Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Discrete Mathematics for Computing MAT1348A

Midterm Examination α

2 March 2015

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Instructions:

- This is an 80-minute *closed-book* exam; no notes are allowed. Calculators are not allowed.
- The exam consists of 11 questions on 9 pages. Page 9 is for additional work. *Please do not detach it.*
- Questions 1-4 are multiple-choice. You must enter the letter corresponding to each correct answer in the table preceding Question 1. No partial marks will be given for other work.
- Questions 5-9 are short-answer. You must write your final answer in the answer box and show your work below, justifying the answer.
- Questions 10-11 are long-answer. You must clearly show all relevant steps and justify your solution to receive full marks. Clearly indicate the final answer.
- Be sure to read carefully and follow the instructions for the individual problems.
- For rough work, you may use the back pages. Do not use scrap paper of your own.
- Use proper mathematical notation and terminology.
  Note: for functions, injective = one-to-one, surjective = onto.
- If you require clarification, raise your hand.
- Good luck!

Last name:____________________

First name:____________________

Student number:____________________

Signature:____________________

Question	$1-4$	$5-6$	$7-8$	9	10	11	Total
Max	4×2	$2+2$	2+3	4	4	5	30
Marks							

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Questions 1–4 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4
Answer				

[2pts] 1. Let A and B be finite non-empty sets. Which of the following statements are **false**?

(i) If $|A| > |B|$, then no function $f : A \to B$ is one-to-one.

(ii) If there exists a bijection $f : A \to B$, then $|A| = |B|$.

(iii) If $|A| \geq |B| > 1$, then every function $f : A \to B$ is onto.

(iv) If $|A| > |B|$, then no function $f : A \to B$ is onto.

(v) If there exists a one-to-one function $f : A \to B$, then $|A| \leq |B|$.

A. only (iv) **B.** (i) and (iii) **C.** only(iii) **D.** (iii) and (iv)

E. (iv) and (v) **F.** (ii) and (v) **G.** None of the previous answers is correct.

[2pts] 2. Let P be a complex proposition, and consider a complete truth tree with P in the root. Which of the following statements are **true**?

(i) If the truth tree for P has no active (open) paths, then P is a tautology.

(ii) If the truth tree for P has no active (open) paths, then $\neg P$ is a tautology.

(iii) If the truth tree for P has no inactive (closed) paths, then P is a tautology.

(iv) The number of complete active paths is equal to the number of counterexamples to the statement $\neg P$ *is a tautology*.

(v) Each complete active path corresponds to one or more counterexamples to the statement P *is a contradiction*.

A. (iii) and (v) **B.** (i) and (iii) **C.** only (ii) **D.** (ii) and (iv)

E. (iv) and (v) **F.** (ii) and (v) **G.** None of the previous answers is correct.

3. Consider the following three compound propositions:

$$P:\ (a \to b) \to c, \qquad Q:\ a \to (b \to c), \qquad \text{and} \qquad R : (a \wedge b) \to c$$

[2pts] Which of the propositions P, Q, and R are **equivalent**?

A. None of them. **B.** only P and Q **C.** only P and R **D.** only Q and R

E. All of them.

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4. The truth table of a compound proposition p with atomic propositions A, B, and C is as follows:

A	B	C	p
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

[2pts] Which of the following propositions are **disjunctive normal forms** of p?

(i) $A \vee B \vee C$

(ii) $(A \wedge B \wedge C) \vee (B \wedge \neg C)$

(iii) $(A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C)$

(iv) $(A \vee B \vee C) \wedge (A \vee B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

(v) $(A \wedge B) \vee (\neg A \wedge B \wedge \neg C)$

(vi) $(A \wedge B \wedge C) \vee (A \wedge B \wedge \neg C)$

A. (i), (iii), and (v) **B.** (ii), (iii), and (v) **C.** only (iii) **D.** (iii), (iv), and (v)

E. only (iii) and (v) **F.** None of the previous answers is correct.

In each of the following five questions, write your final answer in the answer box.

To receive full marks, you must show your work below the box, justifying the answer.

[2pts] 5. Let $A = \{0, 1, \{0, 1\}\}$ and $B = \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$. What is the **cardinality of the power set of** $A \times B$?

$|\mathcal{P}(A \times B)| =$

[2pts] 6. On the Island of Knights and Knaves, as you know, there are two types of natives, indistinguishable by sight: knights, who always tell the truth, and knaves, who always lie.

Strolling on the island, we meet two inhabitants A and B. Person A says: "B is a knave if and only if I am a knight." What is person B?

Answer: B is a

[2pts] 7. Define the following atomic propositions:

S: "Parking regulations are very strict."

Q: "Parisians question parking regulations."

M: "Michel is the mayor of Paris."

T: "Traffic in Paris is improved."

Translate the following sentence into a compound proposition using propositional variables S, Q, M, and T:

Traffic in Paris improves and Parisians do not question parking regulations
if and only if
parking regulations not being very strict is necessary for Michel to be the mayor of Paris.

Compound proposition:

[3pts] 8. Is the following argument valid? If not, give a counterexample.

$$\begin{array}{l} a \leftrightarrow b \\ \neg a \leftrightarrow c \\ \hline \therefore b \leftrightarrow c \end{array}$$

Answer: The argument is (circle): valid invalid

Counterexample (if applicable):

[4pts] 9. Consider the following functions:

$f : \mathbb{Q}^2 \to \mathbb{Q}^2$ defined by $f(x, y) = (x + y, x)$

$g : \mathbb{R}^+ \to \mathbb{R}$ defined by $g(x) = 2x^2 - 5$

$h : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{R}$ defined by $h(x, y) = \frac{2x}{y}$

$\ell : A \to A$, where $A = \{1, 2, 3, 4\}$, defined by $\ell(1) = 3$, $\ell(2) = 1$, $\ell(3) = 2$, $\ell(4) = 3$

Which of these functions are one-to-one? Which are onto?

One-to-one functions:

Onto functions:

10. Let q be a rational number and r an irrational number. Using a **proof by contradiction**, show that $q^2 - 5r$ is irrational.

[4pts]

11. Let A and B be two sets. Consider the following two statements; each one is either true or false. If a statement is true, give a rigorous proof. If a statement is false, give a counterexample, using the universal set $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. *Fully justify your answers.*

[5pts]

(a) $\overline{A \cap \overline{B}} = \overline{A} \cup \overline{B}$

(b) $A \subseteq B$ implies $A \subseteq A \cap B$.

Additional work space. Do not detach this page.