# Carleton University

## School of Mathematics and Statistics

## STAT 3502: Probability and Statistics - Assignment 4

Sections A due Wednesday, Mar 23, 2016 In Tutorial

Section B due Monday, Mar 21, 2016 In Tutorial

Section C due Thursday, Mar 24, 2016 In Tutorial

**INSTRUCTIONS:**

I) Assignments are to be handed in prior to beginning of tutorial on the due dates listed above.
II) For written questions, show all of your work. No credit will be given for answers without justification.
III) No late assignments will be accepted.
IV) Total mark=50

***Last Name*** ____________ ***First Name*** ____________ ***Student Number*** ____________

**Question** 1. [8 Marks] The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean 8.2 minutes and standard deviation 1.4 minutes. Suppose that a random sample of $n = 49$ customers is observed. The average waiting time($\overline{X}$) for these 49 customers is computed.

1) [1 Mark] What is the mean of the sampling distribution of $\overline{X}$?
2) [1 Mark] What is the standard deviation of the sampling distribution of $\overline{X}$?
3) [3 Marks] Can we say that the sampling distribution of $\overline{X}$ is approximately normally distributed? Why?
4) [3 Marks] What is the probability that the sample mean is between 7.7 minutes and 8.7 minutes?
5) [1 Mark] If the waiting time is normally distributed, can we say that the sampling distribution of the average waiting time of 7 randomly selected customer is normally distributed? Why?

**Question** 2. [9 Marks] Let the random variable X represent the number of defective components in a lot of components. Assume that X can take on four values: 0, 1, 2, 3. The probability distribution of X is shown in the table below:

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | 0.4 | 0.2 | 0.1 | 0.3 |

1) [7 Marks] Randomly pick two lots of components, what is the sampling distribution of average number of defective components in a lot.
2) [2 Marks] Find $\mathbf{Pr}(\overline{X} > 2)$.

**Question** 3. [5 Marks] If $X$ and $Y$ are independent, normal random variables with $\mathbf{E}\,X = 3$, $\mathbf{Var}(X) = 4$, $\mathbf{E}\,Y = -2$, $\mathbf{Var}(Y) = 1$. Determine the following:

1) [1 Mark] $\mathbf{E}(2X - 3Y)$.
2) [1 Mark] $\mathbf{Var}(2X - 3Y)$.

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3) [3 Marks] $\mathbf{Pr}(2X - 3Y \geq 2)$

**Question** 4. [8 Marks] Suppose that $X_1, X_2, \cdots X_9$ denote a random sample from from a population having mean $\mu$ and standard deviation $\sigma$. Consider the following four estimators of $\mu$

$$\hat{\mu}_1 = X_2,\ \hat{\mu}_2 = \frac{X_1 + X_2}{2},\ \hat{\mu}_3 = \frac{2X_1 - X_2 + 3X_6}{3},\ \hat{\mu}_4 = \frac{\sum_{i=1}^{9} X_i}{9}.$$

1) [2 Marks] Which of these estimators are unbiased?
2) [4 Marks] Among the unbiased estimators found in the preceding part, which one has the smallest variance?
3) [2 Marks] Among the unbiased estimators found in the preceding part, which estimator is best? In what sense is it best?

**Question** 5. [8 Marks] Let $X$ be a random variable with the following p.d.f.

$$f(x) = \begin{cases} \frac{1+\alpha x}{2}, & -1 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

and $-1 \leq \alpha \leq 1$

1) [4 Marks] Find the method of moments estimator of $\alpha$ based on a random sample of size n.
2) [2 Marks] Is the method of moments estimator in 1) an unbiased estimator of $\alpha$? Explain.
3) [2 Marks] A random sample of five observations yields following data.

$$-0.7,\ 0.4,\ 0.5,\ -0.2,\ 0.3$$

Assume the given data follow the distribution in (1). Using the given data, find a method of moments estimate of $\alpha$.

**Question** 6. [12 Marks] Let $X$ be a random variable with the following p.d.f.

$$f(x) = \begin{cases} (\theta + 1)x^{\theta}, & 0 \leq x \leq 1; \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

1) [2 Marks] Find the likelihood function based on a random sample of size n.
2) [2 Marks] Find the log-likelihood function based on a random sample of size n.
3) [6 Marks] Find the maximum likelihood estimator of $\theta$.
4) [2 Marks] A random sample of six observations yields following data.

$$0.1,\ 0.3,\ 0.5,\ 0.2,\ 0.15$$

Assume the given data follow the distribution in (2). Using the given data, find a maximum likelihood estimate of $\theta$.