CONCORDIA UNIVERSITY
Department of Building, Civil and Environmental Engineering
BCEE 345 - STRUCTURAL DESIGN II (Reinforced Concrete Design)

SOLUTION OF ASSIGNMENT 2

Question 1 :

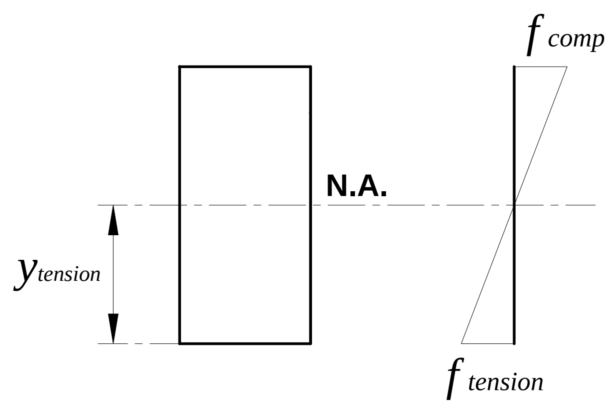
$f_c^{'} = 30\ MPa\ ,\ \ f_y = 400\ MPa$

a) Calculation of M_{cr} :

1) Code Procedure:

$f_r = 0.6\lambda\sqrt{f_c^{'}} = 0.6\ (1.0)\sqrt{30} = 3.29 MPa = f_t$

The cracking moment is the moment that causes the concrete to reach its tensile strength (f_t)



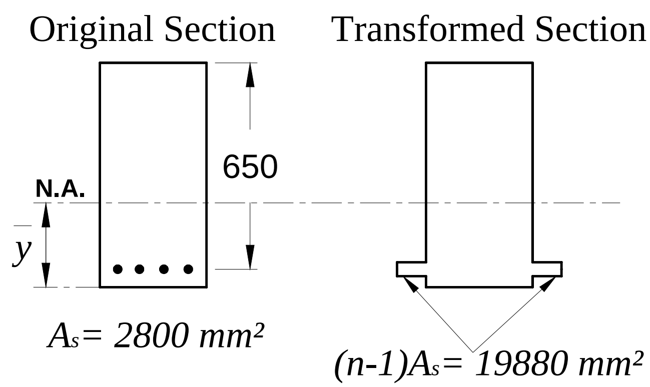
$$f_t = \frac{M_{cr} \times Y_{ten}}{I_{gross}}$$

$$I_{gross}\left(neglecting\ A_s\right) = \frac{300 \times 700^3}{12} = 8.575 \times 10^9\ mm^2$$

$$y_{ten} = \frac{h}{2} = \frac{700}{2} = 350\,mm$$

$$M_{cr} = \frac{3.29 \times \left(8.575 \times 10^9\right)}{350} = 80.61 \times 10^6\ N.mm = 80.61 KN.m$$

2) <u>Procedure using transformed section properties:</u>



$$n = \frac{E_s}{E_c} = \frac{200.000}{4500\sqrt{f'_c}} = 8.1$$

$$\bar{y} = \frac{300 \times 700 \times \left(\frac{700}{2}\right) + 19880(50)}{300 \times (700) + 19880} = 324mm = y_{trans.}$$

$$\bar{I}_{trans.} = \frac{300 \times 700^3}{12} + 300 \times 700 \times (350 - 324)^2 + 19880 \times (324 - 50)^2 = 1.02 \times 10^{10}$$

$$M_{cr} = \frac{I \times F_t}{y_{trans}} = \frac{3.29 \times (1.02 \times 10^{10})}{324} = 103.7 \times 10^6 \ N.mm = 103.7 \ kN.m$$

b) <u>Calculation of Ms :</u>

Stresses in extreme fibres of concrete and reinforcement at M_s = 1.2 M_{cr}

We will use M_{cr} for the transformed section, so M_s = 1.2 (103.7) = 124.4 kN.m

Assuming the section is in the elastic range, so the stress distribution is linear.

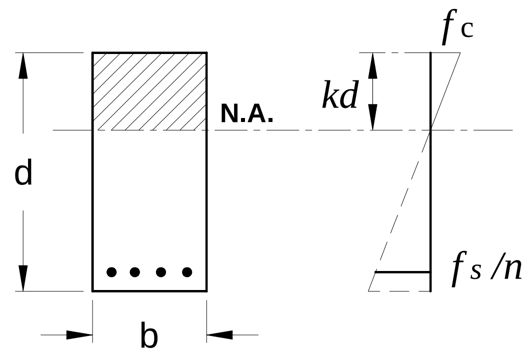
Taking the first moment of are S_{nv} = 0

$$300\,(kd)\,\frac{(kd)}{2} = n\,A_s\,(d - kd)$$

$$300\ \frac{(kd)^2}{2} = 8.1(2800)(650 - kd)$$

$$(kd)^2 + 151.2\,(kd) - 98280 = 0$$

$$\therefore \quad (kd) = 246.9\ mm$$



$$I_{cr@N.A.} = \frac{b\times(kd)^3}{12} + n\,A_s\,(d-(kd))^2$$

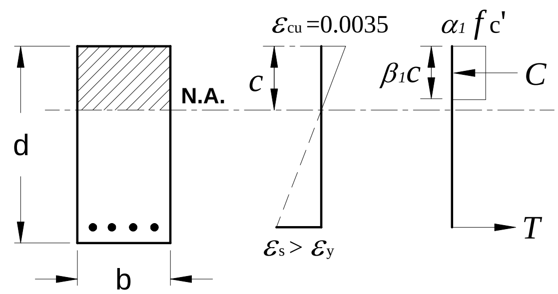
$$= \frac{300\times(246.9)^3}{12} + 8.1(2800)(650-(246.9))^2 = 5.19\times10^9\,mm^4$$

$$f_c = \frac{M_s\times(kd)}{I_{cr}} = \frac{124.4\times10^6\times246.9}{5.19\times10^9} = \underline{5.92\,MPa}$$

$$\frac{f_s}{n} = \frac{M_s\times(d-(kd))}{I_{cr}} = \frac{124.4\times10^6\times(650-246.9)}{5.19\times10^9} = 9.66\,MPa$$

$$\therefore\ f_s = 9.66\times8.1 = \underline{78.3\,MPa}$$

c) <u>Nominal Moment Capacity :</u>



T = C

$$\alpha_1 = 0.85 - 0.0015\,(f_c^{'}) = 0.85 - 0.0015\,(30) = 0.805$$

$$\beta_1 = 0.97 - 0.0025\,(f_c^{'}) = 0.97 - 0.0025\,(30) = 0.895$$

$$A_s F_y = (\alpha_1 f_c^{'})(\beta_1 c)(b)$$

$$c = 172.7\,mm$$

$$M_r = T\left(d - \frac{\beta_1 c}{2}\right) = (2800\times400)\times\left(650 - \frac{0.895(172.7)}{2}\right) = 6.41\times10^8\,N.mm = 641.4\,kN.m$$

d) <u>Factored Moment Capacity:</u>

T = C

$$\phi_s A_s F_y = (\phi_c \alpha_1 f_c^{'})(\beta_1 c)(b)$$

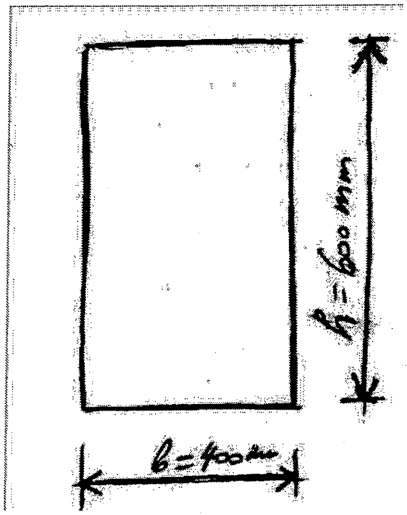
$$\phi_s = 0.85,\ \phi_c = 0.65,$$

$$c = 225.87\,mm$$

$$M_r = T\left(d - \frac{\beta_1 c}{2}\right) = (0.85\times2800\times400)\times\left(650 - \frac{0.895(225.87)}{2}\right) = 5.226\times10^8\,N.mm = 522.6\,kN.m$$

5.4.

a) Find the minimum required number of 25M rebars for the beam section required to support the given load.



Perform the load analysis.

Dead load:

- self-weight $= b \times h \times \gamma_w = 0.4m \times 0.6m \times 24.0\frac{kN}{m^3} = 5.8kN/m$

- superimposed dead load $= 5kN/m$

$DL = 10.8kN/m$

Live load: $LL = 35kN/m$

Factored load (NBC 2005 Table 4.1.3.2)

$$w_f = 1.25DL + 1.5LL = 1.25 \times 10.8 + 1.5 \times 35.0 \cong 66.0\frac{kN}{m}$$

Span $l = 6m$

$$M_f = \frac{w_f \times l^2}{8} = \frac{\left(66\frac{kN}{m}\right)(6.0m)^2}{8} = 297.0kNm$$

- Estimate d value, assuming one layer of reinforcement.

- interior exposure: $cover = 30mm$ (Table A.2)

- stirrup diameter: $d_s = 10mm$ (10M bar)

- bar diameter: $d_b = 25mm$ (25M bar)

$$d = h - cover - d_s - \frac{d_b}{2} = 600 - 30 - 10 - \frac{25}{2} = 547mm$$

$$K_r = \frac{M_r \times 10^6}{bd^2} = \frac{297 \times 10^6}{400\,(547)^2} = 2.48$$

for $f_c' = 30\,MPa$, from table

$\rho = 0.008 < \rho_{max} = 0.026$ (under-reinforced section)

$\therefore A_s = \rho bd = 0.008 * 400 * 547 = 1751\ mm^2$

use 4-25M bars $\Rightarrow A_s = 2000\ mm^2$

- Check CSA A23.3 minimum tension reinforcement requirement (Cl.10.5.1.2)

$$A_{smin} = \frac{0.2\sqrt{f_c'}}{f_y} b_t h \qquad [5.7]$$

$$= \frac{0.2\sqrt{30MPa}}{400MPa} \times 400mm \times 600mm = 657mm^2$$

(Note: $b_t = b$ rectangular sections)

Since

$A_s = 2000mm^2 > 657mm^2$ okay

- Check whether 4-25M bars can fit in one layer. CSA A23.1 (Cl.6.6.5.2) specifies that the clear spacing between the bars (s_{min}) should be at least equal to

$1.4 \times d_b = 1.4 \times 25mm = 35mm$ (25M bar)

$1.4 \times a_{max} = 1.4 \times 20mm = 28mm$ (20 mm maximum aggregate size)

$30mm$

It follows that $s_{min} = 35mm$ governs (largest value).

- Find actual bar spacing.

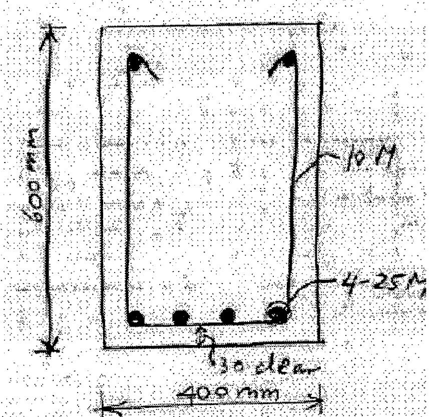
$$s = \frac{400 - 2 \times 30 - 2 \times 10 - 4 \times 25}{3} = 73mm$$

Since

$s = 73mm > 35mm$ okay

it follows that 4 bars can fit in 1 layer.

b) Provide a design summary.



5.7.

a) Find the minimum required beam depth and the amount of tension reinforcement.

Estimate beam dimensions.

- Width: $b = 400mm$ (given)
- Estimate beam depth (h) based on CSA A23.3 Cl.9.8.2.1(see Table A.3 in Appendix A).

Find clear span (l_n).

Span $l = 8m$ and support width is 400 mm, hence

$$l_n = 8000 - 400 = 7600mm$$

For a simply supported beam:

$$h \geq \frac{l_n}{16} = \frac{7600mm}{16} = 475mm$$ (satisfies deflection requirements).

Calculate h that satisfies the acting moment (M_f).

Dead load:

$$DL = 40\frac{kN}{m}$$ (including self-weight)

Live load:

$$LL = 40\frac{kN}{m}$$

- Factored load (NBC 2005 Table 4.1.3.2):

$$w_f = 1.25DL + 1.5LL = 1.25 \times 40 + 1.5 \times 40 = 110\frac{kN}{m}$$

- Factored bending moment (simply supported beam, see Table A.16)

$$M_f = \frac{w_f \times l^2}{8} = \frac{110\frac{kN}{m} \times (8.0m)^2}{8} = 880kNm$$

Set $M_r = M_f$

Knowing that $\rho \leq 0.75\,\rho_b$ where,

$\rho_b = 0.022$ for $f_c' = 25MPa$

$\therefore \rho \leq 0.0165 \Rightarrow K_r = 4.418$

$$\therefore d^2 = \frac{M_r \times 10^6}{b \cdot K_r} = \frac{880 \times 10^6}{400 * 4.418} = 497963$$

$\therefore d = 705$ mm $\Rightarrow h = 800$ mm

$\therefore d = 800 - 90 = 710$ mm. (assuming 2 layers)

$\therefore A_s = \rho b d = 0.0165 * 400 * 710 = 4686 \text{ mm}^2$

using 25 M $(A_{bar} = 500 \text{ mm}^2)$

$\therefore$ use 10 – 25 M $= 5000 \text{ mm}^2$ $(\rho > 0.0165)$

$\therefore$ we should increase h to 850 and retry.

$d = h - 30 - 11.3 - 25 - 25 \times \frac{1.4}{2} = 766 \text{ mm}$

(assuming two layers of 25M & 10M stirrups)

$$K_r = \frac{M_r \times 10^6}{bd^2} = \frac{880 \times 10^6}{400 * (766)^2} = 3.75$$

$\therefore \rho = 0.0133 < 0.0165$ OK

$\therefore A_s = \rho b d = 0.0133 \times 400 \times 766 = 4076 \text{ mm}^2$

$\therefore$ use $\underline{\underline{9 - 25 \text{ M}}} = 4500 \text{ mm}^2$ $(\rho = 0.0147)$ OK

- Check the minimum CSA A23.3 tension reinforcement requirement (Cl.10.5.1.2).

$$A_{smin} = \frac{0.2\sqrt{f_c'}}{f_y} b_t h \qquad [5.7]$$

$$= \frac{0.2\sqrt{25MPa}}{400MPa} \times 400mm \times 900mm = 900mm^2$$

(Note: $b_t = b$ rectangular sections)

Since

$A_s = 4000mm^2 > 900mm^2$ okay

- Check bar spacing:

cover = 30 mm (Table A.2, beam not exposed)

CSA A23.1 (Cl.6.6.5.2) specifies that the clear spacing between the bars (s_{min}) should be at least equal to

$1.4 \times d_b = 1.4 \times 25mm = 35mm$ (25M bar) ✓ (governs)

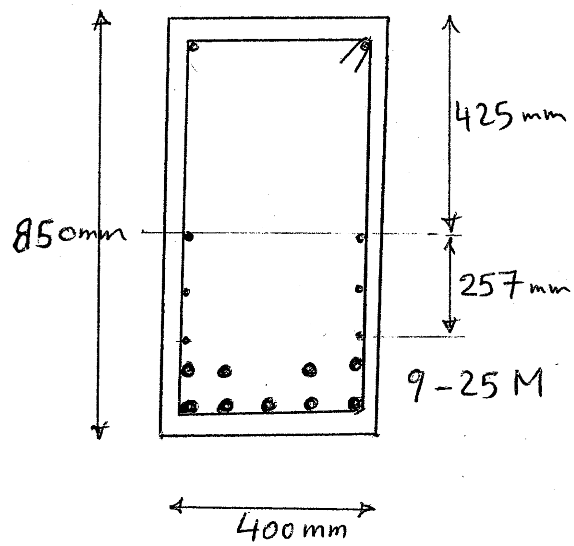
$1.4 \times a_{max} = 1.4 \times 20mm = 28mm$ (20 mm maximum aggregate size)

$30mm$

Check whether 5 bars can fit in one layer :

$$S_{act} = \frac{400 - 2*30 - 2*11.3 - 5*25}{4} = 48.1 \text{ mm}$$

$\therefore S_{act} > S_{min}$ (OK)
$= 48.1$ $= 35$ mm



Design Summary

- Check crack control:-

$$Z = f_s \sqrt[3]{d_c A}$$

$f_s = 0.6 f_y = 240$ MPa

$d_c = 30 + 11.3 + \frac{25}{2} = 53.8$ mm

$y = h - d = 850 - 766 = 84$ mm

$$\therefore A = \frac{2yb}{n} = \frac{2*84*400}{9} = 7466.7 \text{ mm}^2$$

$$\therefore Z = 240 \sqrt[3]{53.8 * 7466.7} = 17708 \text{ N/mm}$$

$< 30,000$ (OK)

- Skin reinforcement: ($h > 750$ mm)

$x = 30 + 11.3 + \frac{11.3}{2} = 47$ mm (assume skin reinf 10M)

$$\therefore \frac{A_{cs}}{2} = \left[\frac{h}{2} - 2(h-d)\right] \cdot 2x = (425 - 2*84) * 2 * 47$$

$$= 24158 \text{ mm}^2$$

$$\therefore A_{skin} / \text{strip} = 0.008 \frac{A_{cs}}{2} = 194 \text{ mm}^2 \quad (2\text{-}10M)$$

$\therefore$ use 3-10M for spacing requirement [$S \not> 200$ mm]
as shown in the design summary.

$\boxed{5.10}$

a) Design of the beam Section with A_s & A_s'

$h = 0.75 * 850 = 638$ mm

$\therefore$ use $h = 650$ mm $\Rightarrow d = h - 90 = 560$ mm.

$M_f = 880$ KN.m (Problem 5.7)

- Calculate M_{rb}:

$M_{rb} = \phi_c \alpha_1 f_c' \beta_1 C_b b \left(d - \frac{\beta_1 C_b}{2}\right)$, where $\frac{C_b}{d} = \frac{700}{700+f_y}$

$= 0.65 * 0.8125 * 25 * 0.9075 * 356 * 400 \left(560 - \frac{0.9075*356}{2}\right)$

$= 680$ KN.m

$\therefore M_f > M_{rb}$ (A_s' is needed)

$A_{sb} = \frac{\phi_c \alpha_1 f_c' \beta_1 C_b b}{\phi_s f_y} = 5018 \text{ mm}^2.$

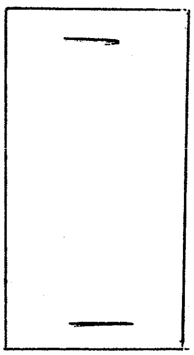
$\Delta M = M_f - M_{rb} = 880 - 680 = 200$ KN.m

$\Delta T = \frac{\Delta M}{d - d'} = \frac{200 * 10^6}{560 - 55} = 396040$ N

where $d' = 55$ mm (assuming one layer)

$\therefore A_s' = \frac{\Delta T}{\phi_s f_y - \phi_c \alpha_1 f_c'} = \frac{396040}{0.85 * 400 - 0.65 * 0.8125 * 25}$

$= 1212 \text{ mm}^2.$



$A_s' = 1212 \text{ mm}^2$

$A_s = A_{s_b} + A_s'$

$= 5018 + 1212$

$= 6230 \text{ mm}^2 \Rightarrow \text{use } \underline{\underline{(9-30M)}} = 6300 \text{ mm}^2$

$\therefore A_{s\,revised}' = 1212 + (6300 - 6230) = 1282 \text{ mm}^2$

$\therefore \text{use } A_s' = \underline{\underline{2-30M}} = 1400 \text{ mm}^2.$

Checks:-

① $c < c_b$: assume A_s, A_s' yielded

$\therefore C_c + C_s = T$

$\phi_c \alpha_1 f_c' \beta_1 c\, b + \phi_s A_s' f_y = \phi_s A_s f_y$

$\therefore 0.65 * 0.8125 * 25 * 0.9075 * c * 400 + 0.85 * 1400 * 400$

$= 0.85 * 6300 * 400$

$\therefore c = 347.6 \text{ mm} < c_b = 356 \text{ mm}$ OK

② $\epsilon_s' > \epsilon_y$

$$\epsilon_s' = \frac{\epsilon_{cu}(c - d')}{c} = \frac{0.0035 * (347.6 - 56)}{347.6} = 0.0029$$

$> \epsilon_y = 0.002$ OK

(3) $M_r > M_f$:

$$M_r = C_c\left(d - \frac{\beta_1 c}{2}\right) + C_s\left(d - d'\right)$$

$$= 0.65 * 0.8125 * 25 * 0.9075 * 347.6 * 400\left(560 - \frac{0.9075 * 347.6}{2}\right)$$

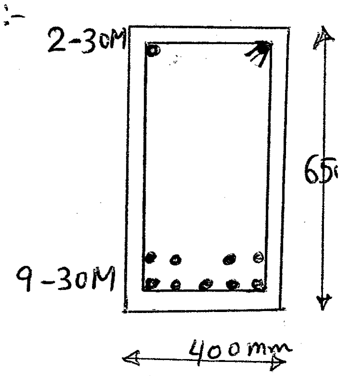
$$+ \; 0.85 * 1400 * 400\left(560 - 56\right)$$

$$= 670.1 + 239.9 = 910 \text{ KN.m} > M_f = 880 \text{ KN.m}$$

OK

- Check bar Spacing :-

Check whether 5-30M can fit in one layer :-



$S_{min} = 1.4\, d_b = 1.4 \times 30 = 42$ mm ✓ (governs)

$1.4\, a_{max} = 35$ mm

30 mm

$$S_{act} = \frac{400 - 2*30 - 2*11.3 - 5*29.9}{4} \approx 42 \text{ mm}$$

OK

b) Compare the beams from Problem 5.7 and 5.10

Beam	5.7	5.10
$A_{conc.}$	0.34 m^2	0.26 m^2
Total A_s	4500 mm^2	7700 mm^2
% $A_{s\,total}$	1.32 %	2.96 %