Multiple Choice – 30 pts total – 3 pts each

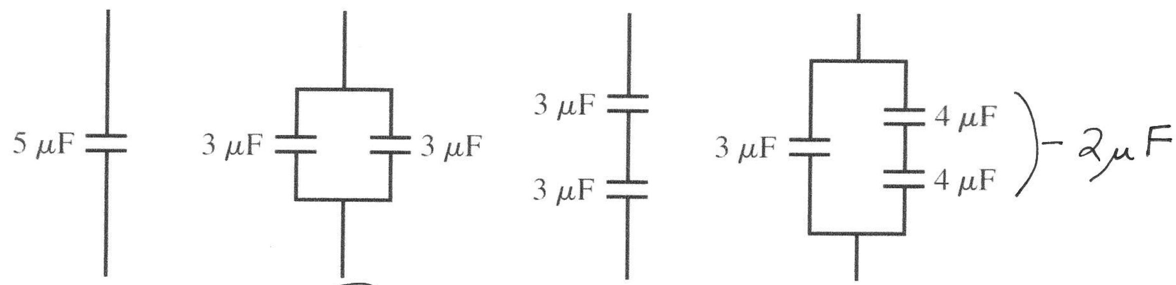
1) A parallel-plate capacitor initially has 320 J of stored energy and is then disconnected from the charging battery. If the plate spacing is now cut in half without changing the amount of charge on the plates, what is the new value of the stored energy?

$U = 320\,J = \frac{1}{2}CV^2 = \frac{1}{2} q^2/C$

$d \to d/2 \quad C \to 2C \quad$ so $U' = \frac{1}{2}\frac{q^2}{2C} = \frac{1}{2}U$

a) 160 J
b) 320 J
c) 640 J
d) 1280 J
e) none of the above

2) Which of the following multiple capacitor combinations has the largest capacitance?



(a) 5μF

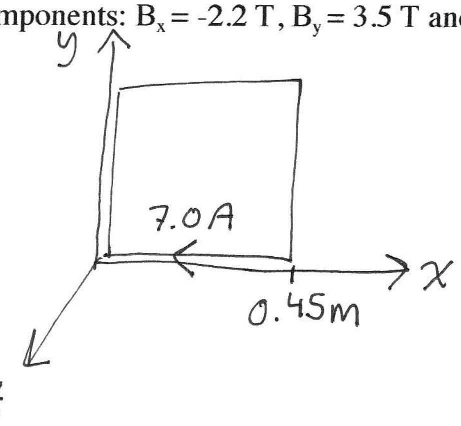
(b) 6μF

(c) 1.5μF

(d) 5μF

3) A square loop lies in the x-y plane such that one side lies along the x-axis. This side is 45 cm long and a current of 7.0 A runs in this side in the negative x-direction. What will be the components of the magnetic force on this side of the loop due to a magnetic field with the following components: $B_x = -2.2$ T, $B_y = 3.5$ T and $B_z = 0$?

a) $F_x = F_y = 0$, $F_z = -11.0$ N
b) $F_x = F_y = 0$, $F_z = 11.0$ N
c) $F_x = F_y = 0$, $F_z = -6.9$ N
d) $F_x = F_y = 0$, $F_z = 6.9$ N
e) None of the above



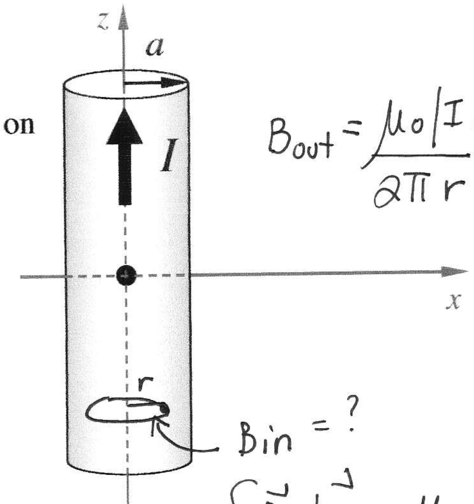
$\vec{F} = I\vec{L}\times\vec{B}$

$= (7.0A)(-0.45m\,\hat{i}) \times (-2.2T\,\hat{i} + 3.5T\,\hat{j})$

$= (7.0A)(-0.45m)(3.5\,T)\,\hat{i}\times\hat{j}$

$= -11.0\,N\,\hat{k}$

4) A uniformly distributed current flows in the +z direction in a cyndrical wire as shown in the figure to the right. Which of the following graphs best describes the magnitude of the magnetic field due to the wire at points on the positive x-axis?



$B_{out} = \frac{\mu_0 |I|}{2\pi r} \propto \frac{1}{r}$

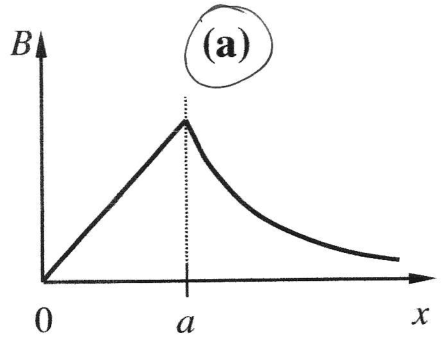
$B_{in} = ?$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I^{enc}$

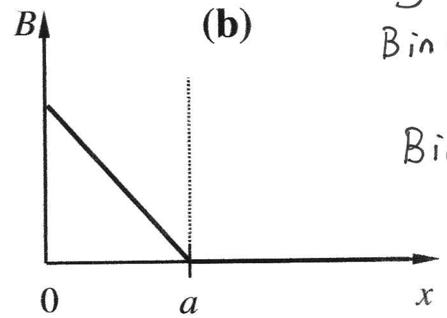
$B_{in} 2\pi r = \mu_0 I \frac{\pi r^2}{\pi a^2}$

$B_{in} = \mu_0 I \frac{r}{2\pi a^2} \propto r$

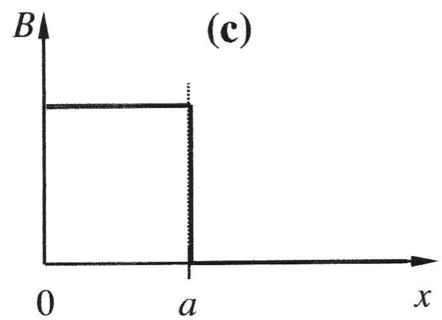
(a)



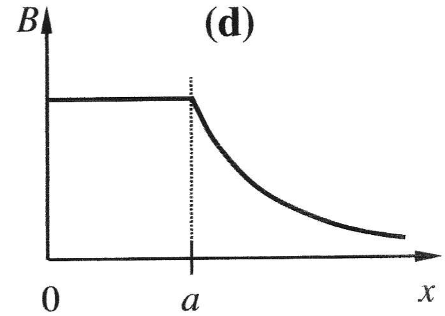
(b)



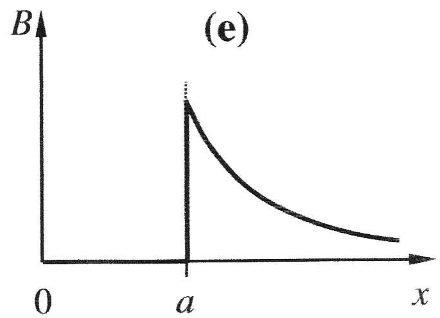
(c)



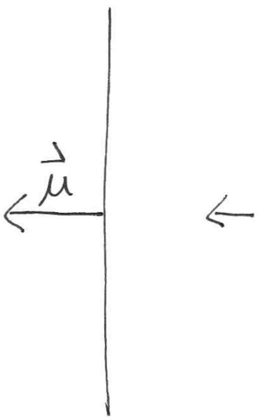
(d)

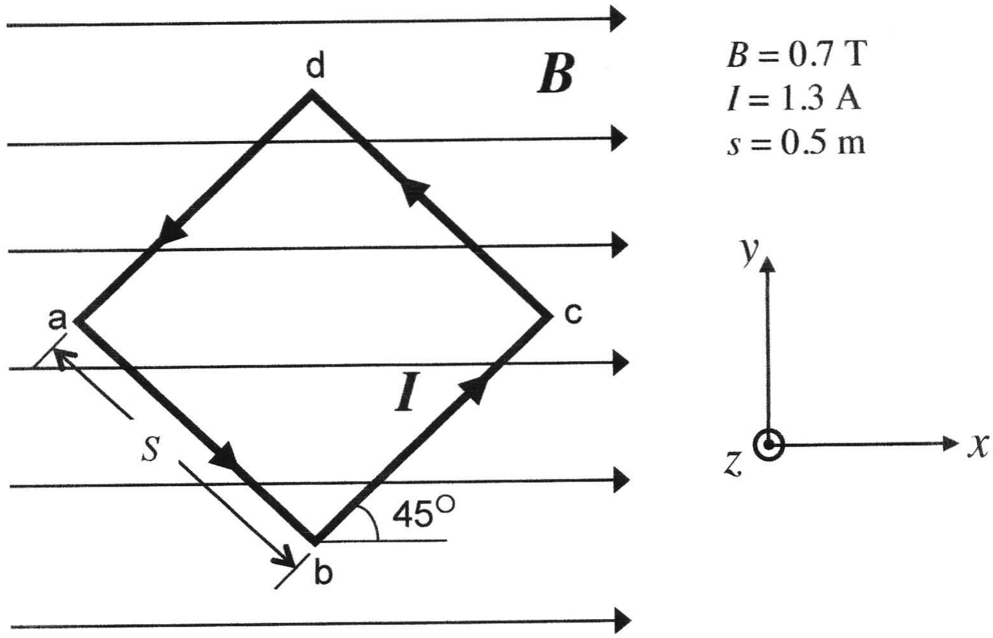


(e)



5) A square wire loop of side $s = 0.5$ m lies in the x-y plane and carries a current of $I = 1.3$ A flowing in the counter-clockwise direction (as viewed from $z > 0$). A constant, uniform magnetic field of magnitude $B = 0.7$ T points in the $+x$ direction, as shown in the figure.





The amount of work required to rotate this loop to an orientation which has the maximal potential energy is:

$U(\theta) = -\vec{\mu}\cdot\vec{B}$ $\quad W = \Delta U = U_f - U_i$

$= -\mu B\cos\theta \rightarrow W = \mu B - 0$

$W = NIAB = (1)(1.3A)(0.5m)^2(0.7T)$

$W = 0.23\ J$

a) 0 J
b) 0.23 J
c) 0.33 J
d) 0.46 J
e) None of the above

6) An RCL circuit contains an inductor with L = 5.00 mH, a capacitor with C = 10.0 μF and a resistor with R = 30.0 Ω all in series. It is driven with a sinusoidal emf: $\varepsilon = 200 \sin \omega t$ (V). At what frequency, f, is I_{rms} maximum?

I_{rms} max when $X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$\omega^2 = \frac{1}{LC}$

$\omega = \frac{1}{\sqrt{(5.00mH)(10.0\mu F)}}$

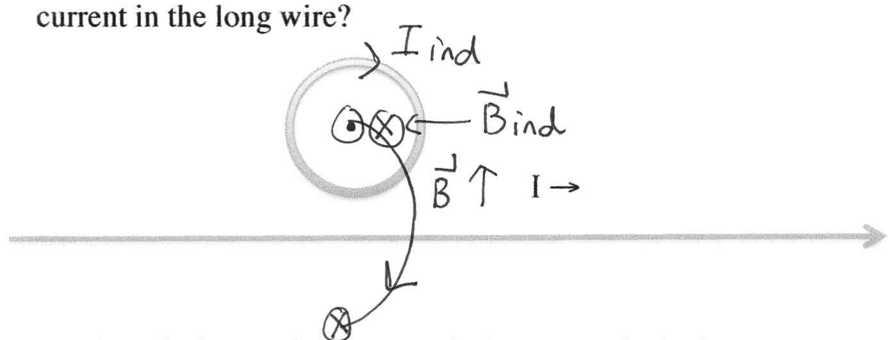
$\omega = 4472\ \frac{rad}{s}$

$f = \frac{\omega}{2\pi} = \frac{4472}{2\pi}$

$f = 712\ Hz$

a) 0 Hz
b) 712 Hz
c) 4.47 x 10^3 Hz
d) 3.18 x 10^6 Hz
e) None of the above

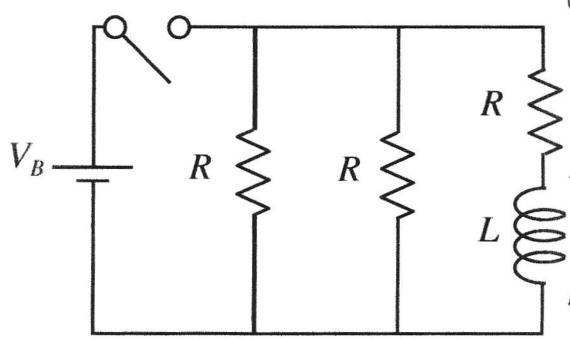
7) The current is increasing in a long straight wire running next to a wire loop as shown. What would be the directions of the induced magnetic field B(ind) inside the loop and the induced current I(ind) in the wire loop induced by the changing current in the long wire?



a) B(ind) out of the page, I(ind) counterclockwise
b) B(ind) out of the page, I(ind) clockwise
c) B(ind) into the page, I(ind) counterclockwise
d) B(ind) into the page, I(ind) clockwise
e) None of the above

8) Three identical resistors are connected to a battery, an inductor, and a switch as shown in the figure below. (The values of all circuit elements are given below the figure.) The switch has been open for a very long time, and then it is closed at time $t = 0$.

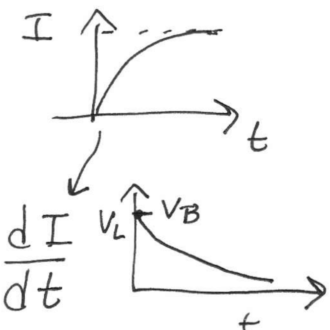
@t=0



$V_R = 0$

$I = 0$

$V_L = +L\frac{dI}{dt}$



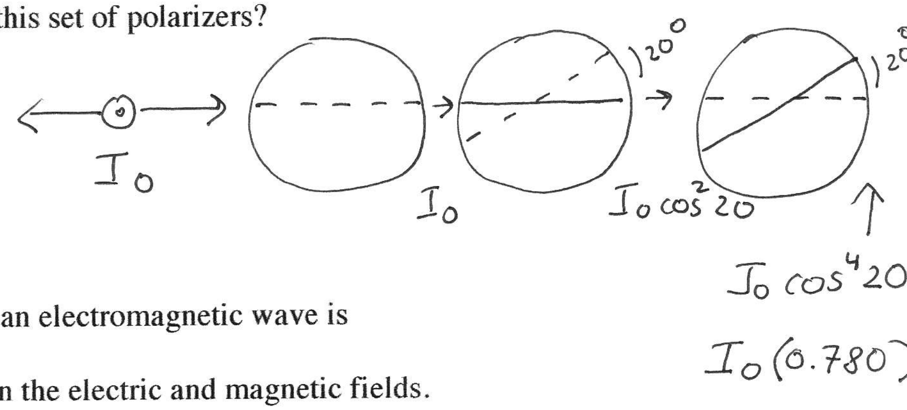
$V_L = 6\ V = V_B$

$V_B = 6$ V, $R = 17\ \Omega$, $L = 53\ \mu$H

Immediately after the switch is closed, the voltage across the inductor, V_L, is

a) $V_L = 0$ V
b) $V_L = 3$ V
c) $V_L = 6$ V
d) $V_L = 12$ V
e) None of the above

9) Incident light of intensity I_0 and polarized horizontally passes through three polarizers. The first and third have horizontal transmission axes, but the second's transmission axis is oriented 20.0^0 to the horizontal. In terms of I_0, what is the intensity of the light that exits this set of polarizers?



a) 0
b) I_0
c) $0.220\ I_0$
d) $0.883\ I_0$
e) None of the above

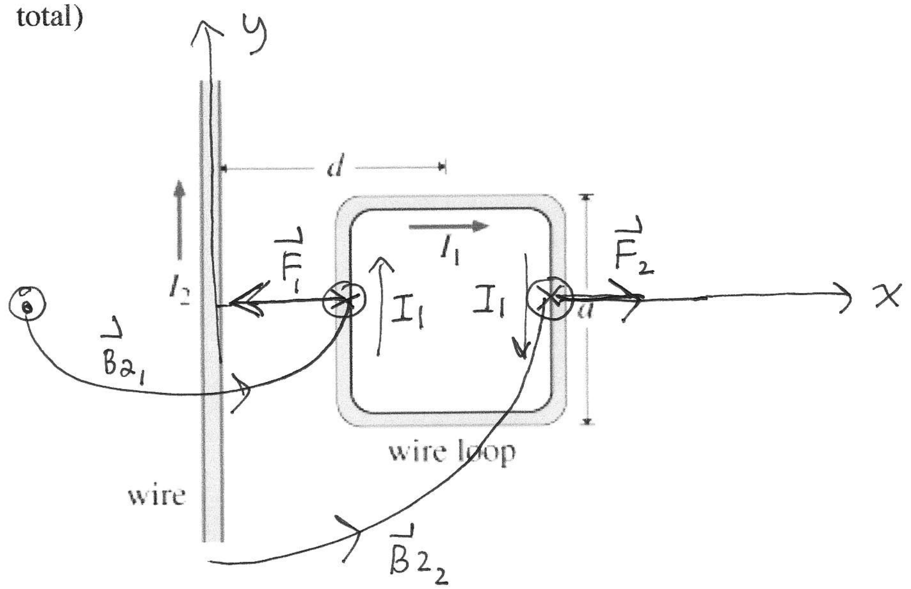
10) The energy per unit volume in an electromagnetic wave is

a) equally divided between the electric and magnetic fields.
b) almost entirely in the electric field.
c) indeterminate.
d) almost entirely in the magnetic field.
e) always equal to zero.

Longer Problems – 70 pts total

1) A square wire loop carrying a current $I_1 = 7.50$ A is placed next to a long straight wire carrying current $I_2 = 3.25$ A as shown in the figure below where $a = d = 28.0$ cm. (20 pts total)

38.0cm ver 2.



a) Determine the magnitude and direction of the force on the side of the square wire loop closest to and parallel with the long wire due to the magnetic field created by the long straight wire. (Assume the current I_1 travels from the bottom to the top of the page in this side of the loop.) (5 pts)

$$B_{2_1} = \frac{\mu_0 |I_2|}{2\pi (d/2)} = \frac{(4\pi \times 10^{-7} \frac{Tm}{A})(3.25A)}{2\pi(0.14m)} = 4.64 \times 10^{-6}\,T$$

(0.19m) $= 3.42 \times 10^{-6}\,T$

$$F_1 = |I_1| L B_2 = (7.50A)(0.28m)(4.64 \times 10^{-6}\,T) = \boxed{9.75 \times 10^{-6}\,N}$$

$(7.50A)(0.38m)\,3.42 \times 10^{-6}$

$= 9.75 \times 10^{-6}$ same answer!

towards the long wire $(-x)$

b) Determine the magnitude and direction of the force on the side of the square wire loop farthest from and parallel with the long wire due to the magnetic field created by the long straight wire. (Assume the current I_1 travels from the top to the bottom of the page in this side of the loop.) (5 pts)

$$B_{2_2} = \frac{(4\pi \times 10^{-7} \frac{Tm}{A})(3.25A)}{2\pi(0.42m)} = 1.55 \times 10^{-6}\,T$$

(0.57m) 1.14×10^{-6}

$$F_2 = (7.50A)(0.28m)(1.55 \times 10^{-6}\,T) = \boxed{3.26 \times 10^{-6}\,N}$$

$(0.38m)\,(1.14 \times 10^{-6})$

away from the long wire $(+x)$

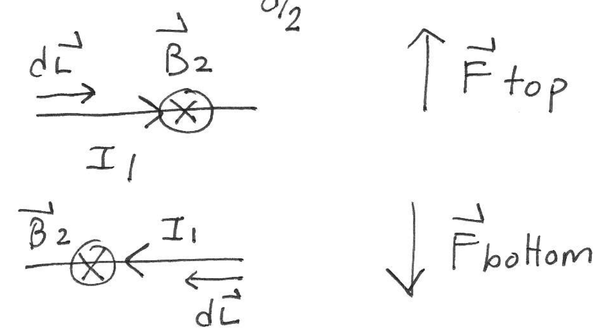
c) The magnetic forces on the top and bottom sides of this loop are the same size, but in opposite directions. Explain why. (2 pts)

$$\vec{F} = |I| \int d\vec{L} \times \vec{B} \Rightarrow |I_1| \int_{d/2}^{3d/2} \frac{\mu_0 |I_2|\, dx}{2\pi x}$$

← magnitude of $\vec{F}$ for top and bottom sides

↳ same

Right-hand rule:



forces in opposite directions because I_1's ($d\vec{L}$) in opp. direction

d) Determine the magnitude and direction of the net force on the square wire loop due to the magnetic field created by the long straight wire. (2 pts)

$$F_{net} = F_1 - F_2 = 9.75 \times 10^{-6}\, N - 3.26 \times 10^{-6}\, N$$

$$= \boxed{6.49 \times 10^{-6}\, N} \quad (-x \text{ direction})$$

e) Determine the magnitude of the magnetic flux thru the wire loop due to the magnetic field created by I_2. Ignore the contribution from I_1. (6 pts)

$$\Phi^{mag} = \int \vec{B} \cdot d\vec{A} \qquad B = \frac{\mu_0 I_2}{2\pi x}$$

$$= \int_0^a \int_{d/2}^{3d/2} \frac{\mu_0 I_2}{2\pi x}\, dx\, dy = \frac{a \mu_0 I_2}{2\pi} \int_{d/2}^{3d/2} \frac{1}{x}\, dx$$

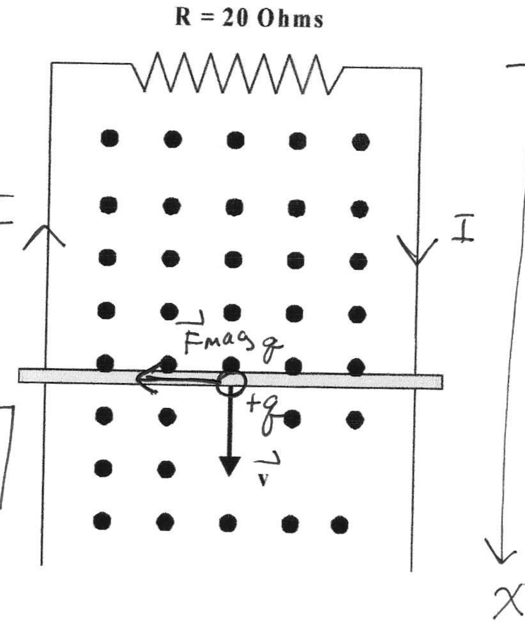
$$= \frac{a \mu_0 I_2}{2\pi} \ln(x) \Big|_{d/2}^{3d/2} = \frac{a \mu_0 I_2}{2\pi} \ln(3)$$

(0.38m)

$$= \frac{(0.28m)(4\pi \times 10^{-7}\, \frac{Tm}{A})(3.25A)}{2\pi} \ln(3)$$

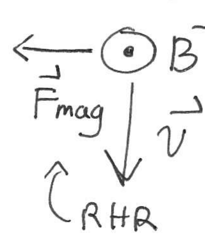
$$= \boxed{2.00 \times 10^{-7}\, Tm^2} \qquad 2.71 \times 10^{-7}\, Tm^2$$

2) Two fixed conductors are connected by a resistor R = 20.0 Ω. The two fixed conductors are separated by $L = 255$ cm and lie horizontally. A moving conductor of mass m slides on them at a constant speed v, producing a current of 1.75 amps. A magnetic field (shown by the black dots in the figure) with magnitude 6.67 T points out of the page.



a) In which direction does the current flow through the moving conductor when the bar is sliding in the direction shown? Justify your answer. (2 pts)

$\vec{F}_{mag_q} = q\vec{v} \times \vec{B}$



force on charge +q in the bar

→ current flows from $\boxed{\text{right to left}}$

b) Determine the speed of the moving bar. (4 pts)

$$|\varepsilon| = \frac{d\Phi_{mag}}{dt} = B\frac{dA}{dt} = BL\frac{dx}{dt} = BLv \rightarrow v = \frac{|\varepsilon|}{BL}$$

$$|\varepsilon| = IR = (1.75A)(20.0\Omega) = 35.0\,V$$

$$v = \frac{35.0\,V}{(6.67T)(2.55m)} = \boxed{2.06\,m/s}$$

c) Determine the magnetic force (magnitude and direction) on the bar. (4 pts)

$$\vec{F}_{mag_I} = I\vec{L} \times \vec{B}$$

$$F_{mag_I} = ILB = (1.75A)(2.55m)(6.67T) = \boxed{29.8\,N}$$

force on curent in bar

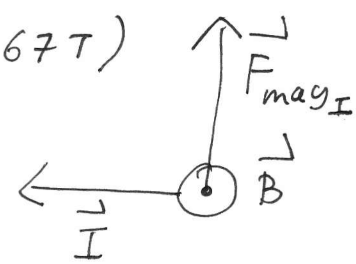
($-x$ direction)

or

(opposite to velocity direction)

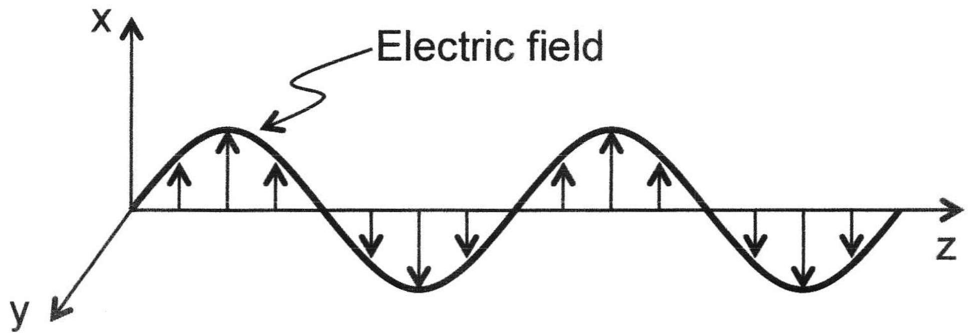
or

(upwards)



3) An electromagnetic plane wave is propagating in empty space. The electric field at time t = 0 over two wavelengths is sketched in the figure below. The E-field is given by

$\vec{E}(x,y,z,t) = \hat{i}E_o \sin(kz + \omega t)$.



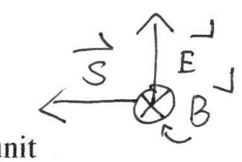
a) In which direction is this wave propagating? Justify your answer. (2 pts)

It propagates in the $-z$ direction because the argument of the sine is: $(kz+\omega t)$. The + sign indicates moving in the $-z$ dir.

b) Write an expression for the magnetic field as a function of time. (3 pts)

$\vec{B}(x,y,z,t) = -\hat{j}\, B_o \sin(kz+\omega t)$

$\vec{S} = \frac{1}{\mu_o}\vec{E}\times\vec{B}$ → so $\vec{B}$ direction begins in $-y$ direction:



b) If the average intensity of the wave is 4.33 Watt/m^2, what is the average energy per unit volume (energy density) $<u>$ of the wave? (3 pts)

$$\langle u\rangle = \varepsilon_o E_{rms}^2 = \frac{I}{c} = \frac{4.33\ \text{Watt/m}^2}{3.00\times10^8\ \text{m/s}} = \boxed{1.44\times10^{-8}\ \text{J/m}^3}$$

$$\frac{5.33}{3.00\times10^8} = 1.78\times10^{-8}$$

c) If the average intensity of the wave is 4.33 Watt/m^2, what is the peak magnetic field of the wave? (3 pts)

$$I = \frac{1}{c\mu_o}E_{rms}^2 = \frac{1}{2c\mu_o}E_o^2 = \frac{1}{2c\mu_o}c^2B_o^2 = \frac{cB_o^2}{2\mu_o}$$

$$B_o = \sqrt{\frac{2\mu_o I}{c}} = \sqrt{\frac{2(4\pi\times10^{-7}\ \frac{Tm}{A})(4.33\ W/m^2)}{3.00\times10^8\ m/s}} = \boxed{1.90\times10^{-7}\ T}$$

$= 2.[illegible]\times10^{-7}\ T$

5.33

d) If the average intensity of the wave is 4.33 Watt/m², what is the root-mean-square (rms) electric field of the wave? (3 pts)

$$I = \frac{1}{c\mu_0} E_{rms}^2 \qquad E_{rms} = \sqrt{c\mu_0 I}$$

$$= \sqrt{(3.00\times10^8\,\mathrm{m/s})(4\pi\times10^{-7}\,\tfrac{\mathrm{Tm}}{\mathrm{A}})(4.33\,\tfrac{\mathrm{W}}{\mathrm{m}^2})}$$

$\underline{\underline{or}}$

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{cB_0}{\sqrt{2}} \qquad = \boxed{40.4\ \mathrm{N/C}} \qquad 44.8\ \mathrm{N/C}$$

e) If the wave number k of this wave is 62.8 m⁻¹, what is the frequency f? (3 pts)

$$k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{62.8\,\mathrm{m}^{-1}} = 0.100\,\mathrm{m}$$

$$f = \frac{c}{\lambda} = \frac{3\times10^8\,\mathrm{m/s}}{0.100\,\mathrm{m}} = \boxed{3.00\times10^9\ \mathrm{Hz}}$$

or// $f = \frac{\omega}{2\pi} = \frac{kc}{2\pi}$

f) If the magnitude of the peak electric field of this wave were doubled, what would the resulting intensity be? (3 pts)

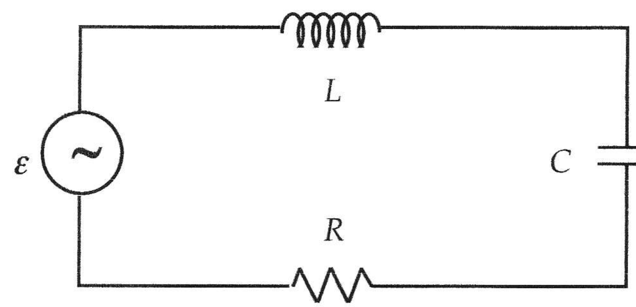
$$E_0 \rightarrow 2E_0 \qquad I = \frac{1}{2c\mu_0}E_0^2 \rightarrow \frac{1}{2c\mu_0}(2E_0)^2$$

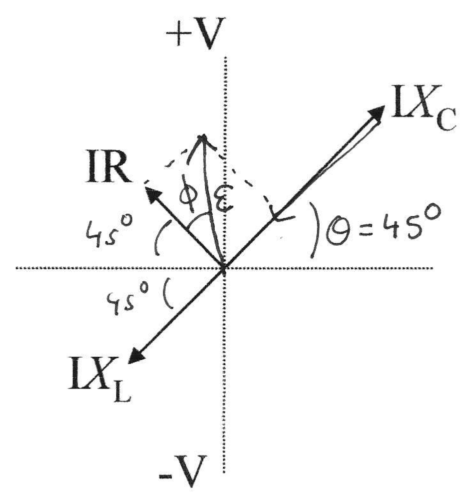
$$I' = 4I = 4(4.33\,\mathrm{W/m^2})$$

$$= \boxed{17.3\ \mathrm{W/m^2}}$$

$21.3\ \mathrm{W/m^2}$

4) An LRC circuit and its associated phasors are shown below. An AC voltage source with angular frequency $\omega = 150$ rad/s and root-mean-square voltage $\varepsilon_{rms} = 120$ V drives the circuit.





$L = 200$ mH $C = 150$ μF $R = 28$ Ω
24

a) Find the capacitive reactance X_C. (2.5 pts)

$$X_C = \frac{1}{\omega C} = \frac{1}{(150\,rad/s)(150\times10^{-6}\,F)} = \boxed{44.4\,\Omega}$$

b) Find the inductive reactance X_L. (2.5 pts)

$$X_L = \omega L = (150\,rad/s)(200\times10^{-3}\,H) = \boxed{30.0\,\Omega}$$

c) Find the impedance Z of the circuit. (2.5 pts)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(28\,\Omega)^2 + (30.0\,\Omega - 44.4\,\Omega)^2}$$

24 Ω

$$= \boxed{31.5\,\Omega}$$

28.0 Ω

d) Find the phase constant ϕ of the circuit. (2.5 pts)

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{30.0\,\Omega - 44.4\,\Omega}{28.0\,\Omega} = -0.514$$

24.0 Ω

$$\boxed{\phi = -27.2^\circ}$$

-31.0°

e) What is the root-mean-square current in this circuit? (2.5 pts)

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{120\,V}{31.5\,\Omega} = \boxed{3.81\,A}$$

$$\frac{110\,V}{28.0\,\Omega} = 3.93$$

f) Draw a phasor respresenting the voltage of the AC source ε in the correct location on the phasor diagram on the previous page. (2.5 pts)

See previous page.

g) If the phasor representing the voltage across the capacitor makes a 45^0 angle with the x-axis in the phasor diagram shown, determine the values of the voltages across the resistor, the capacitor and the inductor at the instant depicted in the diagram. (5 pts)

$$V_C = I X_C \sin 45^\circ \leftarrow I = \sqrt{2}\, I_{rms} = 5.39\,A$$

5.56 A

$$V_C = (5.39\,A)(44.4\,\Omega)\sin 45^\circ = \boxed{169\,V}$$

174.6 V = 175 V

$$V_R = I R \sin 45^\circ = (5.39\,A)(28\,\Omega)\sin 45^\circ = \boxed{107\,V}$$

94.4

$$V_L = -I X_L \sin 45^\circ = -(5.39\,A)(30.0\,\Omega)\sin 45^\circ = \boxed{-114\,V}$$

-118

Extra Credit: (1 pt) What should Dr. Johnson get for her two sons (ages 12 and 14) for their upcoming birthdays?

A watch and ?