<u>**CVG2140 – Solution to Assignment No. 6 (Deflections)**</u>

<u>Problem 1.</u> For the beam and loading shown in Fig. 1, determine:the equation of the elastic curve (deflection curve) for segment *BC* of the beam;

(b) the deflection midway between *B* and *C*; and,

(c) the slope at *C*.

Assume *EI* is constant for the beam.

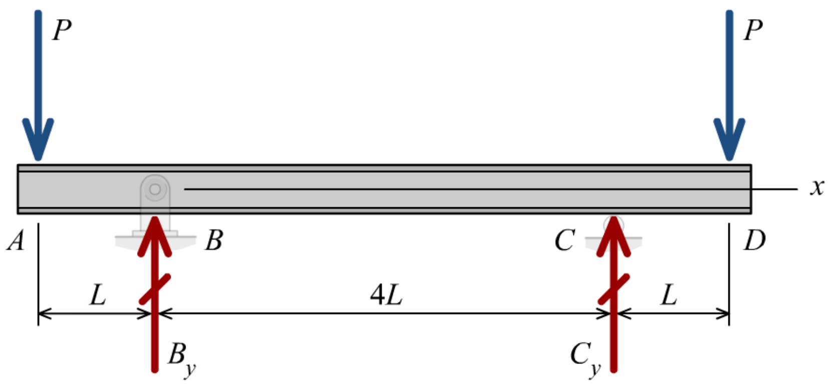

Fig. 1

Establishing equilibrium:

$$\left.\begin{aligned}\sum F_y &= B_y - 2P + C_y = 0\\ \sum M_B &= (P\times L) - (P\times 5L) + (B_y \times 4L) = 0\end{aligned}\right\} \Rightarrow \begin{cases} B_y = P \\ C_y = P\end{cases}$$

Using discontinuity functions, the moment equation is written as:

$$M(x) = -P\langle x-0\rangle + P\langle x-L\rangle + P\langle x-5L\rangle$$

$$EI\frac{d^2v}{dx^2} = M(x) = -Px + P\langle x-L\rangle + P\langle x-5L\rangle$$

$$EI\frac{dv}{dx} = -\frac{P}{2}x^2 + \frac{P}{2}\langle x-L\rangle^2 + \frac{P}{2}\langle x-5L\rangle^2 + C_1 \qquad \text{(Eq. 1)}$$

$$EIv = -\frac{P}{6}x^3 + \frac{P}{6}\langle x-L\rangle^3 + \frac{P}{6}\langle x-5L\rangle^3 + C_1x + C_2 \qquad \text{(Eq. 2)}$$

The boundary conditions of the problem are:

$$x = L \quad \Rightarrow \quad v = 0$$

$$x = 5L \quad \Rightarrow \quad v = 0$$

The constants of integration C_1 and C_2 are then determined by applying the boundary conditions to Eq. 2:

$$x = L \quad \Rightarrow \quad EIv = 0 = -\frac{P}{6}L^3 + C_1L + C_2$$

$$x = 5L \quad \Rightarrow \quad EIv = 0 = -\frac{125P}{6}L^3 + \frac{64P}{6}L^3 + 5C_1L + C_2$$

From the two equations above, $C_1 = 5PL^2/2$ and $C_2 = -7PL^3/3$.

The equations for the slope and deflection are therefore given by:

$$EI\frac{dv}{dx} = -\frac{P}{2}x^2 + \frac{P}{2}\langle x-L\rangle^2 + \frac{P}{2}\langle x-5L\rangle^2 + \frac{5PL^2}{2} \qquad \text{(Eq. 3)}$$

$$EIv = -\frac{P}{6}x^3 + \frac{P}{6}\langle x-L\rangle^3 + \frac{P}{6}\langle x-5L\rangle^3 + \frac{5PL^2}{2}x - \frac{7PL^3}{3} \qquad \text{(Eq. 4)}$$

(a) The equation of the elastic curve (deflection curve) for segment *BC* (L ≤ *x* ≤ 5L) of the beam is given by:

$$EIv = -\frac{P}{6}x^3 + \frac{P}{6}\langle x-L\rangle^3 + \frac{5PL^2}{2}x - \frac{7PL^3}{3}$$

(b) The deflection at midspan (*x* = 3L) is given by:

$$v_{x=3L} = \frac{1}{EI}\left[-\frac{27P}{6}L^3 + \frac{8P}{6}L^3 + \frac{15P}{2}L^3 - \frac{7PL^3}{3}\right] = \frac{2PL^3}{EI}$$

(c) The slope at *C* (*x* = 5L) is given by:

$$\theta_{x=5L} = \left(\frac{dv}{dx}\right)_{x=5L} = \frac{1}{EI}\left[-\frac{25P}{2}L^2 + \frac{16P}{2}L^2 + \frac{5PL^2}{2}\right] = -\frac{2PL^2}{EI} \text{ (rotation is clockwise)}$$

Problem 2. For the cantilever steel beam (*E* = 200 GPa; *I* = 129×10⁶ mm⁴) shown in Fig. 2, determine the deflection at *A*. Assume that *L* = 2.5 m, *P* = 50 kN, and *w* = 30 kN/m.

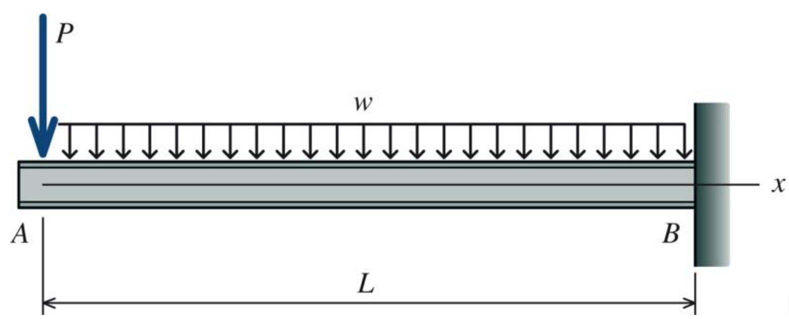

Fig. 2

Establishing equilibrium:

$$\left.\begin{aligned}\sum F_y &= B_y - P - (w\times L) = 0\\ \sum M_B &= (P\times L) + \left(w\times L\times\frac{L}{2}\right) + M_B = 0\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}B_y &= P + wL\\ M_B &= -PL - \frac{wL^2}{2}\end{aligned}\right.$$

Using discontinuity functions, the moment equation is written as:

$$M(x) = -P\langle x-0\rangle - \frac{w}{2}\langle x-0\rangle^2 = -Px - \frac{wx^2}{2}$$

$$EI\frac{d^2v}{dx^2} = M(x) = -Px - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = -\frac{P}{2}x^2 - \frac{wx^3}{6} + C_1 \qquad \text{(Eq. 1)}$$

$$EIv = -\frac{P}{6}x^3 - \frac{wx^4}{24} + C_1x + C_2 \qquad \text{(Eq. 2)}$$

The boundary conditions of the problem are:

$$x = L \quad \Rightarrow \quad \theta = 0$$

$$x = L \quad \Rightarrow \quad v = 0$$

The constants of integration C_1 and C_2 are then determined by applying the boundary conditions to Eqs. 1 and 2:

$$x = L \quad \Rightarrow \quad EI\theta = 0 = -\frac{P}{2}L^2 - \frac{wL^3}{6} + C_1 \quad \Rightarrow \quad C_1 = \frac{P}{2}L^2 + \frac{wL^3}{6}$$

$$x = L \quad \Rightarrow \quad EIv = 0 = -\frac{P}{6}L^3 - \frac{wL^4}{24} + \frac{P}{2}L^3 + \frac{wL^4}{6} + C_2 \quad \Rightarrow \quad C_2 = -\frac{P}{3}L^3 - \frac{wL^4}{8}$$

The equations for the slope and deflection are therefore given by:

$$EI\frac{dv}{dx} = -\frac{P}{2}x^2 - \frac{wx^3}{6} + \frac{P}{2}L^2 + \frac{wL^3}{6} \qquad \text{(Eq. 3)}$$

$$EIv = -\frac{P}{6}x^3 - \frac{wx^4}{24} + \frac{P}{2}L^2x + \frac{wL^3}{6}x - \frac{P}{3}L^3 - \frac{wL^4}{8} \qquad \text{(Eq. 4)}$$

The deflection at A (x = 0) is given by:

$$v_{x=0} = \frac{1}{EI}\left[-\frac{P}{3}L^3 - \frac{wL^4}{8}\right]$$

Substituting for the corresponding values in the above equation results in:

$$v_{x=0} = \frac{1}{200\times10^3\times129\times10^6}\left[-\frac{50\times10^3\times2{,}500^3}{3} - \frac{30\times2{,}500^4}{8}\right] = \underline{-15.8\text{ mm}} \text{ (downwards)}$$

Problem 3. The simply supported beam shown in Fig. 3 consists of a W200×59 structural steel wide-flange shape (E = 200 GPa; I = 60.8×10^6 mm^4). For the loading shown, determine the n along the beam's span.

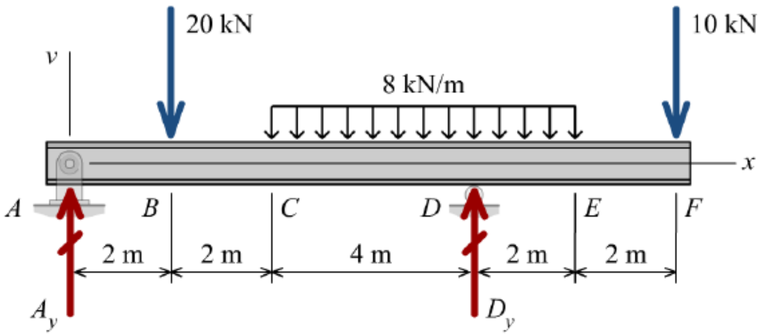

Fig. 3

Establishing equilibrium:

$$\left.\begin{array}{l}\sum F_y = A_y - 20 - (8\times6) + D_y - 10 = 0 \\ \sum M_A = -(20\times2) - (8\times6\times7) + (D_y\times8) - (10\times12) = 0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}A_y = 16\text{ kN} \\ D_y = 62\text{ kN}\end{array}\right.$$

Using discontinuity functions, the moment equation is written as:

$$M(x) = 16\langle x-0\rangle - 20\langle x-2\rangle - \frac{8}{2}\langle x-4\rangle^2 + \frac{8}{2}\langle x-10\rangle^2 + 62\langle x-8\rangle$$

$$= 16x - 20\langle x-2\rangle - \frac{8}{2}\langle x-4\rangle^2 + 62\langle x-8\rangle + \frac{8}{2}\langle x-10\rangle^2$$

$$EI\frac{d^2v}{dx^2} = M(x) = 16x - 20\langle x-2\rangle - \frac{8}{2}\langle x-4\rangle^2 + 62\langle x-8\rangle + \frac{8}{2}\langle x-10\rangle^2$$

$$EI\frac{dv}{dx} = 8x^2 - 10\langle x-2\rangle^2 - \frac{4}{3}\langle x-4\rangle^3 + 31\langle x-8\rangle^2 + \frac{4}{3}\langle x-10\rangle^3 + C_1 \qquad \text{(Eq. 1)}$$

$$EIv = \frac{8}{3}x^3 - \frac{10}{3}\langle x-2\rangle^3 - \frac{1}{3}\langle x-4\rangle^4 + \frac{31}{3}\langle x-8\rangle^3 + \frac{1}{3}\langle x-10\rangle^4 + C_1x + C_2 \qquad \text{(Eq. 2)}$$

The boundary conditions of the problem are:

$$x = 0 \quad\Rightarrow\quad v = 0$$

$$x = 8\text{ m} \quad\Rightarrow\quad v = 0$$

The constants of integration C_1 and C_2 are then determined by applying the boundary conditions to Eq. 2:

$$x = 0 \quad\Rightarrow\quad EIv = 0 = C_2$$

$$x = 8 \quad\Rightarrow\quad EIv = 0 = \frac{8^4}{3} - \frac{10}{3}6^3 - \frac{1}{3}4^4 + 8C_1 \quad\Rightarrow\quad C_1 = -70\text{ kN}\cdot\text{m}^2$$

The equations for the slope and deflection are therefore given by:

$$EI\frac{dv}{dx} = 8x^2 - 10\langle x-2\rangle^2 - \frac{4}{3}\langle x-4\rangle^3 + 31\langle x-8\rangle^2 + \frac{4}{3}\langle x-10\rangle^3 - 70 \qquad \text{(Eq. 3)}$$

$$EIv = \frac{8}{3}x^3 - \frac{10}{3}\langle x-2\rangle^3 - \frac{1}{3}\langle x-4\rangle^4 + \frac{31}{3}\langle x-8\rangle^3 + \frac{1}{3}\langle x-10\rangle^4 - 70x \qquad \text{(Eq. 4)}$$

The deflection at the overhang *F* (*x* = 12 m) is given by:

$$v_F = \frac{1}{EI}\left[\frac{8}{3}12^3 - \frac{10}{3}10^3 - \frac{1}{3}8^4 + \frac{31}{3}4^3 + \frac{1}{3}2^4 - (70\times 12)\right]$$

$$= -\frac{264}{200\times 10^6 \times 60.8\times 10^{-6}} = -2.17\times 10^{-2}\ \text{m} = -21.7\ \text{mm}$$

The maximum deflection between *AD* occurs where θ = 0. Equating Eq. 3 to zero and solving for *x* for segment *AD*:

$$EI\frac{dv}{dx} = 0 = 8x^2 - 10\langle x-2\rangle^2 - \frac{4}{3}\langle x-4\rangle^3 + 31\langle x-8\rangle^2 - 70 \quad \Rightarrow \quad x = 3.29\ \text{m}$$

The deflection at *x* = 3.29 m is given by:

$$v_{x=3.29} = \frac{1}{EI}\left[\frac{8}{3}3.29^3 - \frac{10}{3}1.29^3 - (70\times 3.29)\right] = -\frac{142.5}{200\times 10^6 \times 60.8\times 10^{-6}}$$

$$= -1.17\times 10^{-2}\ \text{m} = -11.7\ \text{mm}$$

Therefore, the maximum deflection occurs at *F* (<u>v_F = -21.7 mm, downwards</u>).