Solutions for April 2010
Final Exam

#1 (a) $-\sqrt{64} + 8\sqrt{36} - 3\sqrt{144}$

$$= -8 + 8(6) - 3(12) \quad = -8 + 48 - 36$$

$$= \boxed{4}$$

(b) $\log_2 20 - \log_2 45 + \log_2 36$

$$= \log_2(4\cdot 5) - \log_2(9\cdot 5) + \log_2(9\cdot 4)$$

$$= 2\underbrace{\log_2 2}_{(=1)} + \cancel{\log_2 5} - \cancel{2\log_2 3} - \cancel{\log_2 5} + \cancel{2\log_2 3} + 2\log_2 2$$

$$= 2 + 2 \quad = \boxed{4}$$

#2 (a) $$\frac{2-\sqrt{5}}{2+3\sqrt{5}} \cdot \frac{2-3\sqrt{5}}{2-3\sqrt{5}} = \frac{4 - 8\sqrt{5} + 15}{4 - 45} = \frac{19 - 8\sqrt{5}}{-41}$$

$$= \boxed{\frac{8\sqrt{5} - 19}{41}}$$

(b) $$\frac{\sqrt{3}}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{5\sqrt{3} + \sqrt{6}}{25 - 2} = \boxed{\frac{5\sqrt{3} + \sqrt{6}}{23}}$$

#3 (a) $8 - 8x^3 + 4 + 4x + 4x^2 + 4x^3$

$$= 12 + 4x + 4x^2 - 4x^3$$

(b) $$\frac{x - x^2}{x^2 + x - 2} = \frac{x(1-x)}{(x+2)(x-1)} = \frac{-x\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = -\frac{x}{x+2}$$

#4 (a) $6x^2+8x+2 = 2(3x^2+4x+1)$

grouping: $\left[\begin{array}{l} ac = 3 \\ (3 \times 1) = 3 \\ \underline{3} + \underline{1} = 4 \end{array}\right]$

So the expression becomes: $2(3x^2+3x+x+1)$

$$= 2[3x(x+1)+(x+1)]$$

$$= \boxed{2(3x+1)(x+1)}$$

(b) $2-8x^2 = 2(1-4x^2) = \boxed{2(1-2x)(1+2x)}$

#5

$$\frac{x}{x^2+x} - \frac{x+4}{x^2+2x+1} = \frac{x}{x(x+1)} - \frac{x+4}{(x+1)(x+1)}$$

(LCM: $x(x+1)^2$)

$$= \frac{x(x+1)-x(x+4)}{x(x+1)^2}$$

$$= \frac{x^2+x-x^2-4x}{x(x+1)^2}$$

$$= \boxed{-\frac{3\cancel{x}}{\cancel{x}(x+1)^2}} = \boxed{-\frac{3}{(x+1)^2}}$$

#6 (a) $\frac{2x}{x^2-4} + \frac{3}{x+2} = \frac{4}{x^2-4}$

$$(x+2)(x-2)\left[\frac{2x}{(x-2)(x+2)} + \frac{3}{x+2}\right] = \left[\frac{4}{(x+2)(x-2)}\right](x+2)(x-2)$$

← multiplied by LCM

$2x + 3(x-2) = 4$

$2x + 3x - 6 = 4$

$5x = 10$ → $x = 2$

But, for initial expression, $x \neq 2$, $x \neq -2$. So there is $\boxed{\text{no solution}}$

(b) $\log_5 x + \log_5 (x-4) = \log_5 (x+6)$

$\log_5[x(x-4)] = \log_5(x+6)$

$x(x-4) = x+6$

$x^2 - 4x - x - 6 = 0$

$x^2 - 5x - 6 = 0$

$(x+1)(x-6) = 0$

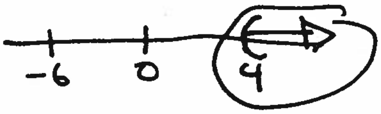
$x = -1$ or $x = 6$

Check:

domain: $x > 0$

$\boxed{x > 4}$ → only need this.

$x > -6$



Then only solution is:

$\boxed{x = 6}$

⊛Alternate solution at end⊛

(c) $4^{1-2x} = 2$

$2^{2(1-2x)} = 2$

Then, $2(1-2x) = 1$

$2 - 4x = 1$

$1 = 4x$

$\boxed{x = 1/4}$

Check: $4^{1-2(\frac{1}{4})} = 4^{1-\frac{1}{2}} = 4^{1/2}$

$= \sqrt{4} = 2$ ✓

$\boxed{\#7}$ (a) $2 \le 3x + 7 < 13$

$-5 \le 3x < 6$

$-5/3 \le x < 2$

Then $\boxed{\{x \mid -\frac{5}{3} \le x < 2\}}$

or

$\boxed{[-5/3, 2)}$

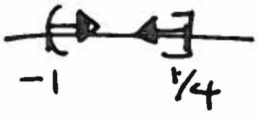
(b) $|1-4x| - 7 < -2$

$|1-4x| < 5$

Case ①: $1 - 4x \ge 0$, then $1 - 4x < 5$

$-4x \ge -1$ $\qquad$ $-4x < 4$

$x \le 1/4$ -and- $x > -1$



Combining $x > -1$ and $x \leq \frac{1}{4}$ we get:

$-1 < x \leq 1/4$

Case ②: $1 - 4x < 0$, then $-1 + 4x < 5$

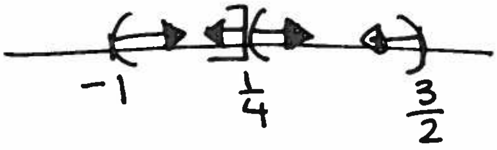
$-4x < -1$ $\quad$ $4x < 6$

$x > 1/4$ $\quad$ $x < 6/4$

$x < 3/2$

So here, $\frac{1}{4} < x < \frac{3}{2}$

Combining Cases ① and ②:



We get: $-1 < x < \frac{3}{2}$

So, $\boxed{\{x \mid -1 < x < \frac{3}{2}\}}$ or $\boxed{(-1, \frac{3}{2})}$

#8 $2x^2 + y^2 = 1$ — ①
$2x - y = -1$ — ②

From ②: $y = 2x + 1$

Plug y into ①:

$2x^2 + (2x+1)^2 - 1 = 0$

$2x^2 + 4x^2 + 4x + 1 - 1 = 0$

$6x^2 + 4x = 0$

$2x(3x+2) = 0$

Then, $x = 0$ or $3x + 2 = 0$

$3x = -2$

$x = -2/3$

If $x = 0$: then $y = 2(0) + 1$ (from ②)

$y = 1$

If $x = \frac{-2}{3}$, then $y = 2\left(\frac{-2}{3}\right) + 1$

$= -\frac{4}{3} + \frac{3}{3} = -\frac{1}{3}$

So, we have two possible solutions: (check)

$(0,1)$: ① $2(0)^2 + (1)^2 = 1$ ✓

② $2(0) - 1 = -1$ ✓

$\left(-\frac{2}{3}, -\frac{1}{3}\right)$: ① $2\left(-\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = 2\left(\frac{4}{9}\right) + \frac{1}{9} = \frac{8}{9} + \frac{1}{9} = 1$ ✓

② $2\left(-\frac{2}{3}\right) - \left(-\frac{1}{3}\right) = -\frac{4}{3} + \frac{1}{3} = -\frac{3}{3} = -1$ ✓

Then, there are two solutions to this system:

$(0,1)$

$\left(-\frac{2}{3}, -\frac{1}{3}\right)$ or

$x = 0$ and $y = 1$

$x = \frac{-2}{3}$ and $y = -1/3$.

$\boxed{\#9}$ (a) $A(4,2)$; $B(3,5)$; $C(2,5)$

$$d(A,C) = \sqrt{(4-2)^2 + (2-5)^2} = \sqrt{4+9} = \sqrt{13}$$

$$d(B,C) = \sqrt{(3-2)^2 + (5-5)^2} = \sqrt{1} = 1$$

$1 < \sqrt{13}$

So point $\boxed{B(3,5)}$ is closer to $C(2,5)$

(b) $x^2+y^2-2x-4y=4$

$(x^2-2x)+(y^2-4y)=4$

$(x^2-2x+1)+(y^2-4y+4)=4+1+4$

$\boxed{(x-1)^2+(y-2)^2=9} \rightarrow$ equation of a circle.

$$\boxed{\begin{array}{l} \left(\frac{-2}{2}\right)^2=1 \\ \left(\frac{-4}{2}\right)^2=4 \end{array}}$$

Center: $\boxed{(1,2)}$

Radius: $\sqrt{9}=\boxed{3}$

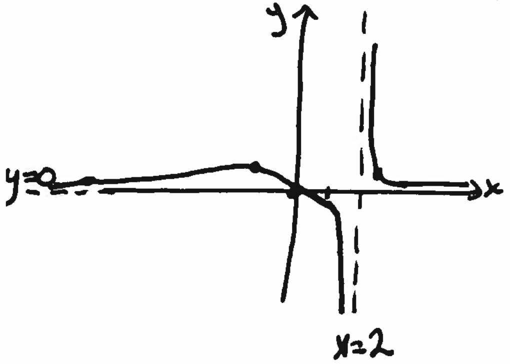
$\boxed{\#10}$ (a) $f(x)=\dfrac{x}{x^3-8}=\dfrac{x}{x^3-2^3}=\dfrac{x}{(x-2)(x^2+2x+4)}$

Only vertical asymptote is: $x=2$ since x^2+2x+4 has no solution.

This is a proper rational fn., so there is a horizontal asymptote $y=0$ → so $f(x)$ crosses x-axis.

$x=0$ is an x-intercept. (mult. 1)

$f(0)=0$, so, $y=0$ is a y-int. $\Big\}$ ⇒ This function passes through $(0,0)$.



Test numbers:

$\boxed{(-\infty,0)}$: $f(-10)=\dfrac{-10}{(-10)^3-8}=\dfrac{10}{1008}=\dfrac{5}{504}$

(So, when x becomes small $(x\to-\infty, y\to 0)$ y gets closer to 0)

$f(-1)=\dfrac{-1}{-1-8}=\dfrac{1}{9}$ (notice $\frac{5}{504}<\frac{1}{9}$)

$\boxed{(0,2)}$: $f(1)=\dfrac{1}{1-8}=-\dfrac{1}{7}$

$f(3/2)=\dfrac{3/2}{\left(\frac{3}{2}\right)^3-8}=\dfrac{3/2}{\frac{27}{4}-\frac{32}{4}}=\dfrac{3/2}{-\frac{5}{4}}=-\dfrac{3}{2}\cdot\dfrac{4}{5}$

$=\dfrac{-12}{10}=-\dfrac{6}{5}$

(notice $\frac{-6}{5}<-\frac{1}{7}$)

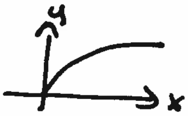
$\boxed{[2,\infty)}$: $f(3) = \frac{3}{3^3-8} = \frac{3}{19} > 0$

$$f\left(\tfrac{5}{2}\right) = \frac{\frac{5}{2}}{\left(\frac{5}{2}\right)^3 - 8} = \frac{\frac{5}{2}}{\frac{125}{8} - \frac{64}{8}} = \frac{\frac{5}{2}}{\frac{61}{8}} = \frac{5}{\cancel{2}} \cdot \frac{\overset{4}{\cancel{8}}}{61} = \frac{20}{61}$$

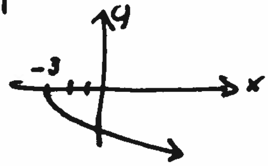
(Notice how $\frac{3}{19} < \frac{20}{61}$)

Range: $(-\infty,\infty)$
Domain: $(-\infty,2)\cup(2,\infty)$

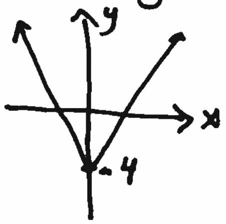
(b) $g(x) = -\sqrt{x+3}$ This is the function $y=\sqrt{x}$ reflected about the x-axis and shifted 3 units to the left



domain: $[-3,\infty)$
range: $(-\infty,0]$



(c) $h(x) = |x| - 4$ This is the absolute value fn $y=|x|$ shifted down 4 units.



domain: $(-\infty,\infty)$
range: $[-4,\infty)$

⊛ Practice on finding domain & range of a rational function:

$$f(x) = \frac{5}{x^2+2x+1} = \frac{5}{(x+1)^2}$$

vertical asymptotes: $x^2+2x+1 = (x+1)(x+1)$
Then there is one vertical asym. $\underline{x=-1}$

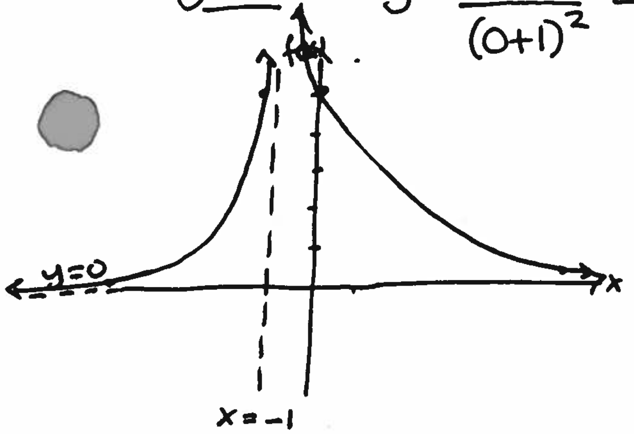
This is a proper rational function. So, it has $\underline{y=0}$ as a horizontal asymptote

x-int.:

$$0 = \frac{5}{(x+1)^2}$$

$0 \neq 5$ → There are no x-intercepts.

y-int.: $y = \frac{5}{(0+1)^2} = 5$ → So, this graph crosses the y-axis at $y=5$ (or $(0,5)$)



Test numbers:

for $(-\infty,-1)$: $f(-10) = \frac{5}{(-9)^2} = \frac{5}{81}$

$f(-2) = \frac{5}{(-1)^2} = 5$

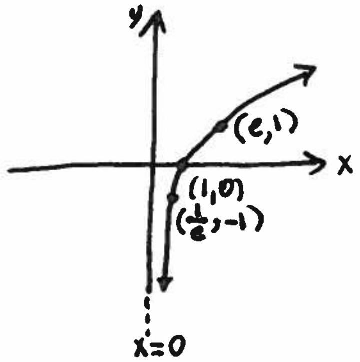
for $(-1,0)$: $f(-0.5) = \frac{5}{(0.5)^2} = 20$

for $(0,\infty)$: $f(10) = \frac{5}{11^2} = \frac{5}{121}$

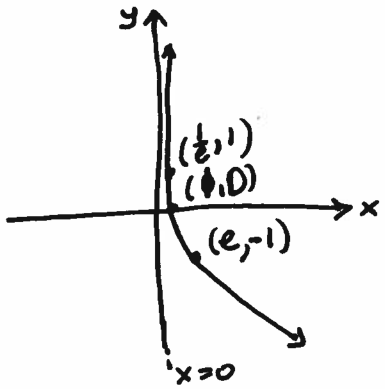
$$\boxed{\begin{array}{l}\text{domain: } (-\infty,-1) \cup (-1,\infty) \\ \text{range: } (0,\infty)\end{array}}$$

$\boxed{\#11}$ $f(x) = -\ln(x-2)$

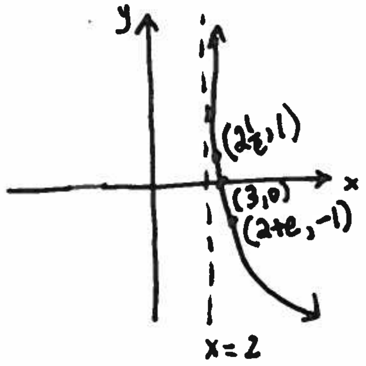
Step①: $g(x) = \ln x$



Step②: $g(x) = -\ln x$ $\rightarrow$ reflect over x-axis



Step③: $f(x) = -\ln(x-2)$ $\rightarrow$ shift to right by 2



$\boxed{\#12}$ $f(x) = \dfrac{2x}{x+3}$; $g(x) = 3-x$

(a) $fg = \left(\dfrac{2x}{x+3}\right)(3-x)$

$$= \frac{2x(3-x)}{x+3} = \boxed{\frac{6x-2x^2}{x+3}} = fg$$

(b) $$\frac{f}{g} = \frac{\frac{2x}{x+3}}{3-x} = \frac{2x}{x+3}\cdot\frac{1}{3-x} = \boxed{\frac{2x}{9-x^2} = \frac{f}{g}}$$

(c) $$f\circ g = \frac{2(3-x)}{(3-x)+3} = \boxed{\frac{6-2x}{6-x}} = f\circ g$$

(d) $$g\circ f = 3 - \frac{2x}{x+3} = \frac{3(x+3)-2x}{x+3} = \boxed{\frac{9+x}{x+3} = g\circ f}$$

$\boxed{\#13}$ (a) $f(x) = \dfrac{3x+4}{2x-3}$

Find inverse: $x = \dfrac{3y+4}{2y-3}$

$$x(2y-3) = 3y+4$$

$$2xy - 3x = 3y + 4$$

$$2xy - 3y = 3x + 4$$

$$y(2x-3) = 3x+4$$

$$y = \boxed{\dfrac{3x+4}{2x-3} = f^{-1}(x)}$$

Vertical asymptote: $2x - 3 = 0$
of f(x)

$\boxed{x = 3/2}$ for f and f^{-1}

Horizontal asymptote:
of f(x)

$$\begin{array}{r} 3/2 \\ 2x-3 \,\overline{)\, 3x+4} \\ 3x - 9/2 \\ \vdots \end{array} \longrightarrow \boxed{y = 3/2}$$ for f. and f^{-1}

Since f and f^{-1} are the same function, the asymptotes for f and f^{-1} are the same.

$\boxed{\#14}$ Let x be the hourly wage earned by Sandra.
hourly wage

Then, we can write:

$$442 = 40x + 8(1.5)x$$

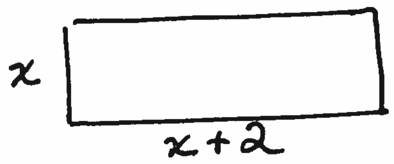
$$442 = 40x + 12x$$

$$442 = 52x$$

$$x = 8.5$$

Sandra makes $\boxed{\$8.50/\text{hour}}$

#15 Area $= 143\ ft^2$



Let x be the length of the width

then $x+2$ is the length of the rectangle.

Now, need dimensions:

$$143 = x(x+2)$$
$$143 = x^2 + 2x$$
$$0 = x^2 + 2x \mp 143$$
$$0 = (x+13)(x-11)$$
$$x = {}^{-}13 \quad \text{or} \quad \boxed{x = 11}\ \text{ft for width.}$$

↳ measurement cannot be negative

$$x+2 = 11+2 = \boxed{13\ ft}\ \text{for length}$$

#16 $P(t) = 500e^{0.02t}$

(a) Initial amount $= \boxed{500\ \text{insects}}$

(b) Double population occurs at 1000 insects.

$$1000 = 500e^{0.02t}$$
$$2 = e^{0.02t}$$
$$\ln 2 = 0.02t(\overset{=1}{\ln e})$$
$$t = \frac{\ln 2}{0.02} \approx \boxed{34.66\ \text{days.}}$$

(c) $800 = 500e^{0.02t}$

$$\ln 1.6 = 0.02t$$
$$t = \frac{\ln 1.6}{0.02} \approx \boxed{23.5\ \text{days}}$$

⊛ Alternate Solution to $\boxed{\#6(b)}$:

$$\log_5[x(x-4)] = \log_5(x+6)$$

$$\log_5(x^2-4x) - \log_5(x+6) = 0$$

$$\log_5\left(\frac{x^2-4x}{(x+6)}\right) = 0$$

Then, $$\frac{x^2-4x}{x+6} = 5^0 \ (=1)$$

So, $$x^2-4x = x+6$$

$$x^2-5x-6=0$$

$$(x+1)(x-6)=0$$

$$x=-1 \text{ or } \boxed{x=6}$$

↖ Since domain is $\{x \mid x>4\}$