Concordia University

Department of Computer Science & Software Engineering

COMP 232/4 **Mathematics for Computer Science** Winter 2016

Assignment 4 Due date: April 15, 2016

1. Write out the addition and multiplication tables for $\mathbb{Z}_6$ (where by addition and multiplication we mean $+_6$ and $\cdot_6$).

2. Let $f : \mathbb{Z}^2 \longrightarrow \mathbb{Z}^2$ be given by $f(m,n) = (m-n, n)$. The composite functions f_k, for $k \in \mathbb{Z}^+$, are defined as $f_1(m,n) = f(m,n)$, and $f_{k+1}(m,n) = f(f_k(m,n))$, for $k \in \mathbb{Z}^+$. Give a formal proof by induction that $f_k(m,n) = (m-kn, n)$, for all $k \in \mathbb{Z}^+$.

3. Use induction to show that for all positive integers n
   (a) $1^3 + 2^3 + 3^3 + \ldots + n^3 = (n(n+1)/2)^2$.
   (b) $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$
   (c) if $n > 6$, then $3^n < n!$

4. Let n be an integer greater than or equal to 2. Prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements.

5. The Fibonacci numbers are defined as: $f_1 = 1$, $f_2 = 1$, and
   $$f_n = f_{n-1} + f_{n-2} , \qquad \text{for } n \geq 3 .$$
   Give a proof by induction to show that $3 \mid f_{4n}$, for all $n \geq 1$.

6. Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. Use strong mathematical induction to show that $P(n)$ is true for $n \geq 18$.

7. Give a recursive definition of each of these sets of ordered pairs of positive integers.
   (a) $S = \{(a,b) : a \in \mathbb{Z}^+, b \in \mathbb{Z}^+, \text{ and } a+b \text{ is odd}\}$
   (b) $S = \{(a,b) \ : \ a \in \mathbb{Z}^+, b \in \mathbb{Z}^+, \ \text{and } a \mid b\}$
   (c) $S = \{(a,b) \ : \ a \in \mathbb{Z}^+, b \in \mathbb{Z}^+, \ \text{and } 3 \mid (a+b)\}$

8. For each of the following relations on the set of all real numbers, determine whether it is reflexive, symmetric, antisymmetric, transitive. Here xRy if and only if:

(a) $x + 2y = 0$	(b) $x = 2y$
(c) $x - y$ is a rational number	(d) $xy = 0$
(e) $xy \geq 0$	(f) $x = 1$ or $y = 1$
(g) x is a multiple of y	(h) $xy = 1$

9. Determine the matrix which represents the transitive closures of the following relations on the set $\{a, b, c, d, e\}$:

   (a) $\{(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)\}$

   (b) $\{(a, b), (a, c), (a, e), (b, a), (b, c), (c, a), (c, b), (d, a), (e, d)\}$

10. List all possible relations on the set $\{0, 1\}$ and determine which of these relations are

    (a) reflexive (b) symmetric (c) antisymmetric (d) transitive

11. Give the equivalence classes of the relation

$$aRb \quad \text{if and only if} \quad a^4 \equiv b^4 (\text{mod } 30) \;,$$

on the set $\{1, 2, 3, ..., 15\}$.

12. Are the relations represented by the following matrices equivalence relations?

$$\text{(a)} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

13. Which of these collections of subsets are partitions of the set of integers?

    (a) the set of even integers and the set of odd integers

    (b) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3

    (c) the set of integers less than -100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100

    (d) the set of integers not divisible by 3, the set of even integers, and the set of integers that leave a remainder of 3 when divided by 6.