Problem Set 1
Review of Dynamics; Units; Fluid Properties

Fluids have mass and respond to applied forces according to Newtonís Laws like all other matter. Review the basic concepts of particle dynamics.

1.1 What acceleration would a vertically upward force of 30 lb_f give to a stone which weighs 10 lb_f, on earth? (Note that values of various physical constants are given in Appendix A).

1.2 What acceleration would that force give to the same stone on the moon where the acceleration due to gravity is 5.31ft/sec^2? (Ans. 91.2 ft/sec^2)

1.3 A mass on the end of a 1 m long string is swinging in a circle about a vertical axis. The half-angle of the cone of revolution generated by the stone is 60 . What is the speed of rotation, in revs/min (RPM), and what is the tension in the string if the mass is 1 kg? (Ans. 42.3 RPM, 19.6 N)

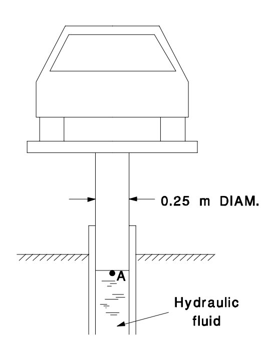
1.4 Decide whether each of the following statements is true or false.
(a) 1 lb_m always weighs 1 lb_f
(b) 1 lb_m weighs 1 lb_f (approximately) on earth
(c) the weight of a quantity of material is independent of its location
(d) the mass of a quantity of material is independent of its location.

1.5 Indicate the correct answer:
A fluid is a substance that:
(a) always expands until it fills any container
(b) is practically incompressible
(c) cannot be subjected to shear forces
(d) cannot remain at rest under action of any shear force
(e) has the same shear stress at a point regardless of its motion

1.6 Complete the following table of equivalents, assuming that the reference atmospheric pressure is 14.696 psia (the value at sea level for the standard atmosphere):

psia	psig	psfa	psfg	kPa(a)	kPa(g)
14.696					
	25				
		985			
			2116		
				101.32	
					10^5

1.7 The drawing shows an automobile held up by a hydraulic lift. The hydraulic piston and the automobile together have a mass of 3000 kg. The hydraulic piston has a diameter of 0.25 m. What is the pressure in the hydraulic fluid at point A? (Ans. 600 kPa)



1.8 An aircraft carries an oxygen cylinder to which a gauge is attached. The gauge reads 100 kPa(g). The air inside the aircraft is pressurized to 8 psi above the ambient pressure outside the aircraft, which is flying at 12,000 m in a standard atmosphere (see data sheets at back). What is the absolute pressure of the cabin air and of the oxygen in the tank?
(Ans. 74.6 kPa(a), 174.6 kPa(a))

1.9 Air is enclosed in a rigid cylinder containing a piston. A pressure gauge attached to the cylinder gives an initial reading of 20 psig. Determine the reading of the gauge after the piston has compressed the air to one third of its initial volume. Assume that the compression process is isothermal (i.e. constant temperature) and that the local atmospheric pressure is 14.7 psia.
(Ans. 89.4 psig)

1.10 As will be shown in lectures, the pressure variation in the vertical direction at any point in any fluid in a gravitational field is given by

$$\frac{dP}{dy} = -\rho g$$

where y is positive vertically upward, ρ is the local density of the fluid, and g is the local acceleration due to gravity.

Consider the change in atmospheric pressure from the bottom to the top of Mt. Tremblant. The bottom of the mountain is at an elevation of 266 m above sea level and the top is at 915 m. On a particular winter day, the atmospheric conditions at the bottom are: T = -20 C, P = 100 kPa. What is the density of the air under these conditions? (Ans. 1.377 kg/m^3)

Estimate the change in atmospheric pressure based on the following assumptions:

(a) Neglecting the variation in both ρ and g with elevation. (Ans. ΔP = -8770 Pa)
(b) Neglecting the variation in g but taking into account the change in ρ due to the change in pressure with elevation. (Ans. ΔP = -8390 Pa)
(c) In the lower part of the earthís atmosphere, the temperature is found to vary essentially linearly with elevation:

$$T = T_0 - B\left(y - y_0\right)$$

where T_0 is temperature at elevation y_0 and B is the "lapse rate" (B = 0.00650 K/m for the standard atmosphere). Estimate again the change in pressure, neglecting the variation in g but taking into account the effect of varying P and T on the density. What is the resulting density at the top of the mountain? (Ans. ΔP = -8460 Pa, 1.282 kg/m^3)

This problem illustrates the "compressibility" of gases: that is, that changes in pressure cause noticeable changes in the density. Here the change in pressure was the result of a change in elevation. As will be shown later, changes in pressure also arise in moving fluids due to changes in velocity. Therefore, caution must be used in assuming "incompressible" conditions (that is, constant density) when dealing with gas flows. On the other hand, liquids are almost always assumed incompressible. The next problem provides an opportunity to examine the validity of the latter assumption.

1.11 The pressure of the water at a depth of 10 km in the ocean is about 100 MPa (recall that sea level atmospheric pressure is about 0.1 MPa). Assuming a value of the bulk modules of elasticity K = 2100 MPa, what is the density of the water at that depth, if the S.G. is 1.03 at the surface?

(Hint: The definition of the bulk modulus is

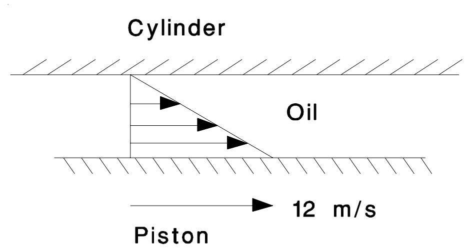
$$K = -V\left(\frac{\partial P}{\partial V}\right)$$

where V is volume. If K is assumed to remain constant as the pressure of the fluid is changed, then we can write

$$K = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

which relates the fractional change in the volume of the fluid, $\Delta V/V$, to the change in applied pressure, ΔP) (Ans. 1081 kg/m^3)

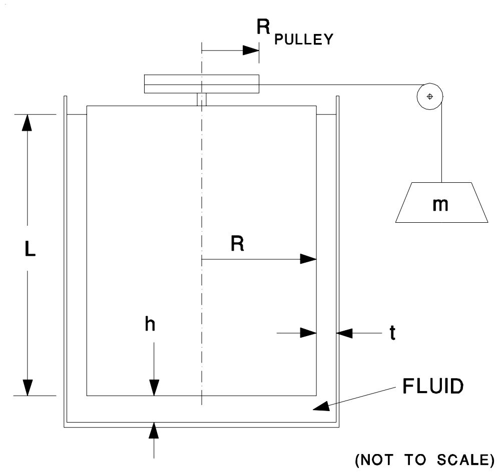
1.12 A smooth-sided piston 5 cm long and of diameter 5.000 cm moves within a cylinder of 5.010 cm diameter. The lubricating oil has a viscosity of 2×10^{-3} N s/m^2. What force is required to move the piston at a speed of 12 m/s? The velocity in the gap between the cylinder and the piston may be assumed to vary linearly, as indicated in the sketch. (Ans. 3.77 N)



1.13 The drawing shows a concentric-cylinder viscometer, a device for measuring the viscosity of a fluid. The fluid fills the gap between the two cylinders. The falling weight unreels a string from the pulley, thus rotating the inner cylinder. The viscosity of the fluid is then inferred from the rotational speed of the cylinder. Assume that the fluid velocity varies linearly both in the vertical gap between the cylinders and in the gap between the flat bottom of the inner cylinder and the bottom of the outer cylinder. The viscometer has the following dimensions: R = 6 cm, R_{PULLEY} = 3 cm, L = 15 cm, and t = h = 0.1 cm. When the falling weight has a mass of 30 gm, the cylinder is observed to rotate at 30 RPM.

(a) Estimate the viscosity of the fluid neglecting the frictional torque due to the fluid in the bottom gap.

(b) Repeat (a) but now include the effect of the fluid in the bottom gap.
(Ans. 1.38×10^{-2} Ns/m^2, 1.26×10^{-2} Ns/m^2)



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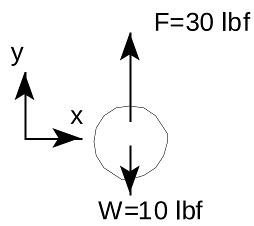
1.1 Vertical acceleration of body

$g := 32.174$ ft/sec^2



$W := 10$ lbf $\quad m := \frac{W}{g} \quad m = 0.3108$ slugs

$F := 30$ lbf

- then applying Newton's 2nd Law in y direction

$\Sigma F_y = m \cdot a_y$

$F - W = m \cdot a_y$ then $a_y := \frac{(F - W)}{m}$

$a_y = 64.35$ ft/sec^2

1.2 Acceleration experienced by body in 1.1 on the moon $\quad g := 5.31$ ft/sec^2

then $\quad W := m \cdot g \quad W = 1.65$ lbf

and thus $\quad a_y := \frac{(F - W)}{m} \quad a_y = 91.21$ ft/sec^2

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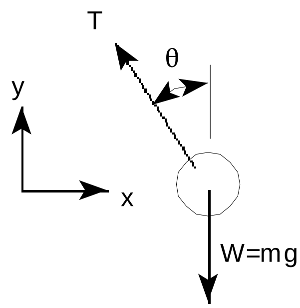
1.3 Mass at end of string

$\theta := 60$ deg. $\quad m := 1$ kg $\quad g := 9.81$ m/s^2



- length of string $\quad L := 1$ m

- thus the mass is moving in a horizontal circle of radius

$R := L \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right) \qquad R = 0.866$ m

- applying Newton's 2nd Law in vertical and horizontal directions

$\Sigma F_y = 0$ since $a_y = 0$

thus $\quad T \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right) - m \cdot g = 0 \qquad T := m \cdot \frac{g}{\cos\left(\frac{1}{180} \cdot \theta \cdot \pi\right)} \qquad T = 19.62$ N

- for the horizontal direction $\quad \Sigma F_x = m \cdot a_x \quad$ and $\quad a_x = -\frac{V^2}{R} = -R \cdot \omega^2$

(acceleration negative since is back towards the centre of the circle, which is in the negative x direction as co-ordinate system is defined)

then $\quad -T \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right) = -m \cdot R \cdot \omega^2$

solving for ω

$\omega := \sqrt{\frac{T \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right)}{m \cdot R}} \qquad \omega = 4.429$ rads/s

- or in RPM

$N := \frac{\omega}{2 \cdot \pi} \cdot 60 \qquad N = 42.3$ RPM

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1.4 True/false statements

(a) F
(b) T
(c) F
(d) T

1.5 Correct answer:

A fluid is a substance that: (d) cannot remain at rest under the action of a shear force.

1.6 Conversion of pressure units: From table of conversion factors: $kPa = psi \cdot 6.8947$

Reference atmospheric pressure: $P_{atm} := 14.696$ psia

$P_{atm} \cdot 144 = 2116.2$ psfa

$P_{atm} \cdot 6.8947 = 101.325$ kPa(a)

psia	psig	psfa	psfg	kPa(a)	kPa(g)
14.696	0	2116.2	0.0	101.325	0.0
39.696	**25.0**	5716.2	3600	273.693	172.368
6.840	-7.856	**985**	-1131.2	47.162	-54.163
23.390	14.694	4232.2	**2116**	202.638	101.313
14.695	0.0	2116.2	0.0	**101.32**	0.0
1.4519×10^4	1.4504×10^4	2.091×10^6	2.089×10^6	1.001×10^5	**10^5**

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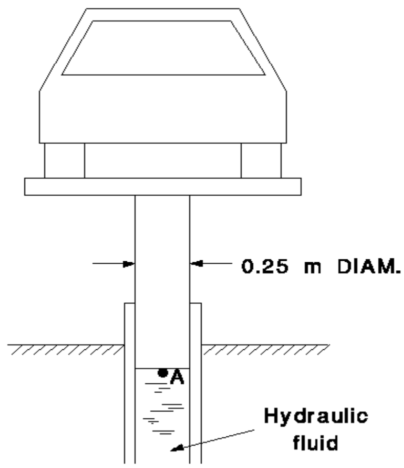
1.7 Pressure in hydraulic fluid



- mass supported by lift $m := 3000$ kg $g := 9.81$ m/s^2
- resulting force on bottom of piston $W := m \cdot g$ $W = 2.943 \times 10^4$ N
- area of piston $D := 0.25$ m $A := \frac{\pi}{4} \cdot D^2$

$A = 0.04909$ m^2

- then pressure at top of hydraulic fluid $P := \frac{W}{A}$ $P = 5.995 \times 10^5$ Pa

(600 kPa)

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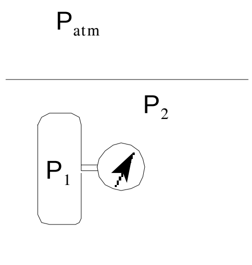
1.8 Pressurized tank inside an aircraft at 12,000 m altitude



- from the standard atmosphere table (Appendix A)

$P_{atm} := 19.399$ kPa at 12,000 m

- gauge on the tank shows the pressure in the tank relative to the surroundings i.e. conditions <u>inside</u> the aircraft

$P_1 - P_2 = 100$ kPa(g)

- the aircraft is pressurized to 8 psi above the outside (ambient) pressure

$\text{kPa} = \text{psi} \cdot 6.8947$ from table of conversion factors

thus $P_2 := P_{atm} + 8 \cdot 6.8947$ $P_2 = 74.56$ kPa(a)

and $P_1 := 100 + P_2$ $P_1 = 174.56$ kPa(a)

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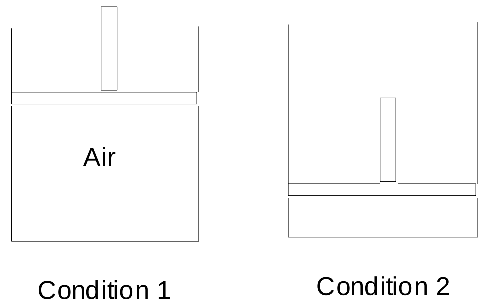
1.9 Change in pressure due to change in volume:



$P_{atm} := 14.7$ psia

Given: - initially (condition 1) $P_1 := 20$ psig

$P_1 := P_1 + P_{atm}$ $\qquad P_1 = 34.7$ psia

- after compression $V_2 = \frac{V_1}{3}$ (V = volume)

$T_2 = T_1$ (isothermal process specified)

- at both conditions, the pressure, temperature and density of the air in the cylinder are related through the Perfect Gas Law:

eg. at condition 1 $\qquad P_1 = \rho \cdot R \cdot T_1 = \frac{m}{V_1} \cdot R \cdot T_1$

- since same mass of air is present in the cylinder at the two conditions, can write

$$P_2 = \frac{m}{V_2} \cdot R \cdot T_2 = \frac{m}{\left(\frac{V_1}{3}\right)} \cdot R \cdot T_1 = 3 \cdot \left(\frac{m}{V_1} \cdot R \cdot T_1\right) = 3 \cdot P_1$$

thus $\qquad P_2 := 3 \cdot P_1 \qquad P_2 = 104.1$ psia

or $\qquad P_2 - P_{atm} = 89.4$ psig

Note: (1) In this course, whenever you need the density of a gas and it is not specified, you should always look for information which allows it to be calculated from the Perfect Gas Law. Only if there is insufficient information to calculate it should an approximate value (such as the density at sea level on a standard day) be used.
(2) N.B. The P and T used in the Perfect Gas Law must always be absolute values.

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1.10 Pressure variation with altitude at Mt. Tremblant

- top elevation: $y := 915$ m

- base elevation $y_0 := 266$ m

- base atmospheric conditions (winter's day with T = -20 C)

$T_0 := 253$ K $P_0 := 100000$ Pa $R := 287$

corresponding density $\rho := \dfrac{P_0}{R \cdot T_0}$ $\rho = 1.3772$ kg/m3

Estimating change in P over a change in elevation $\Delta y := y - y_0$ $\Delta y = 649$ m

(a) Neglecting change in ρ and g

$$\frac{dP}{dy} = -\rho \cdot g$$

- separating variables and integrating $\rho = 1.377$ kg/m3 $g := 9.807$ m/s2

$$\Delta P_1 := -\rho \cdot g \cdot \Delta y$$

$\Delta P_1 = -8765.5$ Pa

(b) Neglecting change in g but including change in ρ due to change in P, using perfect gas law (neglecting change in T)

$$\frac{dP}{dy} = -\frac{P}{R \cdot T} \cdot g \qquad \int \frac{1}{P}\, dP = \int -\frac{g}{R \cdot T}\, dy$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{g}{R \cdot T_0} \cdot \Delta y$$ taking exponentials $$\frac{P}{P_0} = e^{-\frac{g}{R \cdot T_0} \cdot \Delta y}$$

but $$\Delta P = P_0 \cdot \left(\frac{P}{P_0} - 1\right)$$ and thus $$\Delta P = P_0 \cdot \left(e^{-\frac{g}{R \cdot T_0} \cdot \Delta y} - 1\right)$$

$$\Delta P_2 := P_0 \cdot \left(e^{-\frac{g}{R \cdot T_0} \cdot \Delta y} - 1\right)$$ $\Delta P_2 = -8392.3$ Pa

(c) Neglecting change in g but including change in ρ due to change in P and change in T in the atmosphere

$$T = T_0 - B\cdot(y - y_0)$$ where B is the "lapse rate" $B := 0.00650$ K/m

- estimated T at top of mountain $T := T_0 - B\cdot(y - y_0)$ $T = 248.78$ K

$$\frac{dP}{dy} = -\frac{P}{R\cdot\left[T_0 - B\cdot(y - y_0)\right]}\cdot g \qquad \int_{P_0}^{P} \frac{1}{P}\,dP = \int_{y_0}^{y} -\frac{g}{R\cdot\left[T_0 - B\cdot(y - y_0)\right]}\,dy$$

integrating

$$\ln\left(\frac{P}{P_0}\right) = -\left(\frac{g}{R}\right)\cdot\left(-\frac{1}{B}\right)\cdot\ln\left[\frac{T_0 - B\cdot(y - y_0)}{T_0}\right]$$

and take exponentials of both sides

then $$\frac{P}{P_0} = \left[\frac{T_0 - B\cdot(y - y_0)}{T_0}\right]^{\frac{g}{B\cdot R}}$$ and finally $$\Delta P = P_0\cdot\left[\left[\frac{T_0 - B\cdot(y - y_0)}{T_0}\right]^{\frac{g}{B\cdot R}} - 1\right]$$

$$\Delta P_3 := P_0\cdot\left[\left[\frac{T_0 - B\cdot(y - y_0)}{T_0}\right]^{\frac{g}{B\cdot R}} - 1\right] \qquad \Delta P_3 = -8460 \text{ Pa}$$

- corresponding pressure at top of mountain $P := P_0 + \Delta P_3$ $P = 91540$ Pa

and density at top of mountain $\rho := \frac{P}{R\cdot T}$ $\rho = 1.2821$ kg/m^3

Summarizing ΔP results:

Neglecting changes in ρ and g:	$\Delta P_1 = -8765.5$	Pa
Neglecting changes in T and g:	$\Delta P_2 = -8392.3$	Pa
Neglecting changes in g:	$\Delta P_3 = -8460$	Pa

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1.11 Density at depth in the ocean - application of bulk modulus:

- from definition of bulk modulus

$$K = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} \qquad K := 2100 \cdot 10^{6} \text{ Pa} \qquad \text{for water}$$

- then consider moving a reference volume of water from the surface to 10 km depth

$$P_1 := 0.1 \cdot 10^{6} \text{ Pa} \qquad P_2 := 100 \cdot 10^{6} \text{ Pa}$$

$$\Delta P := P_2 - P_1 \qquad \Delta P = 9.99 \times 10^{7} \text{ Pa}$$

- consider 1 m^3 at the surface $\rho := 1030$ kg/m^3 (since S.G = 1.03)

$$V := 1 \text{ m}^3 \qquad \text{thus} \qquad m := \rho \cdot V \qquad m = 1030 \text{ kg}$$

- then change in volume as this mass is moved from the surface to depth

$$\Delta V := \frac{-\Delta P \cdot V}{K} \qquad \Delta V = -0.04757 \text{ m}^3$$

resulting new volume $V_{depth} := V + \Delta V$

$$V_{depth} = 0.95243 \text{ m}^3$$

- but mass present is unchanged, hence

$$\rho_{depth} := \frac{m}{V_{depth}} \qquad \rho_{depth} = 1081 \text{ kg/m}^3$$

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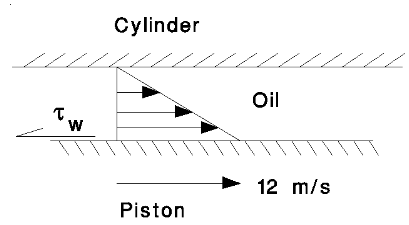
1.12 Force to move piston in cylinder

$D_{piston} := \frac{5.000}{100}$ $\quad D_{piston} = 0.05$ m

$D_{cylinder} := \frac{5.010}{100}$ $\quad D_{cylinder} = 0.0501$ m

$L_{piston} := \frac{5}{100}$ $\quad L_{piston} = 0.05$ m



- then gap width $\quad H := \frac{1}{2}\cdot\left(D_{cylinder} - D_{piston}\right)$ $\quad H = 5 \times 10^{-5}$ m

- piston feels a resisting force because of the fluid friction at its surface
- shear stress at any plane in a viscous (Newtonian) fluid is given by

$\tau = \mu\cdot\frac{du}{dy}$ here $\mu := 2\cdot 10^{-3}$ Ns/m2 (given)

or at the surface of the piston $\quad \tau_w = \mu\cdot\left(\frac{du}{dy}\right)$ with du/dy evaluated at the wall

- since velocity varies linearly, the velocity gradient is the same at every plane in the gap

$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{u_{piston} - u_{cylinder}}{H}$ (using no-slip condition)

$u_{piston} := 12$ $\quad u_{cylinder} := 0$

- thus the shear stress (tangential force per unit area) experienced at the surface of the cylinder

$\tau_w := \mu\cdot\frac{u_{piston} - u_{cylinder}}{H}$ $\quad \tau_w = 480$ N/m2

- this stress is applied to the wetted surface area of the piston

$A := 2\cdot\pi\cdot\frac{D_{piston}}{2}\cdot L_{piston}$ $\quad A = 7.853982 \times 10^{-3}$ m2

- shear force is then $\quad F_s := \tau_w\cdot A$ $\quad F_s = 3.77$ N

- this is the force which must be applied to move the piston at 12 m/s

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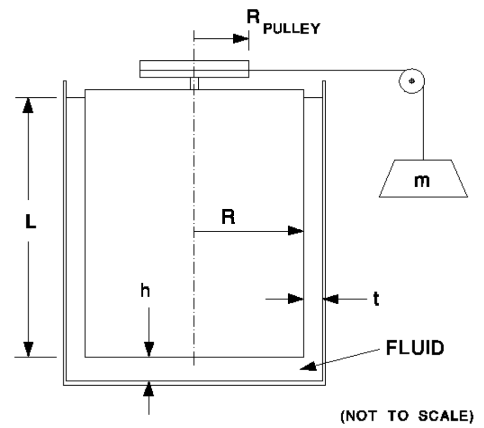
1.13 Viscometer analysis

- dimensions:



$R := \frac{6}{100}$ $R = 0.06$ m

$R_{pulley} := \frac{3}{100}$

$R_{pulley} = 0.03$ m

$L := \frac{15}{100}$ $L = 0.15$ m

$h := \frac{0.1}{100}$ $h = 1 \times 10^{-3}$ m

$t := h$ $t = 1 \times 10^{-3}$ m

- for falling weight $m := \frac{30}{1000}$ $m = 0.03$ kg $g := 9.81$ m/s2

thus $W := m \cdot g$ $W = 0.294$ N

and moment about axis $M_{pulley} := W \cdot R_{pulley}$ $M_{pulley} = 8.829 \times 10^{-3}$ N-m

- when the inner cylinder reaches a steady rotational speed, the moment on the pulley must be exactly balanced by the moment on the cylinder due to the fluid friction

(a) - taking into account the friction in the vertical gap only

- tangential velocity at edge of inner cylinder $V_t = R \cdot \omega$

- velocity varies linearly from this value to zero on surface of the outer cylinder (no-slip condition)

- resulting shear stress on every plane in the gap (and thus at the inner cylinder wall) $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{\Delta u}{\Delta y} = \frac{\mu \cdot R \cdot \omega}{t}$

- this acts on area $A = 2 \cdot \pi \cdot R \cdot L$ (wetted piston area in the gap)

- then total tangential force at surface of piston

$$F_t = \tau \cdot A = \frac{\mu \cdot R \cdot \omega}{t} \cdot 2 \cdot \pi \cdot R \cdot L = 2 \cdot \mu \cdot R^2 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

- this acts at radius R so corresponding moment is

$$M_{gap} = F_t \cdot R = \frac{\mu \cdot R \cdot \omega}{t} \cdot 2 \cdot \pi \cdot R \cdot L \cdot R = 2 \cdot \mu \cdot R^3 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

- equating this to the pulley moment and solving for μ

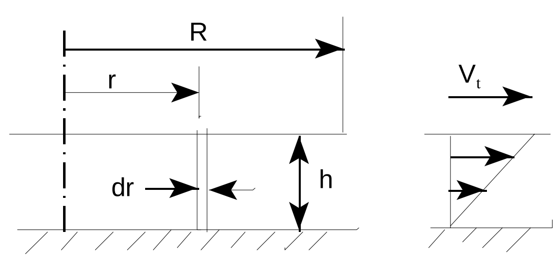
$$M_{pulley} = 2 \cdot \mu \cdot R^3 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

then $\mu = \frac{1}{2} \cdot \frac{M_{pulley}}{\left(R^3 \cdot \omega\right)} \cdot \frac{t}{(\pi \cdot L)}$ since $N := 30$ $\omega := \frac{2 \cdot \pi \cdot N}{60}$ $\omega = 3.142$ rads/s

finally $\mu := \frac{1}{2} \cdot \frac{M_{pulley}}{\left(R^3 \cdot \omega\right)} \cdot \frac{t}{(\pi \cdot L)}$ $\mu = 1.3805 \times 10^{-2}$ Ns/m^2

(b) - including friction on the bottom of the cylinder

- for the gap at the bottom, again assume linear velocity variation across gap



- tangential velocity at radius r $V_t = r \cdot \omega$

then $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{\Delta u}{\Delta y} = \frac{\mu \cdot r \cdot \omega}{h}$

- this acts on a ring of width dr, which has area $dA = 2 \cdot \pi \cdot r \cdot dr$

and resulting tangential force $dF_t = \tau \cdot dA$

- then the total frictional moment on the bottom of the cylinder

$$M_{bottom} = \int r\, dF_t$$

thus $$M_{bottom} = \int_0^R \frac{r\cdot\mu\cdot r\cdot\omega}{h}\cdot 2\cdot\pi\cdot r\, dr$$ or $$M_{bottom} = \int_0^R 2\cdot r^3\cdot\mu\cdot\frac{\omega}{h}\cdot\pi\, dr$$

- since μ, ω and h are not a function of radius, integration gives

$$M_{bottom} = \frac{2\cdot\pi\cdot\mu\cdot\omega}{h}\cdot\frac{R^4}{4}$$

$$M_{pulley} = 2\cdot\mu\cdot R^3\cdot\frac{\omega}{t}\cdot\pi\cdot L$$

- adding to this the moment due to the vertical gap, total frictional moment is

$$M = \frac{2\cdot\pi\cdot\mu\cdot\omega}{h}\cdot\frac{R^4}{4} + 2\cdot\mu\cdot R^3\cdot\frac{\omega}{t}\cdot\pi\cdot L$$

then $$\mu := \frac{M_{pulley}}{\left(\frac{\pi}{2}\right)\cdot\omega\cdot\frac{R^4}{h} + 2\cdot\pi\cdot\omega\cdot L\cdot\frac{R^3}{t}}$$

$$\mu = 1.255\times 10^{-2}$$ Ns/m2