Factoring and Solving Equations Test A2 _____ **Name: SOLUTIONS**

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1) Factor the following expressions completely: (2 marks each)

a) $x^2 + 8x + 15$

Factors of 1 are 1×1

Factors of 15 are 1×15 or 3×5

Playing around we find $1 \times 3 + 1 \times 5 = 8$

$= (x + 5)(x + 3)$

b) $6x^2 - 5x - 1$

Factors of 6 are 1×6 or 2×3

Factors of -1 are -1×1

Playing around we find $1 \times 1 + 6 \times -1 = -5$

$= (x - 1)(6x + 1)$

c) $-14x^5y^3 + 21x^4y^2 + 28x^4y^3$

$= 7x^4y^2(-2xy + 3 + 4y)$

d) $3x^5(9x + 7)^7 - 18x^4(9x + 7)^4$

$= 3x^4(9x + 7)^4(x(9x + 7)^3 - 6)$

e) $4x^2 - 2x - 2$

$= 2(2x^2 - x - 1)$

Factors of 2 are 1×2

Factors of -1 are 1×-1

Playing around we find $1 \times 1 + 2 \times -1 = -1$

$= 2(2x + 1)(x - 1)$

f) $8 - 12x + 4x^2$

$= 4(2 - 3x + x^2)$

Factors of 2 are 1×2

Factors of 1 are -1×-1 (to get a negative middle number)

Playing around we find $1 \times -1 + 2 \times -1 = -3$

$= 4(1 - x)(2 - x)$

OR $= 4(x - 1)(x - 2)$

g) $2x^2 - 72y^2$

common factors should always be done first

$= 2(x^2 - 36y^2)$

difference of squares gives

$= 2(x - 6y)(x + 6y)$

2) Solve for the specified variable. (2 marks each)

a) $E = \frac{mv^2}{2}$ for m

$2E = mv^2$

$m = \frac{2E}{v^2}$

b) $d = t\left(v + \frac{at}{2}\right)$ for v

$d = vt + \frac{at^2}{2}$ or $\frac{d}{t} = v + \frac{at}{2}$

$d - \frac{at^2}{2} = vt$ $\frac{d}{t} - \frac{at}{2} = v$

$v = \frac{d - \frac{at^2}{2}}{t}$ $v = \frac{2d - at^2}{2t}$

$v = \frac{2d - at^2}{2t}$

3) Solve the following equations for the unknown variable x: (2 marks each)

a) $4 + 9x = 3 - 7x$

$7x + 9x = 3 - 4$

$16x = -1$

$x = -\frac{1}{16}$

b) $3(4x + 5) = -2(x + 4) - 9$

$12x + 15 = -2x - 8 - 9$

$12x + 2x = -17 - 15$

$14x = -32$

$x = -\frac{32}{14} = -\frac{16}{7}$

c) $\frac{2}{10x} = \frac{2}{-x-3}$

$LCD = 10x(-x - 3)$

$10x(-x - 3)\left(\frac{2}{10x}\right) = 10x(-x - 3)\left(\frac{2}{-x-3}\right)$

$2(-x - 3) = 2 * 10x$

$-2x - 6 = 20x$

$-6 = 20x + 2x$

$-6 = 22x$

$x = -\frac{6}{22} = -\frac{3}{11}$

d) $\frac{4}{9}x + 3 = 2x + 1$

$\frac{4}{9}x - 2x = 1 - 3$

$9\left(\frac{4}{9}x - 2x\right) = 9(-2)$

$4x - 18x = -18$

$-14x = -18$

$x = \frac{-18}{-14}$

$x = \frac{9}{7}$

e) $\frac{1}{4x} + \frac{4}{3} = \frac{1}{2}$

$LCD = 12x$

$12x\left(\frac{1}{4x} + \frac{4}{3}\right) = 12x\left(\frac{1}{2}\right)$

$3 + 16x = 6x$

$16x - 6x = -3$

$10x = -3$

$x = -\frac{3}{10}$

f) $\frac{1}{4x} + \frac{2}{8(x+1)} = \frac{3}{x+1}$

$LCD = 8x(x + 1)$

$8x(x + 1)\left(\frac{1}{4x} + \frac{2}{8(x+1)}\right) = 8x(x + 1)\left(\frac{3}{x+1}\right)$

$2(x + 1) + 2x = 24x$

$4x + 2 = 24x$

$2 = 24x - 4x$

$2 = 20x$

$x = \frac{2}{20} = \frac{1}{10}$

g) $\frac{4}{2x^2-3x-2} + \frac{2}{2x^2+7x+3} = 0$

$\frac{4}{(2x+1)(x-2)} + \frac{2}{(2x+1)(x+3)} = 0$

$LCD = (2x + 1)(x - 2)(x + 3)$

$(2x + 1)(x - 2)(x + 3)\left(\frac{4}{(2x+1)(x-2)} + \frac{2}{(2x+1)(x+3)}\right) = (2x + 1)(x - 2)(x + 3)(0)$

$4(x + 3) + 2(x - 2) = 0$

$4x + 12 + 2x - 4 = 0$

$6x + 8 = 0$

$x = -\frac{8}{6} = -\frac{4}{3}$

$x = -\frac{4}{3}$

4) Write the solutions to the following pre-factored equations: (1 mark)

$-(x-3)(3x+2)=0$
$(x-3)^2=0$ or $(3x+2)=0$
$x=3$ or $x=-\frac{2}{3}$
$x=3,\ -\frac{2}{3}$

5) Simplify by first factoring and then cancelling common factors. (5 marks)

$$\frac{-5x^2+4x+1}{5x^2+6x+1}\div\frac{2x^2+2x-4}{2x^2+12x+10}$$
$$=\frac{-5x^2+4x+1}{5x^2+6x+1}\times\frac{2(x^2+6x+5)}{2(x^2+x-2)}$$
$$=\frac{(5x+1)(-x+1)}{(5x+1)(x+1)}\times\frac{2(x+1)(x+5)}{2(x+2)(x-1)}$$
$$=\frac{-(x-1)}{1}\times\frac{x+5}{(x+2)(x-1)}$$
$$=-\frac{x+5}{x+2}$$

6) Solve the following trinomials by factoring or using the quadratic equation. If you have taken the module on exponents, simplify the radicals as much as possible without using a calculator. (2 marks)

$7x^2-4x-1=0$
$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(7)(-1)}}{2(7)}$
$x=\frac{4\pm\sqrt{16+28}}{14}$
$x=\frac{4\pm2\sqrt{11}}{14}$
$x=\frac{2(2\pm\sqrt{11})}{14}=\frac{2\pm\sqrt{11}}{7}$