

MATH 1107 B Problem Set 3 (Due 10pm April 5, 2016 in cuLearn)

Name: _____ ID (last 3 digits only): _____

Instructions: Throughout this problem set, $\alpha\beta\gamma$ denote the right-most 3 digits of your student ID. For example, for the ID 123456789, $\alpha = 7$, $\beta = 8$, and $\gamma = 9$. **Using the incorrect values for a problem will result in a grade of zero (0) for the entire problem.**

You must follow all the submission requirements when you submit your solutions for marking. (See the document “Problem Set submission requirements” in cuLearn for details.) **Submissions that do not follow all the requirements will be assigned a grade of zero (0).**

1. (5 marks) What must be the value of q for $2x+q$ to be in the span of $\{x^2+x-3, -x^2-3x+2\}$? Explain.

2. (5 marks) Let $A = \begin{bmatrix} -1 & 3 & -1 \\ 1 & -3 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$. Is b in the column space of A ? Explain.

3. (5 marks) Let $A = \begin{bmatrix} -1 & 2 & 2 \\ 3 & -2 & -3 \\ -6 & 6 & 7 \end{bmatrix}$. You are given that the eigenvalues of A are 1 and 2.

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Show your work. (You do not need to give P^{-1}).

4. (5 marks) Let P_2 denote the vector space of quadratic polynomials in x with real coefficients. Let $T : \mathbb{R}^3 \rightarrow P_2$ be a linear transformation given by

$$T \left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = (a - b - 3c)x^2 + (2b + c)x + (a + b - 2c).$$

- (a) (4 marks) Give a basis for the range of T . Justify your answer.
- (b) (1 mark) Is T surjective? Explain.

5. (5 marks) Let $\Gamma = \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ \alpha \end{bmatrix} \right)$ and $\Omega = \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$ be ordered bases for \mathbb{R}^2 . Give the change-of-basis matrix from Γ to Ω . Show your work.

Acknowledgement