

ECO 1192

Engineering Economics

Project Decision-Making Criteria

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The purpose of this document is to provide detailed calculations for a range of decision-making criteria to determine the

- validity (acceptability) of projects and**
- best among competing alternatives.**

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DISCRETE CASH FLOW AND COMPOUNDING
10.00 % DISCRETE RATE OF INTEREST

| n | (F/P,i%,n) | (P/F,i%,n) | (A/P,i%,n) | (P/A,i%,n) | (A/F,i%,n) | (F/A,i%,n) | (A/G,i%,n) |
|----------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1 | 1.1000 | 0.9091 | 1.1000 | 0.9091 | 1.0000 | 1.0000 | 0.0000 |
| 2 | 1.2100 | 0.8264 | 0.5762 | 1.7355 | 0.4762 | 2.1000 | 0.4762 |
| 3 | 1.3310 | 0.7513 | 0.4021 | 2.4869 | 0.3021 | 3.3100 | 0.9366 |
| 4 | 1.4641 | 0.6830 | 0.3155 | 3.1699 | 0.2155 | 4.6410 | 1.3812 |
| 5 | 1.6105 | 0.6209 | 0.2638 | 3.7908 | 0.1638 | 6.1051 | 1.8101 |
| 6 | 1.7716 | 0.5645 | 0.2296 | 4.3553 | 0.1296 | 7.7156 | 2.2236 |
| 7 | 1.9487 | 0.5132 | 0.2054 | 4.8684 | 0.1054 | 9.4872 | 2.6216 |
| 8 | 2.1436 | 0.4665 | 0.1874 | 5.3349 | 0.0874 | 11.4359 | 3.0045 |
| 9 | 2.3579 | 0.4241 | 0.1736 | 5.7590 | 0.0736 | 13.5795 | 3.3724 |
| 10 | 2.5937 | 0.3855 | 0.1627 | 6.1446 | 0.0627 | 15.9374 | 3.7255 |
| 11 | 2.8531 | 0.3505 | 0.1540 | 6.4951 | 0.0540 | 18.5312 | 4.0641 |
| 12 | 3.1384 | 0.3186 | 0.1468 | 6.8137 | 0.0468 | 21.3843 | 4.3884 |
| 13 | 3.4523 | 0.2897 | 0.1408 | 7.1034 | 0.0408 | 24.5227 | 4.6988 |
| 14 | 3.7975 | 0.2633 | 0.1357 | 7.3667 | 0.0357 | 27.9750 | 4.9955 |
| 15 | 4.1772 | 0.2394 | 0.1315 | 7.6061 | 0.0315 | 31.7725 | 5.2789 |
| 16 | 4.5950 | 0.2176 | 0.1278 | 7.8237 | 0.0278 | 35.9497 | 5.5493 |
| 17 | 5.0545 | 0.1978 | 0.1247 | 8.0216 | 0.0247 | 40.5447 | 5.8071 |
| 18 | 5.5599 | 0.1799 | 0.1219 | 8.2014 | 0.0219 | 45.5992 | 6.0526 |
| 19 | 6.1159 | 0.1635 | 0.1195 | 8.3649 | 0.0195 | 51.1591 | 6.2861 |
| 20 | 6.7275 | 0.1486 | 0.1175 | 8.5136 | 0.0175 | 57.2750 | 6.5081 |
| 21 | 7.4002 | 0.1351 | 0.1156 | 8.6487 | 0.0156 | 64.0025 | 6.7189 |
| 22 | 8.1403 | 0.1228 | 0.1140 | 8.7715 | 0.0140 | 71.4027 | 6.9189 |
| 23 | 8.9543 | 0.1117 | 0.1126 | 8.8832 | 0.0126 | 79.5430 | 7.1085 |
| 24 | 9.8497 | 0.1015 | 0.1113 | 8.9847 | 0.0113 | 88.4973 | 7.2881 |
| 25 | 10.8347 | 0.0923 | 0.1102 | 9.0770 | 0.0102 | 98.3471 | 7.4580 |
| 26 | 11.9182 | 0.0839 | 0.1092 | 9.1609 | 0.0092 | 109.1818 | 7.6186 |
| 27 | 13.1100 | 0.0763 | 0.1083 | 9.2372 | 0.0083 | 121.0999 | 7.7704 |
| 28 | 14.4210 | 0.0693 | 0.1075 | 9.3066 | 0.0075 | 134.2099 | 7.9137 |
| 29 | 15.8631 | 0.0630 | 0.1067 | 9.3696 | 0.0067 | 148.6309 | 8.0489 |
| 30 | 17.4494 | 0.0573 | 0.1061 | 9.4269 | 0.0061 | 164.4940 | 8.1762 |

The Projects

- The Exit Company is studying the purchase of a new palletizer.
- The parameters of the three palletizers under review are shown below.

| | <u>Palletizer Information</u> | | |
|-----------------------------------|-------------------------------|---|---|
| <u>Project Parameters</u> | <u>Alpha</u> | <u>Beta</u> | <u>Gamma</u> |
| 1. Initial Cost (\$) | 75,000 | 85,000 | 100,000 |
| 2. Annual revenues (\$) | 40,000 | 30,000 growing annually thereafter by 2%. | 50,000 |
| 3. Annual costs (\$) | 12,000 | 15,000 | 20,000 growing by \$500 annually thereafter |
| 4. End-of-life salvage value (\$) | -5,000 | 0 | 5,000 |
| 5. Useful life (years) | 4 | 10 | 5 |
| 6. Industry Standard | 3 years | | |
| MARR = 10% | | | |

Section A : Are the palletizers valid?

A1: Annual Equivalent Worth (AEW) Method

AEW (Alpha: N=4 years)

$$= -75,000(A/P, 10\%, 4) + 40,000 - 12,000 - 5000(A/F, 10\%, 4)$$

$$= \$3,262$$

AEW (Beta: N=10 years)

$$= -85,000(A/P, 10\%, 10) + \{[30,000/(0.1-0.02)][1-(1.02/1.1)^{10}]\}(A/P, 10\%, 10)$$

$$- 15,000 + 0(A/F, 10\%, 10)$$

$$= \$3,514$$

AEW (Gamma: N=5 years)

$$= -100,000(A/P, 10\%, 5) + 50,000 - 20,000 - 500(A/G, 10\%, 5) + 5,000(A/F, 10\%, 5)$$

$$= \$3,534$$

| <u>Palletizer</u> | <u>AEW</u> | <u>Acceptable/Unacceptable</u> <u>Is EAW > \$0?</u> |
|-------------------|------------|---|
| Alpha | \$3,262 | Yes; Acceptable |
| Beta | 3,514 | Yes; Acceptable |
| Gamma | 3,534 | Yes; Acceptable |

A2: Net Present Worth (NPW) Method

NPW (Alpha: N=4 years)

$$= -75,000 + (40,000 - 12,000)(P/A, 10\%, 4) - 5,000(P/F, 10\%, 4)$$

$$= \$10,341$$

NPW (Beta: N=10 years)

$$= -85,000 + \{[30,000/(0.1-0.02)][1-(1.02/1.1)^{10}]\} - 15,000(P/A, 10\%, 4)$$

$$= \$2,159$$

NPW (Gamma: N=5 years)

$$= -100,000 + 50,000(P/A, 10\%, 5) - 20,000(P/A, 10\%, 5) - 500(P/G, 10\%, 5)$$

$$+ 5,000(P/F, 10\%, 5)$$

$$= \$13,397$$

| <u>Palletizer</u> | <u>NPW</u> | <u>Acceptable/Unacceptable</u> <u>Is NPW > \$0?</u> |
|-------------------|------------|---|
| Alpha | \$10,341 | Yes; Acceptable |
| Beta | 21,591 | Yes; Acceptable |
| Gamma | 13,397 | Yes; Acceptable |

A3: Net Future Worth (NFW) Method

NFW (Alpha: N=4 years)

$$= -75,000(F/P, 10\%, 4) + (40,000 - 12,000)(F/A, 10\%, 4) - 5000$$
$$= \$15,141$$

NFW (Beta: N=10 years)

$$= -85,000(F/P, 10\%, 10) + \left\{ \frac{30,000}{(0.1 - 0.02)} \left[1 - \left\{ \frac{1.02}{1.1} \right\}^{10} \right] \right\} (F/P, 10\%, 10)$$
$$- 15,000(F/A, 10\%, 10)$$
$$= \$56,000$$

NFW (Gamma: N=5 years)

$$= -100,000(F/P, 10\%, 5) + (50,000 - 20,000)(F/A, 10\%, 5) - 500(F/G, 10\%, 5) + 5,000$$
$$= \$21,576 \quad [(F/G, i\%, N) = (P/A, i\%, N)(A/G, i\%, N)]$$

| <u>Palletizer</u> | <u>NFW</u> | <u>Acceptable/Unacceptable Is NFW > \$0?</u> |
|-------------------|------------|---|
| Alpha | \$15,141 | Yes; Acceptable |
| Beta | 56,000 | Yes; Acceptable |
| Gamma | 21,576 | Yes; Acceptable |

A4: Internal Rate of Return (IRR) Method

Alpha (N=4 years)

Set AE (Cash Outflows) = AE (Cash Inflows)

$$75,000(A/P, i^*, 4) = 40,000 - 12,000 - 5000(A/F, i^*, 4)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 16.40\%$$

OR

Set PW (Cash Outflows) = PW (Cash Inflows)

$$75,000 = (40,000 - 12,000)(P/A, i^*, 4) - 5,000(P/F, i^*, 4)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 16.40\%$$

OR

Set FW (Cash Outflows) = FW (Cash Inflows)

$$75,000(F/P, i^*, 4) = (40,000 - 12,000)(F/A, i^*, 4) - 5,000$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 16.40\%$$

Beta (N=10 years)

Set AE (Cash Outflows) = AE (Cash Inflows)

$$85,000(A/P, i^*, 10) = \{[30,000/(i^*-0.02)][1-\{1.02/(1+i^*)\}^{10}]\}(A/P, i^*, 10) - 15,000 + 0(A/F, i^*, 10)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.28\%$$

OR

Set PW (Cash Outflows) = PW (Cash Inflows)

$$85,000 = \{[30,000/(i^*-0.02)][1-\{1.02/(1+i^*)\}^{10}]\} - 15,000(P/A, i^*, 10)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.28\%$$

OR

Set FW (Cash Outflows) = FW (Cash Inflows)

$$85,000(F/P, i^*, 5) = \{[30,000/(i^*-0.02)][1-\{1.02/(1+i^*)\}^{10}]\}(F/P, i^*, 10)$$

$$- 15,000(F/A, i^*, 10)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.28\%$$

Gamma (N=5 years)

Set AE (Cash Outflows) = AE (Cash Inflows)

$$100,000(A/P, i^*, 5) = 50,000 - 20,000 - 500(A/G, i^*, 5) + 5,000(A/F, i^*, 5)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.07\%$$

OR

Set PW (Cash Outflows) = PW (Cash Inflows)

$$100,000 = (50,000 - 20,000)(P/A, i^*, 5) - 500(P/G, i^*, 5) + 5,000(P/F, i^*, 5)$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.07\%$$

OR

Set FW (Cash Outflows) = FW (Cash Inflows)

$$100,000(F/P, i^*, 5) = (50,000 - 20,000)(F/A, i^*, 5) - 500(F/G, i^*, 5) + 5,000$$

Solve for i^* . (Use Excel or equivalent or a financial calculator)

$$i^* = 15.07\%$$

| <u>Palletizer</u> | <u>IRR</u> | <u>Acceptable/Unacceptable</u> <u>Is IRR > MARR (=10%)?</u> |
|-------------------|------------|---|
| Alpha | 16.40% | Yes; Acceptable |
| Beta | 15.28% | Yes; Acceptable |
| Gamma | 15.07% | Yes; Acceptable |

A5: External Rate of Return (ERR) Method

Alpha

Set FW (Cash Outflows) = FW (Cash Inflows)

$$75,000(F/P, i^*, 4) = (40,000 - 12,000)(F/A, 10\%, 4) - 5,000$$

Solve directly for i^* (= %)

$$i^* = 13.61\%$$

Beta

Set FW (Cash Outflows) = FW (Cash Inflows)

$$85,000(F/P, i^*, 10) = \{30,000 / (0.1 - 0.02)\} [1 - \{1.02 / (1.1)\}^{10}] - 15,000(F/A, 10\%, 10)$$

Solve directly for i^* (= %)

$$i^* = 12.52\%$$

Gamma

Set FW (Cash Outflows) = FW (Cash Inflows)

$$100,000(F/P, i^*, 5) = (50,000 - 20,000)(F/A, 10\%, 5) - 500(F/G, 10\%, 5) + 5,000$$

Solve directly for i^* (= %)

$$i^* = 12.80\%$$

| <u>Palletizer</u> | <u>ERR</u> | <u>Acceptable/Unacceptable</u> <u>Is ERR > MARR (=10%)?</u> |
|-------------------|------------|---|
| Alpha | 13.61% | Yes; Acceptable |
| Beta | 12.52% | Yes; Acceptable |
| Gamma | 12.80% | Yes; Acceptable |

A6: Benefit-Cost Ratio (B/C)

- The AE, PW and FW methods can be used to calculate a project's B/C ratio.
- Each method (AE, PW and FW) gives the same ratio.

| | <u>Projects</u> | | |
|---------------------|-----------------|-------------|--------------|
| | <u>Alpha</u> | <u>Beta</u> | <u>Gamma</u> |
| AEW Benefits | 40,000 | 32,347 | 50,000 |
| AEW Costs | 36,738 | 28,833 | 46,466 |
| PW Benefits | 126,795 | 198,759 | 189,539 |
| PW Costs | 116,453 | 177,169 | 176,142 |

Alpha (N=4 years)

AE (Alpha: Benefits) / AE (Alpha: Costs)
= 40,000/36,738 = 1.09

OR

PW (Alpha: Benefits) / PW (Alpha: Costs)
= 126,795/116,453 = 1.09

Beta (N=10 years)

AE (Beta: Benefits) / AE (Beta: Costs)
= 32,347 / 28,833 = 1.12

PW (Beta: Benefits) / PW (Beta: Costs)
= 198,759 / 177,169 = 1.12

Gamma (N=5 years)

AE (Gamma: Benefits) / AE (Gamma: Costs)
= 50,000 / 46,466 = 1.08

PW (Gamma: Benefits) / PW (Gamma: Costs)
= 189,539 / 176,142 = 1.08

| <u>Palletizer</u> | <u>B/C</u> | <u>Acceptable/Unacceptable</u> <u>Is B/C > 1?</u> |
|--------------------------|-------------------|---|
| Alpha | 1.09 | Yes; Acceptable |
| Beta | 1.12 | Yes; Acceptable |
| Gamma | 1.08 | Yes; Acceptable |

A7: Simple Payback Method

Payback period (n) = Number of years to recover the initial investment (P)
when MARR = 0%

Alpha (N=4 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|-------------------|-----------------------------|--------------------------------|-------------------------------|
| 0 | -75,000 | 0 | -75,000 |
| 1 | 28,000 | 0 | -47,000 |
| 2 | 28,000 | 0 | -19,000 |
| 3 | 28,000 | 0 | 9,000 |

| | | | |
|---|--------|---|--|
| 4 | 23,000 | 0 | |
|---|--------|---|--|

➤ Alpha's Recovery Period = $2 + 19000 / (19,000 + 9000) = 2.68$ years

Beta (N=10 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|------------|----------------------|-------------------------|------------------------|
| 0 | -85,000 | 0 | -85,000 |
| 1 | 15,000 | 0 | -70,000 |
| 2 | 15,600 | 0 | -54,400 |
| 3 | 16,212 | 0 | -38,188 |
| 4 | 16,836 | 0 | -21,352 |
| 5 | 17,473 | 0 | -3,879 |
| 6 | 18,122 | 0 | 14,244 |
| 7 | 18,785 | 0 | |
| 8 | 19,461 | 0 | |
| 9 | 20,150 | 0 | |
| 10 | 20,853 | 0 | |

➤ Beta's Recovery Period = $5 + 3879 / (3879 + 14244) = 5.21$ years

Gamma (N=5 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|------------|----------------------|-------------------------|------------------------|
| 0 | -100,000 | 0 | -\$100,000 |
| 1 | 30,000 | 0 | -\$70,000 |
| 2 | 29,500 | 0 | -\$40,500 |
| 3 | 29,000 | 0 | -\$11,500 |
| 4 | 28,500 | 0 | \$17,000 |
| 5 | 33,000 | 0 | |

➤ Gamma's Recovery Period = $3 + 11,500 / (11,500 + 17,000) = 3.40$ years

| <u>Palletizer</u> | <u>Simple Payback</u> | <u>Acceptable/Unacceptable Longer than Industry Standard?</u> |
|-------------------|-----------------------|---|
| Alpha | 2.68 | Yes; Acceptable |
| Beta | 5.21 | No; Unacceptable |
| Gamma | 3.4 | No; Unacceptable |

A8: Discounted Payback Method

Payback period (n) = Number of years required to recover the initial investment (P) when MARR = 10% (for example)

Alpha (N=4 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|------------|----------------------|-------------------------|------------------------|
| 0 | -75,000 | | -75000 |
| 1 | 28,000 | -7500 | -54,500 |
| 2 | 28,000 | -5450 | -31,950 |
| 3 | 28,000 | -3195 | -7,145 |
| 4 | 23,000 | -714.5 | 15,141 |

➤ Alpha's Recovery Period = $3 + 7,145 / (7,145+15,141) = 3.34$ years

Beta (N=10 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|------------|----------------------|-------------------------|------------------------|
| 0 | -85,000 | | -8500 |
| 1 | 15,000 | -8500 | -7850 |
| 2 | 15,600 | -7850 | -7075 |
| 3 | 16,212 | -7075 | -6161.3 |
| 4 | 16,836 | -6161.3 | -5093.81 |
| 5 | 17,473 | -5093.81 | -3855.89 |
| 6 | 18,122 | -3855.89 | -2429.24 |
| 7 | 18,785 | -2429.24 | -793.673 |
| 8 | 19,461 | -793.673 | 1073.017 |
| 9 | 20,150 | | |
| 10 | 20,853 | | |

➤ Beta's Recovery Period = $7 + 794 / (794+1,073) = 7.45$ years

Gamma (N=5 years; Industry Standard = 3 years)

| <u>EOY</u> | <u>Net Cash Flow</u> | <u>Opportunity Cost</u> | <u>Project Balance</u> |
|------------|----------------------|-------------------------|------------------------|
| 0 | -100,000 | | -100,000 |
| 1 | 30,000 | -10000 | -80,000 |
| 2 | 29,500 | -8000 | -58,500 |
| 3 | 29,000 | -5850 | -35,350 |
| 4 | 28,500 | -3535 | -10,385 |
| 5 | 33,000 | -1038.5 | 21,577 |

➤ Gamma's Recovery Period = $4 + 10,385 / (10,385+21,577) = 4.35$ years

| <u>Palletizer</u> | <u>Discounted Payback</u> | <u>Acceptable/Unacceptable Longer than Industry Standard?</u> |
|-------------------|---------------------------|---|
| Alpha | 3.34 | No: Unacceptable |
| Beta | 7.45 | No; Unacceptable |

| | | |
|--------------|-------------|-------------------------|
| Gamma | 4.35 | No; Unacceptable |
|--------------|-------------|-------------------------|

Section B : Which of the three projects is best?

B1: Annual Equivalent Worth (AEW) Method

Repeating a project without changing its parameter values has no impact on a [project's AEW. From AEW calculations in Section A, each project has an AEW > \$0 and is therefore acceptable.

| <u>Palletizer</u> | <u>AEW</u> | <u>Best Project</u> |
|-------------------|------------|---------------------|
| Alpha | \$3,262 | Gamma |
| Beta | 3,514 | |
| Gamma | 3,534 | |

B2: Net Present Worth (NPW) Method

- A 20-year period of analysis (the least-common denominator) is used to determine the best project.
- There are several ways to determine each project's NPW over 20 years:
 1. Use each project's AEW over 20 years.
 2. Use the NPW method by repeating each project within a 20-year period.

1. Using each project's AEW

| <u>Palletizer</u> | <u>AEW</u> | <u>Acceptable/Unacceptable</u> <u>Is EAW > \$0?</u> |
|-------------------|------------|---|
| Alpha | \$3,262 | Yes; Acceptable |
| Beta | 3,514 | Yes; Acceptable |
| Gamma | 3,534 | Yes; Acceptable |

$$\text{NPW (Alpha: 20 years)} = \text{AEW (Alpha: N=4 years)}(P/A, 10\%, 20) \\ = 3,262(8.5136) = \$27,771$$

$$\text{NPW (Beta: 20 years)} = \text{AEW (Beta: N=10 years)}(P/A, 10\%, 20) \\ = 3,514(8.5136) = \$29,917$$

$$\text{NPW (Gamma: 20 years)} = \text{AEW (Gamma: N=5 years)}(P/A, 10\%, 20) \\ = 3,534(8.5136) = \$30,087$$

| <u>Palletizer</u> | <u>NPW (20 years)</u> | <u>Best Project</u> |
|-------------------|-----------------------|---------------------|
| Alpha | \$27,771 | Gamma |
| Beta | 29,917 | |
| Gamma | 30,087 | |

2. Repeat each project as many times as required for a 20-year period of analysis. Use each project's NPW during its normal life for calculations.

Alpha is repeated 5 times (including the first investment) during a 20-year period.

$$\begin{aligned}
 &\text{NPW (Alpha: 20 years)} \\
 &= \text{NPW(N=4 years)} + \text{NPW(N=4 years)}(P/F, 10\%, 4) \\
 &+ \text{NPW(N= 4years)}(P/F, 10\%, 8) + \text{NPW(N = 4 years)}(P/F, 10\%, 12) \\
 &+ \text{NPW(N= 4years)}(P/F, 10\%, 16) \\
 &= \$27,771
 \end{aligned}$$

Beta is repeated twice (including the first investment) during a 20-year period.

$$\begin{aligned}
 &\text{NPW (Beta: 20 years)} \\
 &= \text{NPW(N=10 years)} + \text{NPW(N=10 years)}(P/F, 10\%, 10) \\
 &= \$29,917
 \end{aligned}$$

Gamma is repeated 4 times (including the first investment) during a 20-year period.

$$\begin{aligned}
 &\text{NPW (Gamma: 20 years)} \\
 &= \text{NPW(N=5 years)} + \text{NPW(Gamma=5 years)}(P/F, 10\%, 5) \\
 &+ \text{NPW(N=5 years)}(P/F, 10\%, 10) + \text{NPW(N=5 years)}(P/F, 10\%, 15) \\
 &= \$30,087
 \end{aligned}$$

B3: Net Future Worth (NFW) Method

- A 20-year period of analysis (the least-common denominator) is used to determine the best project.
- There are several ways to determine each project's NFW over 20 years:
 1. Use each project's AEW over 20 years.
 2. Use the NFW method by repeating each project within a 20-year period.

1. Using each project's AEW

| <u>Palletizer</u> | <u>AEW</u> | <u>Acceptable/Unacceptable</u> <u>Is EAW > \$0?</u> |
|--------------------------|-------------------|---|
| Alpha | \$3,262 | Yes; Acceptable |
| Beta | 3,514 | Yes; Acceptable |
| Gamma | 3,534 | Yes; Acceptable |

$$\text{NFW (Alpha: 20 years)} = \text{AEW (Alpha: N=4 years)}(F/A, 10\%, 20) \\ = 3,262(57.275) = \$186,831$$

$$\text{NFW (Beta: 20 years)} = \text{AEW (Beta: N=10 years)}(F/A, 10\%, 20) \\ = 3,514(57.275) = \$201,264$$

$$\text{NFW (Gamma: 20 years)} = \text{AEW (Gamma: N=5 years)}(F/A, 10\%, 20) \\ = 3,534(57.275) = \$202,410$$

| <u>Palletizer</u> | <u>NFW (20 years)</u> | <u>Best Project</u> |
|-------------------|-----------------------|---------------------|
| Alpha | \$186,831 | Gamma |
| Beta | 201,264 | |
| Gamma | 202,410 | |

2. Repeat each project as many times as required for a 20-year period of analysis. Use each project's NPW during its normal life for calculations.

Alpha is repeated 5 times (including the first investment) during a 20-year period.

$$\text{NFW (Alpha: 20 years)} \\ = \text{NFW}(N=4 \text{ years})(F/P, 10\%, 16) + \text{NFW}(N=4 \text{ years})(F/P, 10\%, 12) \\ + \text{NFW}(N=4 \text{ years})(F/P, 10\%, 8) + \text{NFW}(N=4 \text{ years})(F/P, 10\%, 4) \\ + \text{NFW}(N=4 \text{ years})(F/P, 10\%, 0) \\ = \$186,831$$

Beta is repeated twice (including the first investment) during a 20-year period.

$$\text{NFW (Beta: 20 years)} \\ = \text{NFW}(N=10 \text{ years})(F/P, 10\%, 10) + \text{NFW}(N=10 \text{ years})(F/P, 10\%, 0) \\ = \$201,264$$

Gamma is repeated 4 times (including the first investment) during a 20-year period.

$$\text{NFW (Gamma: 20 years)} \\ = \text{NFW}(N=5 \text{ years})(F/P, 10\%, 15) + \text{NFW}(N=5 \text{ years})(F/P, 10\%, 10) \\ + \text{NFW}(N=5 \text{ years})(F/P, 10\%, 5) + \text{NFW}(N=5 \text{ years})(F/P, 10\%, 0) \\ = \$202,410$$

| <u>Palletizer</u> | <u>NFW (20 years)</u> | <u>Best Project</u> |
|-------------------|---------------------------|---------------------|
| Alpha | \$186,831 | Gamma |
| Beta | 201,264 | |
| Gamma | 202,410 | |

B4: Incremental Internal Rate of Return (IRR) Method

Compare projects Alpha and Beta

- AEW is the preferred method since Alpha (N=4) and Beta (N=5) have different lives.
- If the NPW or the NFW methods are used, a common period of analysis (i.e., 20 years) is required.
- Set AEW (Alpha: 4 years) = AEW (Beta: 10 years)

$$\begin{aligned}
 & -75,000(A/P, i^{**\%}, 4) + 40,000 - 12,000 - 5000(A/F, i^{**\%}, 4) \\
 & = -85,000(A/P, i^{**\%}, 10) + \{ [30,000 / (i^{**} - 0.02)] [1 - (1.02 / (1 + i^*))^{10}] \} (A/P, i^{**\%}, 10) \\
 & - 15,000 + 0(A/F, i^{**\%}, 10) \\
 & \text{Solve for } i^{**\%}. \text{ (Use Excel or equivalent or a financial calculator)}
 \end{aligned}$$

➤ $i^{**\%} = 11.7\%$

Conclusion: Beta is better than Alpha.

Compare projects Beta and Gamma

- AEW is the preferred method since Alpha (N=4) and Beta (N=5) have different lives.
- If the NPW or the NFW methods are used, a common period of analysis (i.e., 20 years) will be required.
- Set AEW (Beta: 10 years) = AEW (Gamma: 5 years)

$$\begin{aligned}
 & -85,000(A/P, i^{**\%}, 10) + \{ [30,000 / (i^{**} - 0.02)] [1 - (1.02 / (1 + i^*))^{10}] \} (A/P, i^{**\%}, 10) \\
 & - 15,000 + 0(A/F, i^{**\%}, 10) \\
 & = -100,000(A/P, i^{**\%}, 5) + 50,000 - 20,000 - 500(A/G, i^{**\%}, 5) + 5,000(A/F, i^{**\%}, 5) \\
 & \text{Solve for } i^{**\%}. \text{ (Use Excel or equivalent or a financial calculator)}
 \end{aligned}$$

➤ $i^{**\%} = 10.5\%$

Conclusion: Gamma is better than Beta; Gamma is the best project..

B5: Incremental External Rate of Return (ERR) Method

- The NFW method **MUST** be used to calculate the incremental ERR.

Compare projects Alpha and Beta using a 20-year period of analysis.

$$\begin{aligned}
 & -75,000[1+(P/F,10\%,4)+(P/F,10\%,8)+(P/F,10\%,12)+(P/F,10\%,16)](F/P,i^{**\%},20) \\
 & + (40,000-12,000)(F/A,10\%,20) \\
 & -5,000[(F/P,10\%,16)+(F/P,10\%,12)+(F/P,10\%,8)+(F/P,10\%,4)+(F/P,10\%,0)] \\
 & = -85,000[1+(P/F,10\%,10)](F/P,i^{**\%},20)-15,000(F/A,10\%,20) \\
 & + \{[30,000/(0.1-0.02)][1-\{1.02/1.1\}^{10}]\}(F/P,10\%,20)+(F/P,10\%,10)
 \end{aligned}$$

Solve for $i^{**\%}$. (Use Excel or equivalent or a financial calculator)

- $i^{**\%} = 10.1\%$ (based on Excel's MIRR function)

Conclusion: Beta is better than Alpha.

Note: If there is more than 1 negative Net Cash Flows between 2 projects (such as Alpha and Beta), Excel's MIRR function provides an incorrect ERR.

Corrective steps:

1. Find the Present Worth (with MARR) of each negative net cash flow and add them to the initial cost (or to the differential between the initial costs if you are calculating an incremental ERR).
2. Replace each of the negative cash flows between the two projects with a \$0 value (actually not required; simply leave the cell empty).
3. The positive net cash flows are not adjusted.
4. Use Excel's MIRR function to calculate the ERR or incremental ERR using the adjusted net cash flows.

| EOY | Net Cash Flows (Beta – Alpha) | PW of Negative Cash Flows (Beta-Alpha) | Adjusted Net Cash Flows (Beta-Alpha) |
|-----|----------------------------------|---|--|
| 0 | -10,000 | -10,000 | -115,012 |
| 1 | -13,000 | -11,818 | 0 |
| 2 | -12,400 | -10,248 | 0 |
| 3 | -11,788 | -8,856 | 0 |
| 4 | 68,836 | | 68,836 |
| 5 | -10,527 | -6,536 | 0 |

| | | | |
|----|---------|---|----------------------|
| 6 | -9,878 | -5,576 | 0 |
| 7 | -9,215 | -4,729 | 0 |
| 8 | 71,461 | | 71,461 |
| 9 | -7,850 | -3,329 | 0 |
| 10 | -92,147 | -35,527 | 0 |
| 11 | -13,000 | -4,556 | 0 |
| 12 | 67,600 | | 67,600 |
| 13 | -11,788 | -3,415 | 0 |
| 14 | -11,164 | -2,940 | 0 |
| 15 | -10,527 | -2,520 | 0 |
| 16 | 70,122 | | 70,122 |
| 17 | -9,215 | -1,823 | 0 |
| 18 | -8,539 | -1,536 | 0 |
| 19 | -7,850 | -1,284 | 0 |
| 20 | -2,147 | -319 | 0 |
| | | Present Worth of Negative Cash Flows = 115,012 | Δ ERR = 10.1% |

Compare projects Beta and Gamma using a 10-year period.

$$\begin{aligned}
&= -85,000(F/P, i^{**\%}, 10) - 15,000(F/A, 10\%, 10) \\
&+ \{ [30,000 / (0.1 - 0.02)] [1 - \{1.02 / 1.1\}^{10}] \} (F/P, 10\%, 10) \\
&= -100,000 [1 + (P/F, 10\%, 5)] (F/P, i^{**\%}, 10) + (50,000 - 20,000) (F/A, 10\%, 10) \\
&- 500 (F/G, 10\%, 5) [1 + (F/P, 10\%, 5)] + 5,000 [1 + (F/P, 10\%, 5)]
\end{aligned}$$

Solve for $i^{**\%}$. (Use Excel or equivalent or a financial calculator)

➤ $i^{**\%} = 10.02\%$

Conclusion: Gamma is better than Beta. Gamma is the best project.

Note: If there is more than 1 negative Net Cash Flows between 2 projects (such as Alpha and Beta), Excel's MIRR function provides an incorrect ERR.

Corrective steps:

1. Find the Present Worth (with MARR) of each negative net cash flow and add them to the initial cost (or to the differential between the initial costs if you are calculating an incremental ERR).
2. Replace each of the negative cash flows between the two projects with a \$0 value (actually not required; simply leave the cell empty).

3. The positive net cash flows are not adjusted.
4. Use Excel's MIRR function to calculate the ERR or incremental ERR using the adjusted net cash flows.

| EOY | Net Cash Flows (Gamma-Beta) | PW of Negative Cash Flows (Gamma-Beta) | Adjusted Net Cash Flows (Gamma-Beta) |
|-----|-----------------------------|---|--------------------------------------|
| 0 | -15,000 | -15,000 | -67,451 |
| 1 | 15,000 | 0 | 15,000 |
| 2 | 13,900 | 0 | 13,900 |
| 3 | 12,788 | 0 | 12,788 |
| 4 | 11,664 | 0 | 11,664 |
| 5 | -84,473 | -52,451 | 0 |
| 6 | 11,878 | 0 | 11,878 |
| 7 | 10,715 | 0 | 10,715 |
| 8 | 9,539 | 0 | 9,539 |
| 9 | 8,350 | 0 | 8,350 |
| 10 | 12,147 | 0 | 12,147 |
| | | Present Worth of Negative Cash Flows = 67,461 | Δ ERR = 10.5% |

B6: Benefit-Cost Ratio (B/C)

- The AE, PW and FW methods can be used to calculate a project's B/C ratio.
- If the PW or FW methods are used to calculate the incremental B/C ratio, a common period of analysis must be used.

| | <u>Projects</u> | | |
|--------------|-----------------|-------------|--------------|
| | <u>Alpha</u> | <u>Beta</u> | <u>Gamma</u> |
| AEW Benefits | 40,000 | 32,347 | 50,000 |
| AEW Costs | 36,738 | 28,833 | 46,466 |
| PW Benefits | 126,795 | 198,759 | 189,539 |
| PW Costs | 116,453 | 177,169 | 176,142 |

Compare projects Alpha & Beta using the AE Method

Δ AE (Benefits: Beta - Alpha) / Δ AE (Costs: Beta - Alpha)

= (32,347 - 40,000) / (28,833 - 36,738) = 0.97

Conclusion: Alpha is better than Beta (?????).

Compare projects Alpha & Gamma using the AE Method

ΔAE (Benefits: Gamma - Alpha) / ΔAE (Costs: Gamma - Alpha)

= (50,000 - 40,000) / (46,466 - 36,738 = 1.03

Conclusion: Gamma is better than Alpha.

B7. The Simple Payback Method

- Use the project recovery periods calculated in section A7.
- Since Alpha is the only valid (acceptable) project (Beta and Gamma are unacceptable), Alpha is the best project.

| <u>Palletizer</u> | <u>Simple Payback</u> | <u>Best Project</u> |
|-------------------|-----------------------|---------------------|
| Alpha | 2.68 | Alpha |
| Beta | 5.21 | |
| Gamma | 3.4 | |

B8. The Discounted Payback Method

- Use the project recovery periods calculated in section A8.
- Since the projects have recovery periods that exceed the industry standard, no project is acceptable.

| <u>Palletizer</u> | <u>Discounted Payback</u> | <u>Best Project</u> |
|-------------------|---------------------------|---------------------|
| Alpha | 3.34 | None. |
| Beta | 7.45 | |
| Gamma | 4.35 | |