CVG 3109 Fall – 2015
SOIL MECHANICS I
ASSIGNMENT #4
Due on Nov 17, 2015

Shear Strength: Mohr Circle and Direct Shear Test

Q1. The stresses acting on a soil element are show in Figure 1. Draw the Mohr Circle and determine:

a) the magnitude of the major and the minor principal stresses and the orientation of the two principal planes.

b) the normal stress, σ_α and the shear stress, τ_α on the plane inclined at an angle α = -20° from the horizontal, as shown in Figure 1.

c) Describe the steps in the graphical solution for a) and b).

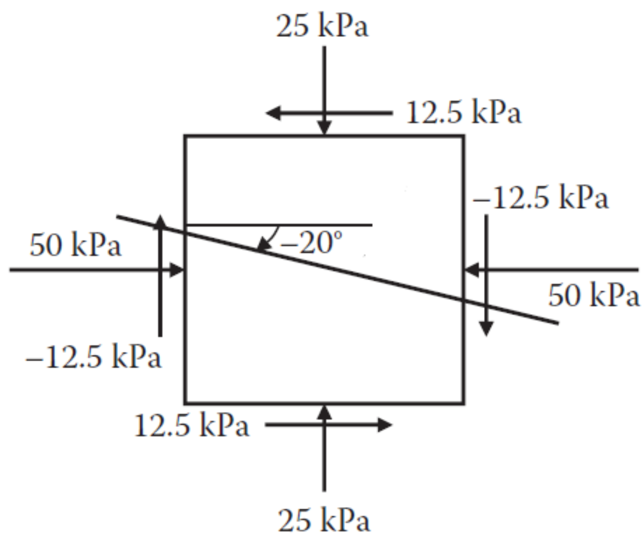

Figure 1

Solution:

Note that the shear stress on the vertical planes is negative as it produces a counterclockwise rotation about a point just outside of the element.

a) Locate the two points of known stresses (25, 12.5) and (50, -12.5). As these stresses act on two perpendicular planes, the line connecting the points (25, 12.5) and (50, -12.5) is the diameter of the Mohr Circle and it intersects the x-axis at the center of the Mohr circle. Next, the Mohr circle is drawn as shown in Figure 1a. It intersects the x-axis at points corresponding to the major and the minor principal stresses. The magnitude of the principle stresses is scaled off the graph as σ_1= 58 kPa and σ_3=17 kPa. Next, starting from the point of known stresses (25, 12.5), a horizontal line is drawn as the stresses (25, 12.5) act on the horizontal plane. The line intersects the Mohr circle at the Pole. To find the orientation of the principal planes, draw lines from the Pole to σ_1 and σ_3; these lines are parallel to the principal planes.

b) Starting from the Pole, -20° line is drawn and at the intersection of this line with the Mohr circle we find the stresses on the plane inclined at an angle α = -20° from the horizontal as σ_α=36kPa and τ_α=18kPa.

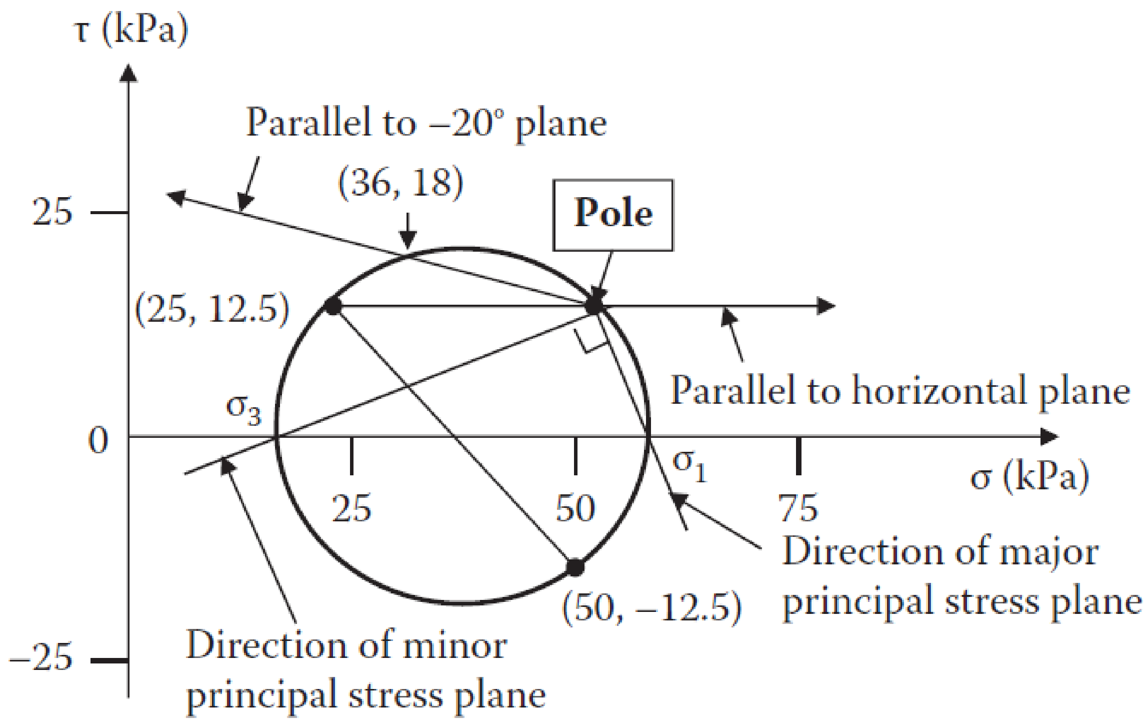

Figure 2a

Q2. The major and the minor principal stresses on a soil element, rotated at an angle 30° from the horizontal, are 120 kPa and 50 kPa, respectively. Using the Mohr Circle method, determine the normal stress, σ_α and the shear stress, τ_α on the plane inclined at an angle α=45° from the base of the element as shown in Figure 2. Describe the steps in the graphical solution.

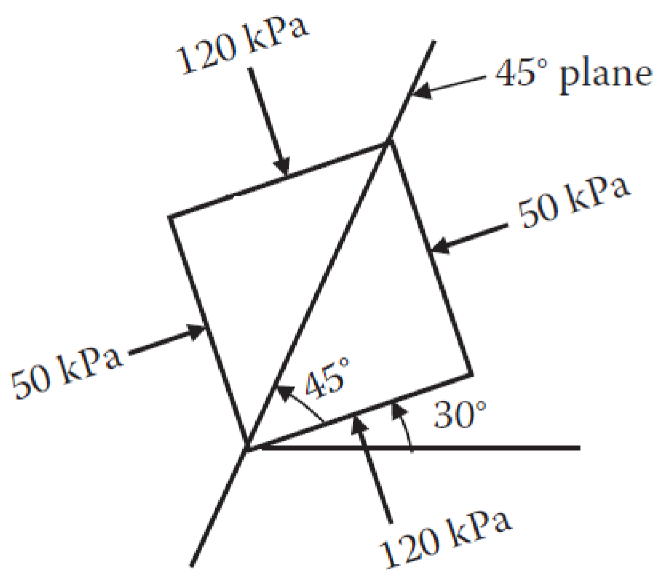

Figure 2

Solution:

Plot the Mohr circle to some convenient scale (Figure 2a).

$$Center\ of\ Morh\ circle = \frac{\sigma_1 + \sigma_3}{2} = \frac{120+50}{2} = 85\,kPa$$

$$Radius\ of\ Mohr\ circle = \frac{\sigma_1 - \sigma_3}{2} = \frac{120-50}{2} = 35\,kPa$$

To establish the Pole, through the point representing (σ_1, 0) draw a line parallel to the plane upon which σ_1 acts (this plane is inclined at angle 30° from the horizontal). By definition, the Pole is where this line intersects the Mohr circle. Next, find the stresses on the α-plane which is inclined at 45° from the base of the element. From the line parallel to the major principal plane, turn an angle in the same direction as in the element, 45°, and the stresses on that plane are defined by the point of intersection of that line with the Mohr circle. Scale off the values of σ_α =85kPa and τ_α=35kPa.

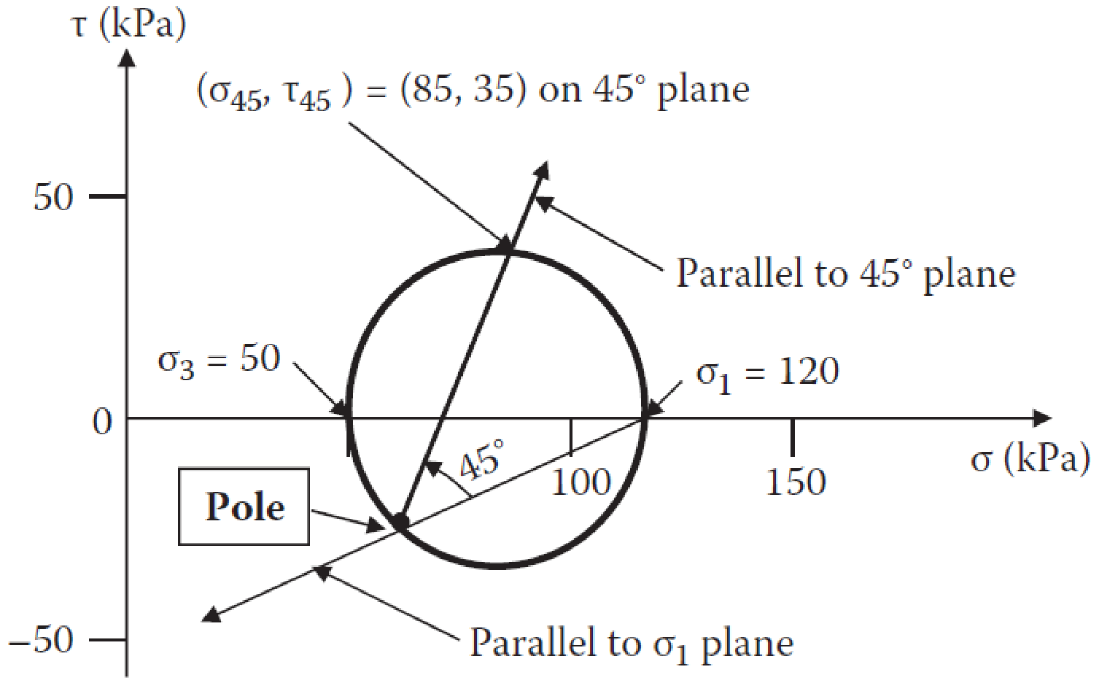
τ (kPa)
(σ45, τ45) = (85, 35) on 45° plane
50
Parallel to 45° plane
σ3 = 50
σ1 = 120
0
45°
100
150
σ (kPa)
Pole
–50
Parallel to σ1 plane

Figure 2a

Q3. A direct shear test is performed on a sample of dry sand. At failure, the measured normal force *N* = 288 N and the shear force, *S* = 173 N. The cross sectional area of the sample is 36 cm^2. Draw the Mohr-Coulomb failure envelope and the Mohr circle at failure. Determine:

a) The angle of internal friction of the sand.
b) The major and the minor principal stresses at failure.
c) The orientation of the failure plane.
d) The orientation of the major and the minor principal planes at failure.
e) Describe the steps in the graphical solution for a), b), c) and d).

Solution:

a) First determine the normal stress and the shear stress at failure:

$$\sigma = \frac{N}{A} = \frac{288}{36} = 8.0\,N/cm^2 = 80\,kN/m^2$$

$$\tau = \frac{S}{A} = \frac{173}{36} = 4.8\,N/cm^2 = 48\,kN/m^2$$

Now we know one point on the Mohr-Coulomb failure envelope. Plot point *A* with coordinates $\sigma = 80\,kN/m^2$ and $\tau = 48\,kN/m^2$. As cohesion *c* = 0 for sand, the Mohr-Coulomb failure envelope, *OM*, passes through the origin. The slope of *OM* gives the angle of internal friction, $\phi = 31^\circ$.

b) Draw a line *AC* normal to the envelope *OM* intersecting the abscissa at point *C*. With *C* as center, and *AC* as radius, draw the Mohr circle which intersect the abscissa at points *B* and *D* corresponding to the major and the minor principal stresses. Scale off the magnitude of the major principal stress = *OB* = σ_1 =163.5 kN/m^2 and the minor principal stress = *OD* = σ_3 =53.5 kN/m^2.

c) The state of stress at failure point *A* is (80, 48) and the failure plane is assumed to be horizontal, as given by the line *AP*. (Recall that in the Direct Shear Test the failure is forced to occur along a horizontal plane).

d) The orientation of the major and the minor principal planes at failure may be found by locating the pole, *P*. *P* is obtained by drawing a horizontal line from point *A*, which line is parallel to the failure plane. Now *PB* and *PD* give the orientation of the major and minor principal planes, respectively.

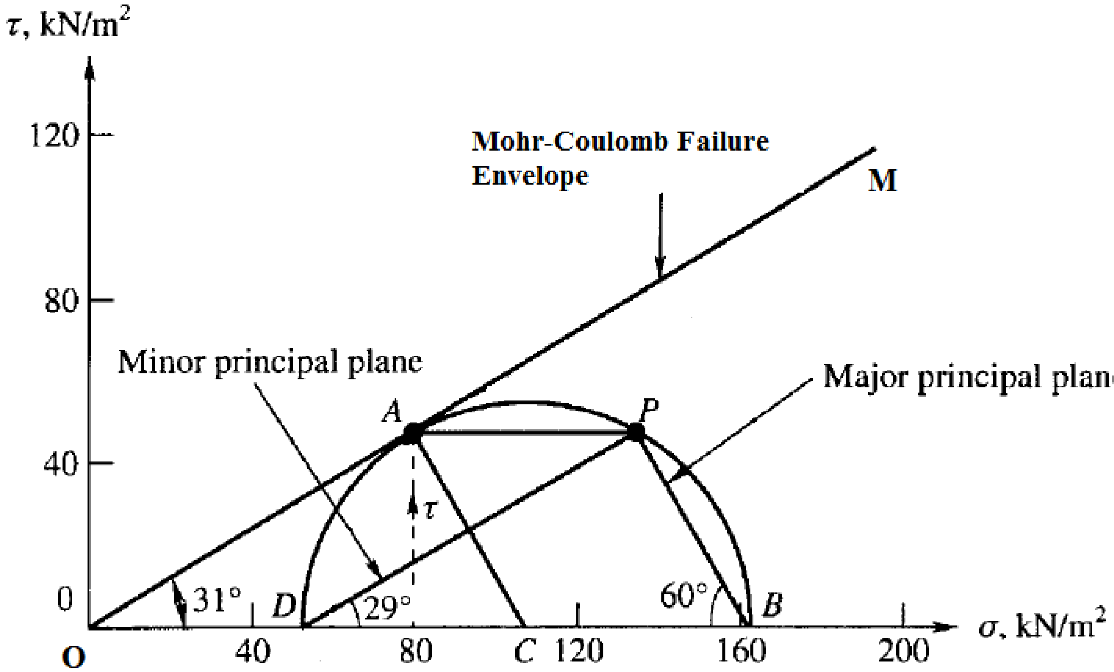
τ, kN/m²
120
80
40
0
O
40
80
C 120
160
200
σ, kN/m²
Mohr-Coulomb Failure
Envelope
M
Minor principal plane
Major principal plane
A
P
τ
31°
D
29°
60°
B

Figure 3a