ITI1100/section C Winter 2016

DIGITAL SYSTEM I

Lectures:

Tuesday, 13:00 – 14:30 room: STE H0104

Thursday, 11:30 –13:00 room: STE H0104

DGD 1-Thursday 17:30 - 19:00 TBT-070

DGD 2-Thursday 14:30 -16:00 CBY-D207

LAB 1 Tuesday 14:30 - 17:30 CBY B302

LAB 2 Friday 8:30 - 11:30 CBY B302

Professor : Dr. A. Karmouch, office CBY A508

Mid-term exam: Saturday March 5, 2016 (10:00-11:30AM)

Course Management

Email

Karmouch@site.uottawa.ca

When emailing me, please format the subject line as follows:

“ITI1100A - <last name> <first initial> - <subject>”

Course Format

- 7.5 Hours of scheduled instruction per week
  - 3 hours of Lecture
  - 1.5 hour of Group Discussions. (starting date: to be announced in the class)
  - 3 hours of Laboratory (starting date: to be announced in the class)

Laboratory

→Each student will have a laboratory session every week. There are six experiments to be performed, each requiring a group preparation and completion report.

→ Laboratory groups will consist of **two** students only.

→ Students are required to stay in the same group and with the same TA for the whole semester.

Laboratory

→Every group performing the experiment is required to record their data on paper and this should be seen and signed by the TA.

→The data should be attached to the submitted report. One lab report is expected from each group after each lab.

→The lab report should be prepared according to the guidelines specified in the lab manual.

→ Lab reports are due one week after experiment.

→ Lab reports must be submitted to the TAs during the lab sessions.

Textbook

Available at the University of Ottawa bookstore/AGORA.

REQUIRED!

Book Title: Digital Design

Authors: M Morris Mano & Michael D. Ciletti

Edition: Fifth Edition

Publisher: Pearson-Prentice Hall

Grading Scheme

Assignments/QUIZ	10%
Laboratories	15%
Mid term exam	25%
Final Exam	50%

Cheating and plagiarism

- Cheating is any act that gives you unfair advantage at the expense of another classmate.
- Examples:
  - copying on exams, homework

- **Plagiarism** see the following URL: **http://www.uottawa.ca/plagiarism.pdf**
- If we detect you are involved in cheating or plagiarism you will be turned over to the Faculty, for investigation and sanctions

Course Outline

Digital Design

1. **Binary Systems.**
   Digital Systems. Binary Numbers. Number Base Conversions. Octal and Hexadecimal Numbers. Complements. Signed Binary Numbers. Binary Codes. Binary Storage and Registers. Binary arithmetic

2. **Boolean Algebra and Logic Gates.**
   Basic Definitions. Basic Theorems and Properties of Boolean Algebra. Boolean Functions. Canonical and Standard Forms. Other Logic Operations. Digital Logic Gates.

Course Outline

Digital Design [2]

3. Gate Level Minimization

The Map Method. Four Variable Map. Product of Sums Simplification. Don't Care Conditions. NAND and NOR, Implementation. Other Two Level Implementations. Exclusive OR Function.

4. Combinational Logic

Combinational Circuits. Analysis Procedure. Design Procedure. Binary Adder Subtractor. Magnitude Comparator. Decoders. Encoders. Multiplexers.

Course Outline

Digital Design [3]

5. **Synchronous Sequential Logic.**
   Sequential Circuits. Latches. Flip Flops. Analysis of Clocked Sequential Circuits. Design Procedure.

6. **Registers ad Counters.**
   Registers. Shift Registers. Ripple Counters. Synchronous Counters. Other Counters.

Chapter 1

BINARY SYSTEMS

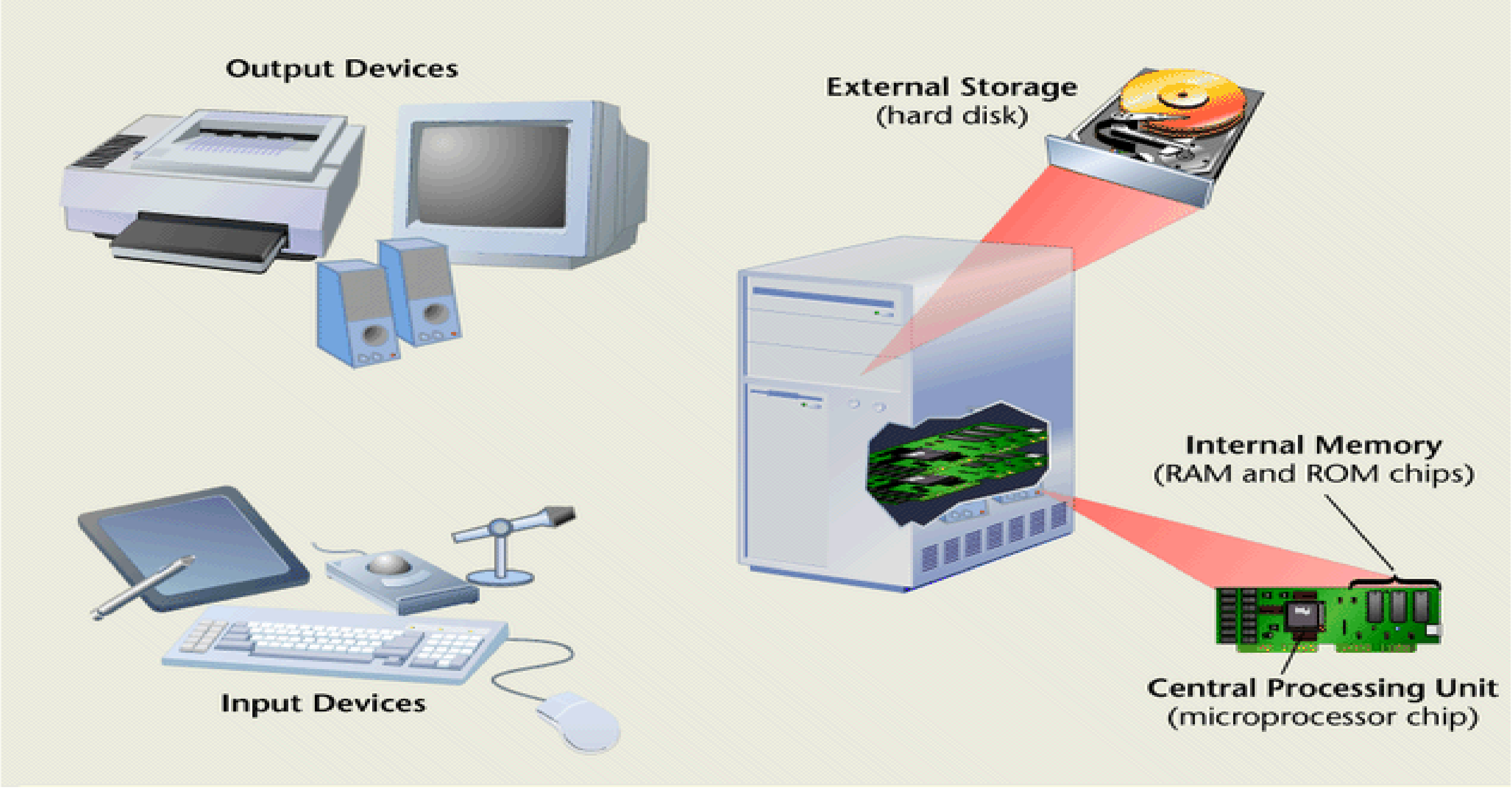
- Are computers analog or digital systems?

Computers are digital systems

Which is easier to design an analog or a digital system?

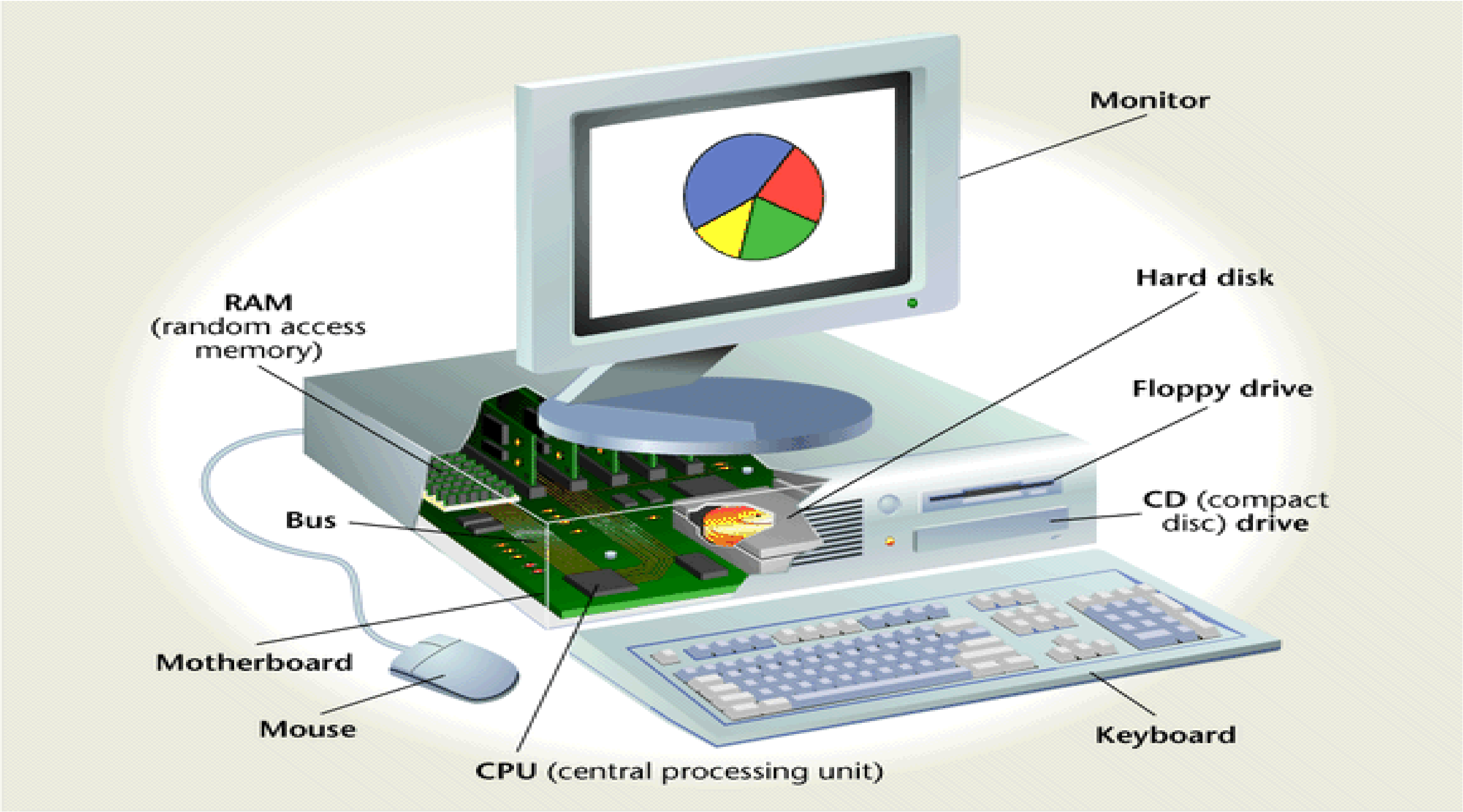
Digital systems are easier to design, because they deal with a limited set of values rather than an infinitely large range of continuous values

The Central Tool of Modern Information Systems



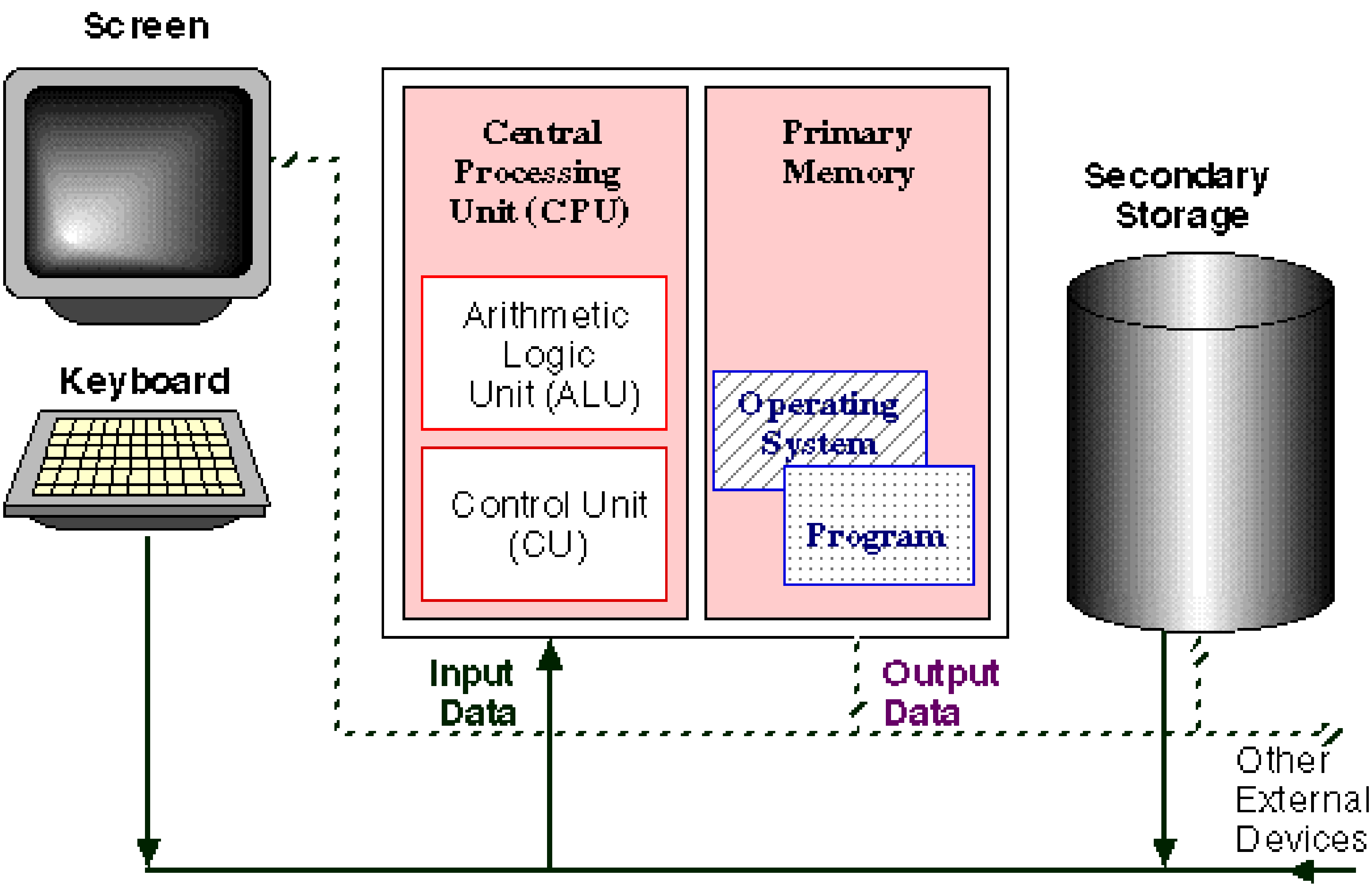
All computers have the same basic components.

Inside the computer



A look inside a computer

Block Diagram of a Digital Computer



Digital Systems

→ Early computers were designed to perform numeric computations

→ They used ***discrete*** elements of information named digits (finite sets)

→DIGITAL SYSTEMS: manipulate ***discrete*** elements of information

--such as the 10 decimal digits or the 26 letters of the alphabet

Binary System and Logic Circuits

- What kind of data do computers work with?
  - Deep down inside, it's all 1s and 0s
- What can you do with 1s and 0s?
  - Boolean algebra operations
  - These operations map directly to hardware circuits (logic circuits)

Different Number Systems

- Decimal (Arabic): (0,1,2,3,4,5,6,7,8,9):
  Example: **$(452968)_{10}$**
- Octal: (0,1,2,3,4,5,6,7):
  Example **$(4073)_8$**
- Hexadecimal(0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)
  Example: $(2BF3)_{16}$
- Binary: (0,1):
  Example: $(1001110001011)_2$

Base in Number systems

- The decimal number system uses **base 10**. The values of the positions are calculated by taking 10 to some power.

1	6	2	.	3	7	5	Digits
100	10	1		1/10	1/100	1/1000	Weights

1	6	2	.	3	7	5	Digits
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	Weights

- Base 10 for decimal numbers?

It uses 10 digits: The digits 0 through 9.

Base in Number systems [2]

- The binary number system is called binary because it uses **base 2**. The values of the positions are calculated by taking 2 to some power.
- Base 2 for binary numbers :

It uses 2 digits. The digits 0 and 1.

Representation of Numbers

→ ***There are two possible ways of writing a number in a given system:***

1- Positional Notation

2- Polynomial Representation

Positional Notation

$$N = (a_{n-1}a_{n-2} \ldots a_1a_0 \, . \, a_{-1}a_{-2} \ldots a_{-m})_r$$

Where

. = radix point

r = radix or base

n = number of integer digits to the left of the radix point

m = number of fractional digits to the right of the radix point

a_{n-1} = most significant digit (MSD)

a_{-m} = least significant digit (LSD)

Positional Notation

The Decimal Numbering System

- The decimal numbering system is a positional number system.
- Example:

$(4\ 6\ 2\ 1)_{10}$

(1000 100 10 1)

1×10^0

2×10^1

6×10^2

4×10^3

Positional Notation

Binary Numbering System

- The Binary Numbering System is also a positional numbering system.
- Instead of using ten digits, 0 - 9, the binary system uses only two digits, the 0 and the 1.

→Example of a binary number & the values of the positions.

1	0	1	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

1	1	0	1	.	0	1	Binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	Weights (in base 10)

Polynomial Notation

$N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots + a_0 \times r^0 + a_{-1} \times r^{-1} \ldots + a_{-m} \times r^{-m}$

$$= \sum_{i=-m}^{n-1} a_i r^i$$

Example:

Positional (N) ***Polynomial (N)***

$N = (651.45)_{10} = 6 \times 10^2 + 5 \times 10^1 + 1 \times 10^0$

$+ 4 \times 10^{-1} + 5 \times 10^{-2}$

Important number systems

→ There are three important number systems

- Binary Number System
- Octal Number System
- Hexadecimal Number System

Binary numbers

Digits = {0, 1}

Positional Polynomial

$(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

– **1 K (kilo) = 2^{10} = 1,024**

– **1M (mega) = 2^{20} = 1,048,576**

– **1G (giga) = 2^{30} = 1,073,741,824**

Converting Decimal to Binary

$$N = (a_{n-1}a_{n-2} \dots a_1a_0 \, . \, a_{-1}a_{-2} \dots a_{-m})_r$$

← *Integer* → ← *Fractional* →

Radix point

→ **Integer** part and **Fractional** part are **converted differently**

Converting the Integer Part

- keep dividing by 2 until the quotient is 0. Collect the remainders *in reverse order*.
- Example: $(162)_{10}$.

162 / 2	= 81	rem 0
81 / 2	= 40	rem 1
40 / 2	= 20	rem 0
20 / 2	= 10	rem 0
10 / 2	= 5	rem 0
5 / 2	= 2	rem 1
2 / 2	= 1	rem 0
1 / 2	= 0	rem 1

- Then $(162)_{10} = (10100010)_2$

Converting the Fraction Part

→ keep multiplying the *fractional part* by 2 until it becomes 0. Collect the integer parts (in forward order).

- However this may not terminate!
- Example: $(0.375)_{10}$

$0.375 \times 2 = 0.750$
$0.750 \times 2 = 1.500$
$0.500 \times 2 = 1.000$

→ So, $(.375)_{10} = (.011)_2$

And $(162.375)_{10} = (10100010.011)_2$

Why does this work?

•This method can be applied to convert from decimal to *any* base

•Try converting 162.375 from decimal to decimal.

162 / 10 = 16 rem 2
16 / 10 = 1 rem 6
1 / 10 = 0 rem 1

•Each division "strips off" the rightmost digit (the remainder). The quotient represents the remaining digits in the number.

Why does this work? [2]

$$0.375 \times 10 = 3.750$$
$$0.750 \times 10 = 7.500$$
$$0.500 \times 10 = 5.000$$

- Each multiplication "strips off" the leftmost digit (the integer part). The fraction represents the remaining digits.

Converting binary to decimal

- To convert binary, or base 2, numbers to decimal we first obtain the polynomial representation of the number, then sum the products.
  - Example: $(1101.01)_2$

1	1	0	1	.	0	1	Binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	Weights (in base 10)

$$(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) =$$

$$8 + 4 + 0 + 1 + 0 + 0.25$$

→ The decimal value is: $(13.25)_{10}$

Octal number system

–Digits = {0, 1, 2, 3, 4, 5, 6, 7}

Positional = Polynomial

$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1}$

- **Octal (base 8) digits range from 0 to 7.**

Since $8 = 2^3$, one octal digit is equivalent to 3 binary digits.

Converting Decimal to Octal

→ Integer part: keep dividing by 8 until the quotient is 0. Collect the remainders *in reverse order*.

→ Fractional Part: keep multiplying the *fractional part* by 8 until it becomes 0. Collect the integer parts (in forward order).

Same method as for the decimal to binary

Converting Octal to Decimal

- To convert Octal, or base 8, numbers to decimal we first obtain the polynomial representation of the number, then sum the products.

Example

$$(127.4)_8 = 1 \text{ x } 8^2 + 2 \text{ x } 8^1 + 7 \text{ x } 8^0 + 4 \text{ x } 8^{-1} = (87.5)_{10}$$

Converting Binary to Octal

- To convert from binary to octal, make groups of 3 bits, starting from the binary point. Add 0s to the ends of the number if needed. Then convert each group of bits to its corresponding octal digit.

Example

$(10110100.001011)_2 \quad =(010\ 110\ 100\ .\ 001\ 011\ )_2$

$= \quad (\ 2 \quad 6 \quad 4\ .\ 1 \quad 3\ )_8$

Converting Octal to Binary

- To convert from octal to binary, replace each Octal digit with its equivalent 3-bit binary sequence.

- Example

$(261.35)_8 = (2 \quad 6 \quad 1 \quad . \quad 3 \quad 5)_8$

$= (010 \quad 110 \quad 001 \quad . \quad 011 \quad 101)_2$

Hexadecimal numbers

-Digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

Positional Polynomial

- $(B65F)_{16} = 11 \times 16^{3} + 6 \times 16^{2} + 5 \times 16^{1} + 15 \times 16^{0}$

- *Hexadecimal* (base 16) digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Since $16 = 2^4$, one hexa digit is equivalent to 4 binary digits.
  - It's often easier to work with a number like B5 instead of 10110101.

Converting Decimal to Hexadecimal

→ **Integer part**: keep dividing by 16 until the quotient is 0. Collect the remainders *in reverse order*.

→ **Fractional Part**: keep multiplying the *fractional part* by 16 until it becomes 0. Collect the integer parts (in forward order).

Same method as for the decimal to binary conversion

Converting Hexadecimal to Decimal

- To convert Hexadecimal, or base 16, numbers to decimal, first obtain the polynomial representation of the number, then sum the products.

Example

$$\mathbf{(B65F)_{16} = 11 \times 16^{3} + 6 \times 16^{2} + 5 \times 16^{1} + 15 \times 16^{0}}$$

$$\mathbf{= (46{,}687)_{10}}$$

Converting Hexadecimal to Binary

- To convert from hexadecimal to binary, replace each hex digit with its equivalent 4-bit binary sequence.
- Example

$$261.35_{16} = (2 \quad 6 \quad 1 \quad . \quad 3 \quad 5)_{16}$$

$$= (0010 \quad 0110 \quad 0001 \quad . \quad 0011 \quad 0101)_2$$

Converting Binary to Hexadecimal

- To convert from binary to hex, make groups of 4 bits, starting from the binary point. Add 0s to the ends of the number if needed. Then convert each group of bits to its corresponding hex digit.
- Example

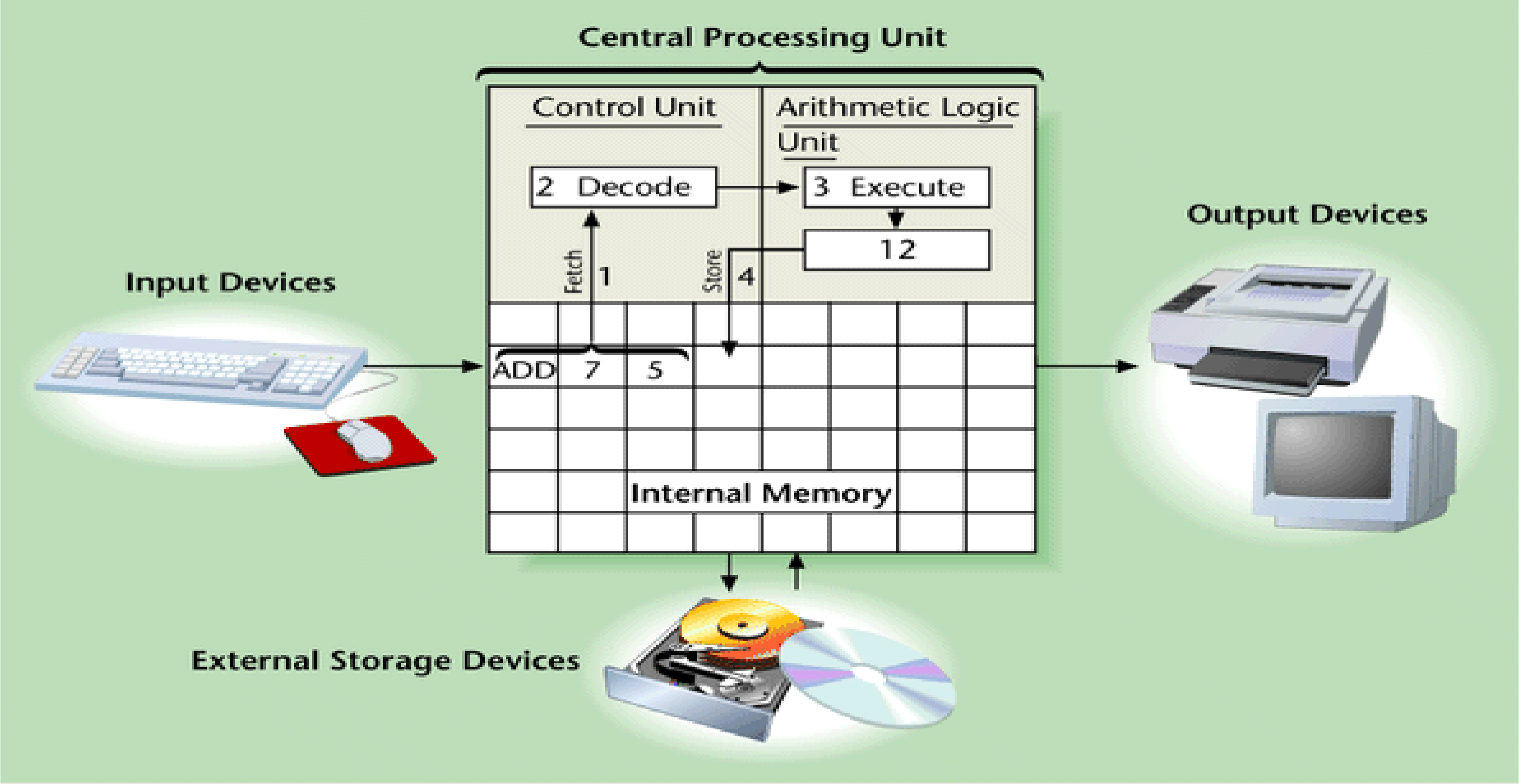
$$(10110100.001011)_2 = (1011\ 0100.\ 0010\ 1100)_2$$

$$= (B\ 4\ .\ 2\ C)_{16}$$

Decimal	Binary	Octal	Hex
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

ARITHMETIC OPERATIONS IN A BINARY SYSTEM

Arithmetic Operations



What happens inside the CPU in one machine cycle executing the operation 7 + 5

Binary Addition

	0	0	1	1
	+0	+ 1	+0	+1
	--	--	--	--
sum	0	1	1	10
				(sum of 0 and carryover of 1)

Examples:

1 0 0 1	0 0 0 1	1 1 0 0
+ 0 1 1 0	+ 1 0 0 1	+ 0 1 0 1
1 1 1 1	1 0 1 0	1 0 0 0 1

carryover

Binary Addition-Examples

		Check your work
Carry	1 1 1 1 0 1	
	1 0 1 1 0 1	$(45)_{10}$
	+ 0 1 1 1 0 1	+ $(29)_{10}$
Sum	1 0 0 1 0 1 0	= 74

$1 \text{ x } 2^5 + 0 \text{ x } 2^4 + 1 \text{ x } 2^3 + 1 \text{ x } 2^2 + 0 \text{ x } 2^1 + 1 \text{ x } 2^0$

$= 32 + 0 + 8 + 4 + 1 = 45$

Binary Addition- Examples

Addition of three Binary Digits

x	y	CarryIn	Sum	CarryOut
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Addition of Large Binary Numbers

- Example showing larger numbers:

```
     1010  0011  1011  0001
  +  0111  0100  0001  1001
  ---------------------------
  1  0001  0111  1100  1010
```

Binary Subtraction

	0	0	1	1
	- 0	- 1	- 0	- 1
	--	--	--	--
difference	0	11	1	0

(Diff. of 1 and carryover of 1)

```
0  10100
   101011

 - 010101
 ---------
 0 010110
```

Binary Multiplication

	0	0	1	1
	x 0	x 1	x 0	x 1
	--	--	--	--
Product	0	0	0	1

A 0 010000.010

x B 0 001000.010

0000000000
0010000010
0000000000
0000000000
0000000000
0000000000
0010000010

1000110.000100

Binary Division

	0	1
	÷ 1	÷ 1
=	0	1

Complements in Numbering Systems

• Complements are used in digital systems (computers) for simplifying the Subtraction operation and for logical manipulation

• There are to type of complements for each ***base* ‘*r*’ *system*:**

1- Radix complement → r’s complement

Ex. ***base 10-→ 10’s complement***

base 2→ 2’s complement

2- Diminished radix complement→ (r-1) complement

Ex. ***base 10→ 9’s complement***

Base 2→ 1’s complement

Radix complement (r's complement)

$$[N]_r = r^n - (N)_r$$

where n is the number of digits in $(N)_r$.

Example

- **2's complement of $(N)_2 = (101001)_2$**

$$[N]_2 = 2^6 - (101001)_2 = (1000000)_2 - (101001)_2 = (010111)_2$$

- **10's complement of $(N)_{10} = (72092)_{10}$**

$$[N]_{10} = (100000)_{10} - (72092)_{10} = (27908)_{10}.$$

Obtaining 2's complement

- **Can be obtained directly from the given number by** ***1-copying each bit of the number starting at the least significant bit and proceeding forward the most significant bit until the first 1 has been copied.***

  2- After the first 1 has been copied replace each of the remaining 0s and 1s by 1s and 0s respectively

(a) $\mathbf{[1010100]_2} = 2^7 - (1010100)_2 = (10000000)_2 - (1010100)_2 = \mathbf{(0101100)_2}$

(b) $\mathbf{[101001]_2} = 2^6 - (101001)_2 = (1000000)_2 - (101001)_2 = \mathbf{(010111)_2}$

(a) 1 0 1 0 1 0 0 **(b) 1 0 1 0 0 1**

2's → **0 1 0 1 1 0 0** **0 1 0 1 1 1**

Diminished radix complement (*r-1*'s complement)

$$[N]_{r-1} = (\boldsymbol{r^n} - 1)r - (N)_r$$

-9's complement of $[546700]_9$

= 999999 – 546700 = 453299

-1's complement of [1011000]

= $(10000000 - 1)_2 - (1011000)_2 = (0100111)_2$

Obtaining 1's complement

- *1 's complement can be obtained directly from the given number by replacing each of 0s and 1s by 1s and 0s of the number (i.e. complement each bit)*

$$[\mathbf{1011000}] = (10000000 - 1)_2 - (1011000)_2$$

$$= (\mathbf{0100111})_2$$

1 0 1 1 0 0 0

1 's complement → **0 1 0 0 1 1 1**

Subtraction with 2's Complement

- **2's complement are used to convert subtraction to addition, which reduces hardware requirements (only adders are needed).**

$$A - B = A + (-B)$$

(add *2*'s complement of *B* to *A*)

- 2's Complement has the properties of the minus sign

A + (-A) = 0 **A + 2's{A} = 0**

- (- A) = A **2's{ 2's{A}} = A**

A - B = A + (-B) **A - B = A + 2's{B}**

Subtraction with 2's Complement [2]

Examples:

A= 1010100 B= 1000011

- 2's complement

A - B = A + (-B)= A+ [B]

= (1010100) + (0111101) = (0010001)

```
            1010100
 +          0111101
 ------------------
          1 0010001
```

Discard end carry

Subtraction with 1's Complement

Examples: note: same proprieties minus sign as 2's complement.

A= 1010100 B= 1000011

- 1's complement

A - B = A+ [B]= (1010100) + (0111100) = (0010001)

```
            1010100
    +       0111100
        -----------
*Add end carry*
        1   0010000
        |
        +------> + 1
      -------------
            0010001
```

Subtraction with 10s/9'sComplements

$(72)_{10} - (32)_{10} = (40)_{10}$

10's Complement

$[32] = 10^2 - (32)_{10} = (68)_{10}$

$(72)_{10} + (68)_{10} = \not{1}\ (40)_{10}$

9's Complement

$[32] = (10^2 - 1) - (32)_{10} = (67)_{10}$

$(72)_{10} + (67)_{10} = (1 + 39)_{10} = (40)_{10}$

Signed Binary numbers

• Recall that digital Systems are made with devices that take on exactly two states : 0 and 1.

•**The only states are “1” and “0”. There is no “-” state!**

→because of hardware limitations computers represent negative numbers by using the leftmost bit for the sign bit.

-- “0” indicates a positive number,

-- while a “1” indicates a negative number

Signed Magnitude

- The leftmost bit indicates the sign of the number. The remaining bits give the magnitude of the number

→Using 8 bits to represent binary number the value in the example is:

sign	**magnitude**						
1	0	0	0	0	0	1	1

-3 = 10000011 = 1/ (sign bit) 0000011

→ Sign Magnitude representation is good for having the ability for a human to read and understand what number is represented

Signed Complement

(a) Signed Magnitude representation

1	0	0	0	0	0	1	1

-3 = 10000011 = 1/ (sign bit) 0000011

(b) Signed 1's representation

1	1	1	1	1	1	0	0

-3 = 10000011 = 1/ (sign bit) 1111100

(c) Signed 2's representation

1	1	1	1	1	1	0	1

-3 = 10000011 = 1/ (sign bit) 1111101

Fixed-Length Registers

- All practical digital devices have fixed-length registers
- This means that numbers in a computer are represented by a fixed number of bits
  - The earliest microprocessors were 4-bit devices
  - Intel 8080 and the 6502 (Apple II) chips were 8-bit
  - Intel 8088 (IBM PC) and Motorola 68000 (Mac) are 16-bit devices
  - Pentium chips and PowerPC chips are 32-bit

Range of a number Overflow during addition

- A fixed-length register can only hold a ***Range*** of numbers

→For a 4-bit device, the *range* of positive integers is 0 - 15

→For an 8-bit device the *range* of positive integers is 0 – 255

→ When adding positive integers, ***Overflow*** occurs when the sum falls outside the range of the register

Overflow in Signed Complements

•when numbers are treated as signed complement, a "carry" of 1 from the addition of the most significant bits **DOES NOT** indicate an overflow, this example uses 2's

3 00011
+ (-3)+11101
= 00000, with a carry of "1" (2'complement) : We know that for addition operation in 2's complement the end-carry is discarded !

•For signed complement, overflow occurs when:

→***The addition of two positive numbers results in a negative number***

OR → ***The addition of two negative numbers results in a positive number***

Overflow Examples

- In a 6-bit register with signed 2's complement

+ 17 = 010001

+ 16 = +010000

=100001

100001 = - (11111) = -(31)$_{10}$ instead of **+ (33)$_{10}$**

- Same with a 7-bit register

+ 17 = 0 010001

+ 16 = +0 010000

= 0100001

0100001 = + 33 No Overflow

Binary codes: *BCD* (1)

- To represent information as strings of alpha-numeric characters.
- ***Binary Coded Decimal (BCD)***
  - Used to represent the decimal digits 0 - 9.
  - 4 bits are used.
  - Each bit position has a weight associated with it (***weighted code***).
  - Weights are: 8, 4, 2, and 1 from MSB to LSB (called 8-4-2-1 code).

Binary codes: *BCD* (2)

– BCD Codes:

0→ 0000	1→ 0001	2→ 0010
3→ 0011	4→0100	5→ 0101
6→ 0110	7→0111	8→ 1000
9→ 1001		

– Used to encode numbers for output to numerical displays

–***Example***: $(9750)_{10} = (1001011101010000)_{BCD}$

Binary codes: *ASCII* [2]

- ***ASCII*** (American Standard Code for Information Interchange) (see table 1.7 of textbook)
  - Most widely used character code.
  - ***Example***: ASCII code representation of the word ***'Digital'***

Character	Binary Code	Hexadecimal Code
D	1000100	44
i	1101001	69
g	1100111	67
i	1101001	69
t	1110100	74
a	1100001	61
l	1101100	6C

Practice Problems Solved in the Class

Examples: Signed Complements 2's

	Sign-bit	
$(9)_{10}$	0	1001
$+(6)_{10}$	0	0110
	0	1111

$(9)_{10}$	0	1001
$-(6)_{10}$	1	1010
	0	0011

$(6)_{10}$	0	0110
$-(9)_{10}$	1	0111
	1	1101

Examples: Signed Complements 1's

	Sign-bit	
$(9)_{10}$	*0*	*1001*
$+(6)_{10}$	0	0110
	0	1111

$(9)_{10}$		*0*	*1001*
$-(6)_{10}$		1	1001
	1	0	0010 = (0010) + (0001) = (0011)

$(6)_{10}$	0	0110
$-(9)_{10}$	*1*	*0110*
	1	1 1 0 0

Problems

Question

(a) Convert le following binary number into (i) Octal, (ii) Decimal, (iii) hexadecimal

10101101.10110

(b) Convert A= 16.25 and B = 8.25 into binary, use 7 bits to represent the integer part and 3 bits to represent the fractional part, then perform the following operations

I) C= A + B
ii) D = A – B

Note: Compute C and D
(a) using non-signed binary numbers and without complements
(b) using signed 2's complement

(c) Convert le following number into (i)Decimal, (ii) Octal, (iii) binary

$(FD8.C2B)_{16}$

Problems

10101101.10110

Answers:

i) Octal→ **(010 101 101.101 100)**

(2 5 5 . 5 4)$_8$

iii) Hexa → **(1010 1101.1011 0000)2**

(A D . B 0)16

ii) Decimal

$(10101101.10110)_2 = 1 \text{ x } 2^7 + 0 \text{ x } 2^6 + 1 \text{ x } 2^5 + 0 \text{ x } 2^4 + 1 \text{ x } 2^3 + 1 \text{ x } 2^2 + 0 \text{ x } 2^1 + 1 \text{ x } 2^0 + 1 \text{ x } 2^{-1} + 0 \text{ x } 2^{-2}\ 1 \text{ x } 2^{-3} + 1 \text{ x } 2^{-4} + 0 \text{ x } 2^{-5}$

$= (173.6875)_{10}$

Problems

Integer part

- Conversion de $(16)_{10}$.

16 / 2	= 8	remainder 0
8 / 2	= 4	remainder 0
4 / 2	= 2	remainder 0
2 / 2	= 1	remainder 0
1 / 2	= 0	remainder 1

- Then $(16)_{10} = (10000)_2$

Problems

Fractional part

– Converting: $(0.25)_{10}$

$0.25 \times 2 = 0.50$
$0.50 \times 2 = 1.00$

• then, $(.25)_{10} = (.01)_2$

and $(16.25)_{10} = (10000.01)_2$

Same as for A

→ B = 8.25: $(8.25)_{10} = (1000.01)_2$

→ Representation using 7 bits and 3 bits

A= $(16.25)_{10}$ = $(0010000.010)_2$

B= $(8.25)_{10}$ = $(0001000.010)_2$

Problems

Non Signed Binary

A	0010000.010
+ B	0001000.010
	0 011000.100

A	0010000.010
- B	0001000.010
	0001000.000

Problems

Signed 2' complement

A	0 010000.010
+ B	0 001000.010
	0 011000.100

A	0 010000.010
+ (-B)	1 110111.110
	0 001000.000

Problems

A	0 010000.010
x B	0 001000.010

```
        0000000000
       0010000010
      0000000000
     0000000000
    0000000000
   0000000000
  0010000010
```

1000110.000100

Problems

A ÷ B

Dividend

10000.010 | 1000 .010 Divider

1000010 | 1.1.. Quotient

01000 0000

1000010

Question 1:

Convert A = $(00010010.0101)_{BCD}$ and B= $(2.25)_{10}$ into pure binary format employing 8 bits for the integer part and 3 for the fractional part, including the sign bit. Perform the following operations in specific signed complement as indicated for each operation.

(i) C = -A - B using signed 2's complement
(ii) D = – A + B using signed 1's complement
(iii) E= A – B using 9's complement (show all intermediate steps)

Problems

A= $(12.50)_{10}$ B= $(2.25)_{10}$ $(12.50)_{10}$ - $(2.25)_{10}$ using 9's complement

9's complement of (2.25)

First step use the same number of position to represent the two numbers (02.25) then obtain the 9's complement

$[02.25]_9 = (10^4 - 1)_{10} - (02.25)_{10} = (99.99 - 02.25)_{10} =$

$(97.74)_{10}$

E = A- B = A+ $[B]_9$ =

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                 12.50
     +           97.74
(end carry) 1   10 .2 4
                    + 1
                 10.  25
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