[8] **5.** Evaluate the given improper integral or show that it diverges:

$$\text{(a)} \int_0^4 \frac{1}{x\sqrt{x}}\,dx \qquad \text{(b)} \int_0^\infty x\,e^{-2x^2}\,dx$$

[17] **6.** **(a)** Sketch the curves $y = x^2 - 2$ and $y = |x|$ and find the area enclosed.

(b) Sketch the region enclosed by $f(x) = \cos^2(x)$ and the x-axes on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and find the volume of revolution of this region about the $x-$axis.

(c) Find the average value of the function $f(x) = \cos(x)\sin(2x)$ on the interval $[0, \frac{\pi}{2}]$.

[9] **7.** Find the limit of the sequence $\{a_n\}$ or explain why the limit does not exist:

$$\text{(a)}\ a_n = \frac{2^n + 5^{n+2}}{6^n} \qquad \text{(b)}\ a_n = \frac{\sqrt{n^4 + n^3}}{n + 3n^2} \qquad \text{(c)}\ a_n = \frac{n^2\cos(\pi n)}{1 + n^2}$$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally, and explain why.

$$\text{(a)} \sum_{n=1}^\infty \frac{(-1)^n n}{1 + 2n} \qquad \text{(b)} \sum_{n=2}^\infty \frac{(-1)^n}{n\ln(n)} \qquad \text{(c)} \sum_{n=2}^\infty \frac{\sqrt{n+4}}{n^2 + 4}$$

[6] **9.** Find the radius and the interval of convergence of the series

$$\sum_{n=1}^\infty \frac{(x-3)^{2n}}{n\,4^n}$$

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^2 e^{2x}$ (HINT: start with the series for e^z where $z = 2x$).

(b) Use differentiability of power series to find the sum $F(x) = \sum_1^\infty \frac{(x-1)^n}{n}$ within its radius of convergence.

[5] **Bonus Question.** Let f be a continuous function on the interval $[1, 4]$. Prove that $\bar{f}_{[1,4]} = \frac{1}{3}\bar{f}_{[1,2]} + \frac{2}{3}\bar{f}_{[2,4]}$, where $\bar{f}_{[1,4]}$, $\bar{f}_{[1,2]}$ and $\bar{f}_{[2,4]}$ are the average values of f on the respective intervals.