CVG2140 – Solution to Assignment No. 5 (Flexural Stresses)

Problem 1. Two vertical forces are applied to a simply supported beam as shown in Fig. 1. The beam has the cross section shown in Fig. 2. For the section subjected to maximum bending moment $|M_{max}|$, determine:

(a) The bending stress at point *H*. State whether the normal stress at *H* is *tension* or *compression*.

(b) The maximum bending stress produced in the cross section. State whether the stress is in *tension* or *compression*.

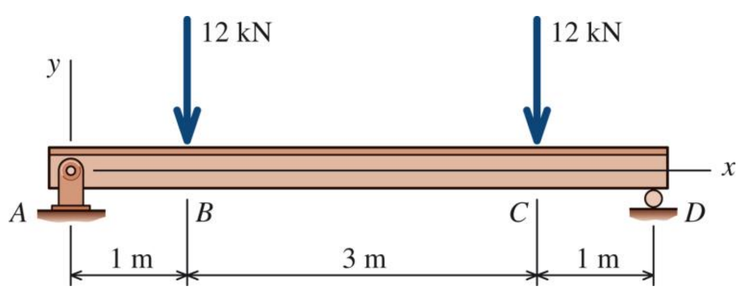


Fig. 1

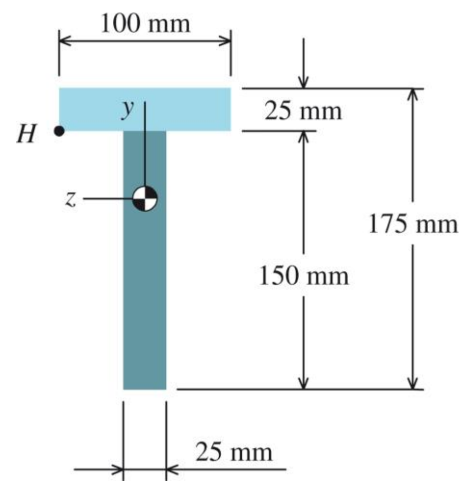
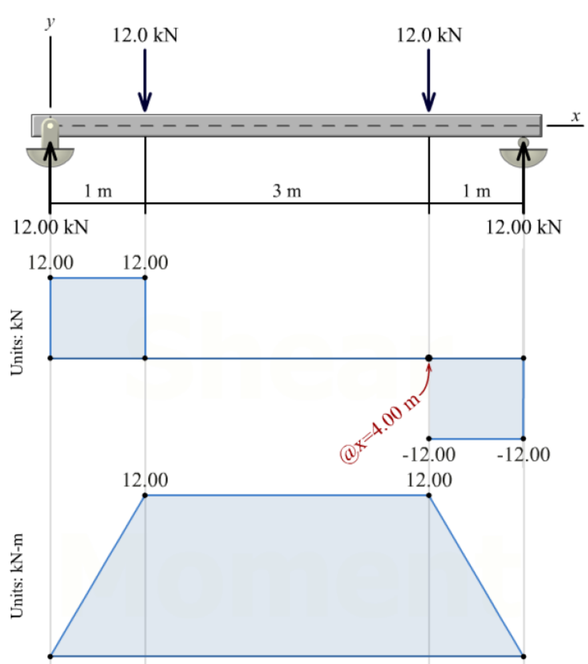


Fig. 2

Shear force & bending moment diagrams



The maximum moment occurs between *B* and *C*. The moment magnitude is <u>12 kN·m</u>.

Cross-sectional properties

	A (mm²)	$\bar{y}_i$ (mm)	$\bar{y}_i \cdot A_i$ (mm³)
Flange	100×25=2,500	150+12.5=162.5	406,250
Web	150×25=3,750	150/2=75	281,250
Σ	6,250		687,500

$$\bar{y} = \frac{\sum \bar{y}_i \cdot A_i}{\sum A_i} = \frac{687,500}{6,250} = 110 \text{ mm} \text{ from the bottom}$$

	I_{zc}^i (mm⁴)	A (mm²)	d_z^i (mm)	I_z^i (mm⁴)
Flange	130,208	2,500	52.5	7,020,833
Web	7,031,250	3,750	35	11,625,000
Σ		6,250	***I*z =**	**18,645,833**

Bending stress at point *H*

$$\sigma_H = -\frac{M_{\max} y_H}{I_z} = -\frac{(12\times10^6)(40)}{18.645833\times10^6} = \text{-25.74 MPa (compressive)}$$

Maximum bending stress produced in the cross section

The maximum bending stress in the cross section will result in the cross sectional fibre that is the furthest from the neutral axis, i.e., at the bottom where *y* = -110 mm.

$$\sigma_{\max} = -\frac{M_{\max} y_{bot}}{I_z} = -\frac{(12\times10^6)(-110)}{18.645833\times10^6} = \text{70.79 MPa (tensile)}$$

Problem 2. A seesaw weighing 5 kg/m of length is occupied by two children, each weighing 40 kg (see Fig. 3). The centre of gravity of each child is 2.5 m from the fulcrum. The board is 5.8-m long, 20-cm wide and 4-cm thick. What is the maximum bending stress in the board?

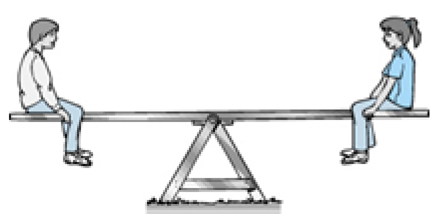
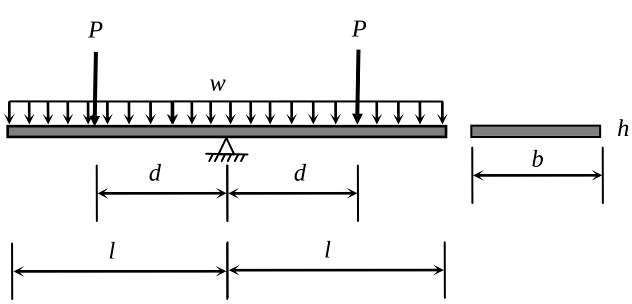


Fig. 3



where: w = 5 kg/m × 9.8 m/s² = 49 N/m, P = 40 kg × 9.8 m/s² = 392 N, d = 2.5 m, l = 2.9 m, b = 20 cm, h = 4 cm

The maximum bending moment occurs at mid-span, and its value is:

$$M_{\max} = Pd + \frac{wl^2}{2} = (392 \times 2.5) + \left(\frac{49 \times 2.9^2}{2}\right) = 1{,}186.1 \text{ N·m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{bh^2}{6} \quad \Rightarrow \quad \sigma_{\max} = \frac{6M_{\max}}{bh^2} = \frac{6 \times 1186.1}{0.2 \times 0.04^2} = \underline{22.2 \text{ MPa}}$$

Problem 3. A small dam of height h = 2.4 m is constructed of vertical wood beams AB of thickness t = 160 mm, as shown in Fig. 4. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is γ = 9.81 kN/m³. Hint: the hydrostatic pressure p in still water is $p = \gamma d$ (Pa), where γ is the weight density of water (N/m³) and d is the depth from the free surface (m). Do the problem for 1 m of the dam width.

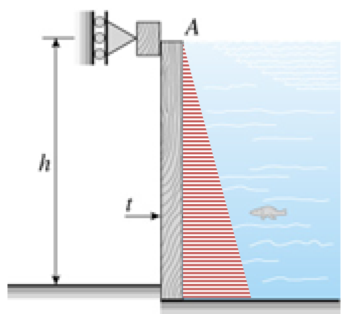
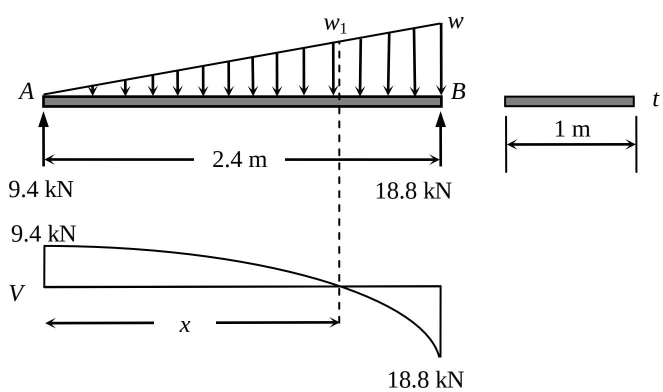
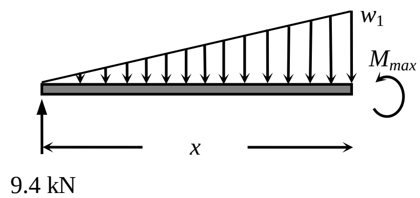


Fig. 4

$$w = \gamma \times h \times width = 9.81 \times 2.4 \times 1 = 23.5 \text{ kN/m}$$



The maximum moment occurs at x, since $V = \frac{dM}{dx} = 0$. By establishing equilibrium at that section:



$$\sum F_y = 9.4 - \left(\frac{1}{2} \times w_1 \times x\right) = 0$$

Since $\frac{w_1}{x} = \frac{w}{2.4}$, $9.4 - \left(\frac{1}{2} \times \frac{23.5}{2.4} \times x^2\right) = 0 \quad \Rightarrow \quad x = 1.39$ m

The moment at x = 1.39 m is then calculated by establishing moment equilibrium at that section:

$$\sum M = -(9.4 \times 1.39) + \left(\frac{1}{2} \times 13.6 \times 1.39 \times \frac{1}{3} \times 1.39\right) + M_{\max} = 0 \quad \Rightarrow \quad M_{\max} = 8.69 \text{ kN·m}$$

$$\sigma_{\max} = \frac{M_{\max}}{S}, \quad S = \frac{I}{y} = \frac{bt^2}{6} \quad \Rightarrow \quad \sigma_{\max} = \frac{6M_{\max}}{bt^2} = \frac{6 \times 8.69 \times 10^3}{1 \times 0.16^2} = \underline{2 \text{ MPa}}$$

<u>Problem 4.</u> The beam shown in Fig. 5 has a rectangular cross section as shown in Fig. 6. Determine the maximum compressive and tensile stresses in the beam.

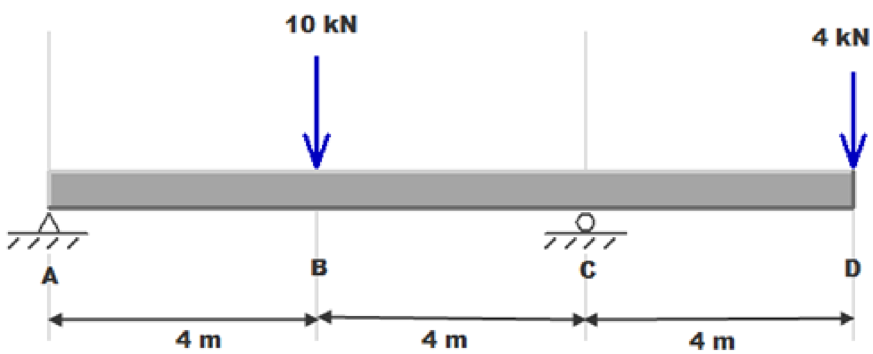


Fig. 5

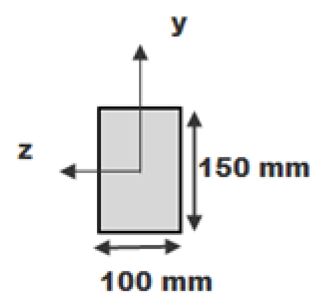


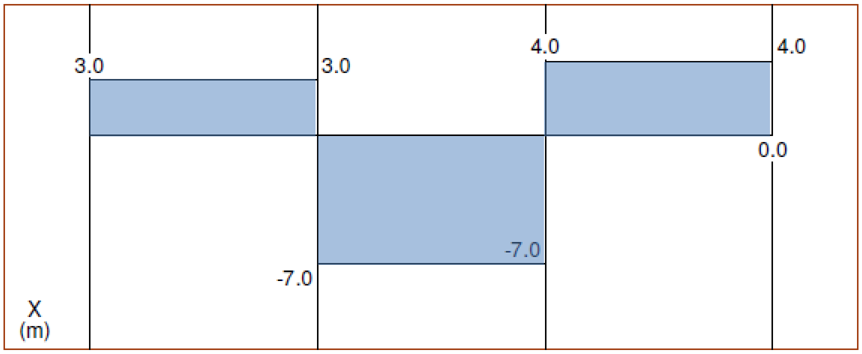
Fig. 6

From equilibrium:

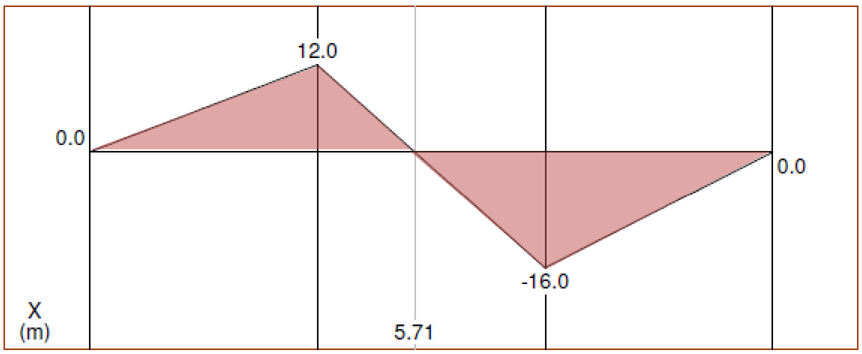
$$\sum F_x = A_x = 0$$

$$\left.\begin{array}{l} \sum F_y = A_y + C_y - 14 = 0 \\ \sum M_A = -(10)(4) + (C_y)(8) - (4)(12) = 0 \end{array}\right\} \Rightarrow \begin{cases} A_y = 3 \text{ kN} \\ C_y = 11 \text{ kN} \end{cases}$$

The shear and bending moment diagrams of the beam are as follows:



Shear Diagram (kN)



Moment Diagram (kN-m)

a) From the BMD, $|M_{max}|$ = 16 kN·m = 16×10^6 N·mm

b) The neutral axis (or centroid) of the cross section is located at mid depth (i.e., 75 mm).

c) The moment of inertia of the cross section with respect to the bending axis *z* (neutral axis) is given by:

$$I_z = \frac{bh^3}{12} = \frac{100\times150^3}{12} = \underline{28.125\times10^6\ \text{mm}^4}$$

d) The maximum compressive and tensile stresses are equal, since both the top and bottom fibres are equidistant from the neutral axis, *y* = ±75 mm. Therefore, the bending stress $|\sigma_{max}|$ is given by:

$$|\sigma_{\max}| = \left|\frac{M_{\max}y_{\max}}{I_z}\right| = \frac{(16\times10^6)(75)}{(28.125\times10^6)} = \underline{42.7\ \text{MPa}}$$