Université d'Ottawa · Univers
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and St

Discrete Mathematics for Computing MAT1348A

Midterm Examination (Version α)

16 February 2012

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Instructions:

- This is an 80-minute *closed-book* exam; no notes are allowed. Calculators (without graphing or programming function) are allowed, but not needed.
- The exam consists of 11 questions on 10 pages. Page 10 is for additional work. Please do not detach it.
- Questions 1-6 are multiple-choice. You must enter the letter corresponding to each correct answer in the table preceding Question 1. No partial marks will be given for other work.
- Questions 7-11 are long-answer. You must clearly show all relevant steps in your solution to receive full marks. Clearly indicate the final answer.
- Be sure to read carefully and follow the instructions for the individual problems.
- For rough work, you may use the back pages. Do not use scrap paper of your own.
- Use proper mathematical notation and terminology.
- If you require clarification, raise your hand.
- Good luck!

Last name:______________________________

First name:______________________________

Student number:__________________________

Signature:______________________________

Question	1 – 6	7	8	9	10	11	Total
Max	12	3	4	5	4	5	33
Marks							

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Questions 1–6 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4	5	6
Answer						

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1. The truth table of a compound proposition p with atomic propositions A, B, and C is as follows:

A	B	C	p
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

Only one of the following propositions is a **disjunctive normal form** of p — which one?

A. $(A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C)$
B. $(A \wedge \neg B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge \neg C)$
C. $\neg A \wedge \neg B \wedge \neg C$
D. $(A \vee \neg B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C)$
E. $\neg A \vee \neg B \vee \neg C$
F. $(A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge \neg C)$

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2. Let A and B be finite sets with $|A| = 5$ and $|B| = 2$. What is the **cardinality of the power set of** $A \times B$?

A. 4 **B.** 10 **C.** 16 **D.** 32 **E.** 512 **F.** 1024.

3. Which of the following statements are **true**?

(i) The compound proposition $(a \to b) \to b)$ is a tautology.
(ii) If the set of premises of an argument is inconsistent, then the argument is valid.
(iii) If X is false, Y is true, and Z is false, then $X \land Y \to Z$ is true.
(iv) The compound propositions $\neg((a \to b) \to c)$ and $a \land b \land \neg c$ are logically equivalent.

A. only (iii) **B.** only (iv) **C.** only (i) **D.** (i) and (iii)

E. (ii) and (iii) **F.** only (ii)

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4. On the Island of Knights and Knaves you meet two inhabitants A and B. Person B says: "A is a knave only if I am a knave." Which of the following statements is **true**?

(i) A is a knight and B is a knave.
(ii) A is a knave and B is a knight.
(iii) A and B are both knaves.
(iv) A and B are both knights.
(v) B is a knight but it is impossible to determine what A is.
(vi) A is a knight but it is impossible to determine what B is.

A. (v) **B.** (iii) **C.** (i) **D.** (vi) **E.** (iv) **F.** (ii)

5. Let $S = \{1, \{2\}, \{1, 2\}, \emptyset\}$. Which of the following statements are **true**?

   (i) $\{\{1\}, \emptyset\} \subseteq S$
   (ii) $\{1, \{2\}\} \in S$
   (iii) $\{1, \{1, 2\}\} \subseteq S$
   (iv) $\{1, 2\} \subseteq S$
   (v) The cardinality of the power set of S is 8.
   (vi) $\{\emptyset\} \in S$

   A. only (iii) **B.** (i) and (iii) **C.** only (v) **D.** (ii) and (vi)

   E. (iii) and (v) **F.** (iv) and (vi)

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6. Which of the following arguments (rules of inference) are **invalid**?

   (i) $\begin{array}{l} a \to b \\ \neg a \\ \hline \therefore \neg b \end{array}$ (ii) $\begin{array}{l} a \to b \\ \neg b \\ \hline \therefore \neg a \end{array}$ (iii) $\begin{array}{l} a \vee b \\ \neg a \vee c \\ \hline \therefore b \vee c \end{array}$

   (iv) $\begin{array}{l} a \vee b \\ \neg b \\ \hline \therefore a \end{array}$ (v) $\begin{array}{l} a \vee b \\ \neg a \vee c \\ \hline \therefore b \wedge c \end{array}$ (vi) $\begin{array}{l} a \to b \\ \neg a \to c \\ \hline \therefore \neg b \to c \end{array}$

   A. (i) and (v) **B.** (ii) and (v) **C.** (iii) and (vi) **D.** (i) and (iv) **E.** only (vi)

7. Let A, B, and C be subsets of the universal set U. Use properties of set operations and set identities to show the following. *You need not name the identities used.*

$$A - (\overline{C} \cap B) = (A \cap C) \cup (A - B)$$

8. Define the following atomic propositions:

   H: "The tiger hides."

   F: "The hunt is finished soon."

   K: "The tiger is killed."

   E: "The hunter is eaten by the tiger."

   N: "The hunt is happening at night."

   Translate each of the following sentences into compound logical propositions using the atomic propositions H, F, K, E, and N as defined above.

   (a) For the hunt to be finished soon, it is necessary that the tiger be killed or the hunter be eaten by the tiger.

   (b) The tiger hides only if the hunt is happening at night.

   (c) For the hunt to be finished soon, it is necessary and sufficient that the hunt be happening at night and the hunter be eaten by the tiger.

   (d) If the tiger hides or the hunt is not happening at night, then (the hunt is not finished soon unless the hunter is eaten by the tiger).

9. Use any method you know to determine whether or not the argument below is valid. If the argument is not valid, give a counterexample.

   $\neg(B \wedge \neg A)$

   $A \leftrightarrow \neg(B \rightarrow A)$

   $C \leftrightarrow B$

   $\therefore \neg(A \rightarrow C)$

10. Let n be an integer. Give an **indirect proof** of the following theorem.

    If $n^2 + 2n - 1$ *is an odd integer, then* n *is an even integer.*

11. Is the function $F : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by

$$F(x, y) = (2y, 3x + y)$$

   (a) one-to-one?
   (b) onto?
   (c) a bijection?

   Fully justify your answer.

Additional work space. Do not detach this page.