

1. Which of the following matrices are in row echelon form?

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For each matrix in row echelon form, use row operations to find the equivalent matrix in reduced row echelon form.

2. Find the reduced row echelon form of the augmented matrix for the system

$$\begin{aligned} x_1 + x_2 - x_3 - x_4 + 3x_5 &= 2 \\ 2x_1 + 2x_2 - x_3 + 5x_5 &= 5 \\ x_1 + x_2 + x_4 + x_5 &= 2 \end{aligned}$$

and hence find all of the solutions.

3. For the matrices  $A_{2 \times 3}$ ,  $B_{2 \times 3}$ ,  $C_{4 \times 3}$ ,  $D_{3 \times 2}$ ,  $E_{3 \times 4}$ , which of the following are defined?

- (i)  $A+B$     (ii)  $A+C$     (iii)  $A^T+D$     (iv)  $B+D^T$     (v)  $(A+B)D$     (vi)  $C+E^T$   
 (vii)  $AEC$     (viii)  $AD+DB$     (ix)  $ADBE$     (x)  $E^TC$     (xi)  $BD+AC^T$     (xii)  $AB^TDA$ .

For each defined term, give the size of the answer, for each undefined term, explain why you think it is not defined.

4. Suppose that  $A$  is a diagonal matrix with main diagonal entries  $a_{11}, a_{22}, \dots, a_{nn}$  and  $B$  is a diagonal matrix with main diagonal entries  $b_{11}, b_{22}, \dots, b_{nn}$ .

Calculate  $AB$ ,  $A^2$  and  $A^3$ .

If none of the  $a_{ii}$  are zero, show that  $A$  invertible and find  $A^{-1}$ . (Hint:  $A^{-1}$  will have to be a diagonal matrix.)

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