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CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2015	3
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[11] **1.** (a) Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{4-x}$. Find $g \circ f$ and $f \circ g$ and determine the domain of each of these composite functions.

(b) Find the range of the function $f = e^{2x} + 2e^x$, the inverse function f^{-1}, and the range of f^{-1}. (HINT: assume $e^x = u$ to see how to find f^{-1})

[10] **2.** Evaluate the limits:

(a) $\displaystyle\lim_{x\to -2} \frac{x^2 - x - 6}{4 - x^2}$ (b) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+9} - 3}{x}$

Do not use l'Hôpital rule.

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{|x|\sqrt{4x^2+1} - 2x^2}{x^2 - 3}$$

[15] **4.** Find the derivatives of the following functions:

(a) $f(x) = \dfrac{2\sqrt{x^5} - x^{3/2}}{x^2}$

(b) $f(x) = \ln \dfrac{x^4}{x-3}$

(c) $f(x) = e^3 + \arctan(e^x - e^{-x})$

(d) $f(x) = \dfrac{e^x}{1 + \cos(x^2)}$

(e) $f(x) = (1 + x^2)^{2x}$ (use logarithmic differentiation)

[15] **5.** **(a)** Verify that the point (2.0) belongs to the curve defined by the equation $y + x\sqrt{1+y^2} + 2 = x^2$, and find the equation of the tangent line to the curve at this point.

(b) A particle is moving along a circle with radius $r = 5\,\text{m}$ described by the equation $x^2 + y^2 = 25$ in the (x, y) plane. At the point $(-4, 3)$ the x-coordinate changes at the rate $\frac{dx}{dt} = 15\,\frac{\text{m}}{\text{sec}}$. How fast is the y coordinate changing at that instant?

(c) Use the l'Hôpital's rule to evaluate the $\lim\limits_{x \to 0} \dfrac{e^x - x - 1}{x^2 + x^3}$.

[6] **6.** Let $f(x) = \dfrac{x}{3x - 1}$.

(a) Find the slope m of the secant line joining the points $(1, f(1))$ and $(3, f(3))$.

(b) Find all points $x = c$ (if any) on the interval [1,3] such that $f'(c) = m$.

[9] **7.** The volume of a sphere with radius r is given by the formula $V(r) = \frac{4\pi}{3} r^3$.

(a) Use the **definition of the derivative** to show that $\frac{dV}{dr} = 4\pi r^2$.

(b) If a is a given fixed value for r, write the formula for the linearization of the volume function $V(r)$ at a.

(c) Use this linearization to calculate the thickness Δr (in centimeters) of a layer of paint on the surface of a spherical ball with radius $r = 52\,\text{cm}$ if the total volume of paint used is $340\,\text{cm}^3$.

[12] **8.** **(a)** Find the absolute extrema of $f(x) = x\,e^{-x^2}$ on the interval $[-\frac{1}{2}, 1]$.

(b) Find the radius r and the height h of the a cylindrical can that is open at the top and has a volume $1000\,\text{cm}^3$, but has the smallest possible surface area.

[16] **9.** Given the function $f(x) = 2x^2 - x^4$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question**

We know that a function f is differentiable on the interval [0,2] and has values $f(0) = 0$, $f(1) = 1$ and $f(2) = -1$. Is this information sufficient to claim, using the Mean Value theorem, that the tangent line to the graph of $f(x)$ must be horizontal at least at one point x in the interval (0,2)? Explain why yes or why not.