0.5}{\frac{\pi}{32}1^4} = 26.9 \text{ ksi}$$

Therefore, the maximum shear stress in the shaft occurs in segment *AC*, and its value is 29.3 ksi.