Math 208 Sample Midterm (SOLUTIONS)

1 (a) [cost] = [fixed cost] + [variable cost]

$\Rightarrow$ $C(x) = 24{,}000 + 7.45x$ (x = # of DVDs)

[revenue] = [# of items] $\cdot$ [price per item]

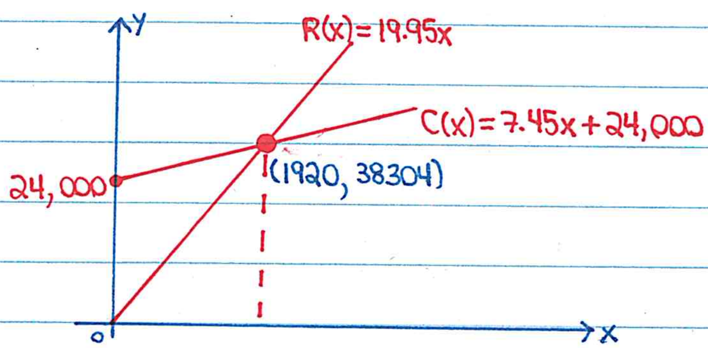
$\Rightarrow$ $R(x) = x(19.95)$

(b) Set $R(x) = C(x)$: $19.95x = 24{,}000 + 7.45x$

$\Rightarrow 12.5x = 24{,}000$

$\Rightarrow x = 1{,}920$ (DVDs).

(c)



2 (a) $(9^2)^{2x} = 9^{x^2-12}$

$\Rightarrow 9^{4x} = 9^{x^2-12}$

$\Rightarrow 4x = x^2 - 12$

$\Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow (x-6)(x+2) = 0$

$\Rightarrow x = 6$ OR $x = -2$

(b) $\log_3 x + \log_3(x-3) = \log_3 10$

$\Rightarrow \log_3[x(x-3)] = \log_3 10$

$\Rightarrow x(x-3) = 10$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x = 5$ ~~OR $x = -2$~~ ← (not valid)

(c) $\log_b x = 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20$

$\Rightarrow \log_b x = \log_b(2^3) + \log_b(25^{1/2}) - \log_b(20)$

$\Rightarrow \log_b x = \log_b(8) + \log_b(5) - \log_b(20)$

$\Rightarrow \log_b x = \log_b\left[\frac{8 \cdot 5}{20}\right]$

$\Rightarrow \log_b x = \log_b 2$ $\Rightarrow$ $x = 2$.

(d) $\frac{1}{4}\log_x 256 = 12$

$\Rightarrow \log_x(256^{1/4}) = 12$

$\Rightarrow \log_x 4 = 12$

$\Rightarrow x^{12} = 4$

$\Rightarrow x = \sqrt[12]{4}$ (since $x > 0$).

③ (a) $S_{22} = a_1 + a_2 + a_3 + \cdots + a_{22}$

$= 5(1.1)^0 + 5(1.1)^1 + 5(1.1)^2 + \cdots + 5(1.1)^{21}$

$*1.1$ $*1.1$ $*1.1$

This is a geometric series with $\begin{cases} a_1 = 5(1.1)^0 = 5 \\ r = 1.1 \\ n = 22 \text{ terms} \end{cases}$

The sum is:

$$S_{22} = \frac{a_1(r^n - 1)}{r-1} = \frac{5[1.1^{22} - 1]}{1.1 - 1} \approx \underline{357.01}$$

(b) $b_1 = -4,\ b_2 = -1,\ b_3 = 2,\ b_4 = 5, \ldots$

This is an arithmetic sequence with $\begin{cases} b_1 = -4 \\ d = 3 \\ n = 54 \text{ (given)} \end{cases}$

$\Rightarrow b_n = b_1 + (n-1)d$

$\Rightarrow b_{54} = -4 + (53)(3)$

$\Rightarrow \underline{b_{54} = 155}$

④

	MONTH 1	MONTH 2	MONTH 3	...	MONTH 60
PAYMENT TO REDUCE LOAN	\$100	\$100	\$100	...	\$100
INTEREST PAYMENT	\$60	\$59	\$58	...	\$1

The total interest is:

$1 + 2 + 3 + \cdots + 60$ (arithmetic series with $a_1 = 1$, $a_n = 60$, $n = 60$)

$= n\left(\frac{a_1 + a_n}{2}\right)$

$= 60\left(\frac{1+60}{2}\right)$

$= \$1,830.$

The total payments to reduce the loan is: $60(\$100) = \$6,000.$

Thus, the total cost of the loan is: $\$1,830 + \$6,000 = \underline{\$7,830.}$

5 **(a)** Since $APY = (1+\frac{r}{m})^m - 1 = 1.551\%$, we get:

$$\left(1+\frac{r}{12}\right)^{12} - 1 = 0.01551$$
$$\Rightarrow \left(1+\frac{r}{12}\right)^{12} = 1.01551$$
$$\Rightarrow 1+\frac{r}{12} = \sqrt[12]{1.01551}$$
$$\Rightarrow \frac{r}{12} = \sqrt[12]{1.01551} - 1$$
$$\Rightarrow r = 12\left(\sqrt[12]{1.01551} - 1\right) \approx 0.0154 = \underline{1.54\%} \text{ (nominal rate)}$$

(b) $PMT = FV\left[\frac{i}{(1+i)^n - 1}\right]$ where $\begin{cases} FV = 10{,}000 \\ i = \frac{0.0154}{12} \left(\frac{r}{m}!\right) \\ n = (4)(12) = 48 \end{cases}$

$$= 10{,}000\left[\frac{0.00128333\ldots}{(1.00128333\ldots)^{48} - 1}\right]$$
$$\approx \underline{\$202.12}$$

6 **(a)** Each monthly payment can be found using:

$PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$ where $\begin{cases} PV = 500 \\ i = 0.01 \\ n = 6 \end{cases}$

$$= 500\left[\frac{0.01}{1-(1.01)^{-6}}\right]$$
$$\approx \underline{\$86.27}$$

(b)

MONTH #	A MONTHLY PAYMENT	B AMOUNT FOR INTEREST	A − B AMOUNT FOR UNPAID BALANCE	NEW UNPAID BALANCE (originally \$500)
1	\$86.27	\$5	\$81.27	\$418.73
2	\$86.27	\$4.19	\$82.08	\$336.65
3	\$86.27	\$3.37	\$82.90	\$253.75
4	\$86.27	\$2.54	\$83.73	\$170.02
5	\$86.27	\$1.70	\$84.57	\$85.45
6	\$86.27	\$0.85	\$85.42	\$0.03 ≈ \$0 (rounding...)